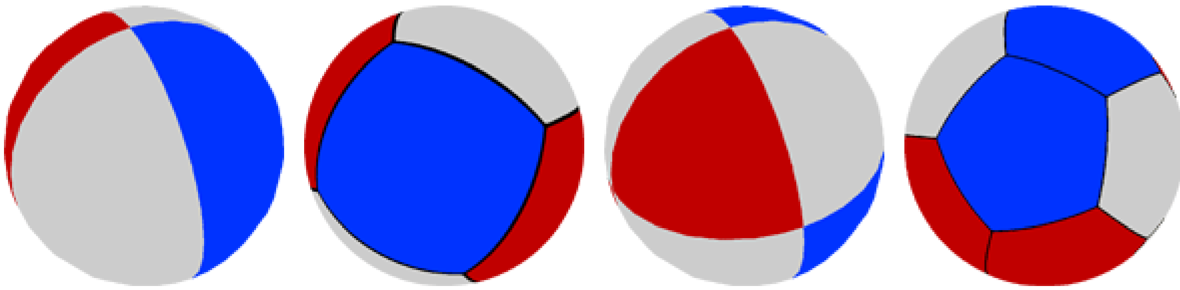
How to Make Spherical Fabric Juggling Beanbags



Technical Instructions and information for the detail-oriented, mathematically-inclined, DIY juggler

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[First Edition, earliest available draft, from 11/8/2012]

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INTRODUCTION

About me and this document

I am not a serious or very skilled juggler, but I have been casually juggling to some extent for as long as I can remember (I am 34). I can juggle up to four balls (I cannot visualize the five-ball pattern) and I can juggle several 3-ball variations. I have not learned to juggle any other objects proficiently.

I also have no expertise in sewing, but I learned the basics when I was young and I have a natural propensity for craft and DIY projects. The stitching and assembly techniques I describe are primarily my own, though I've done some research to improve them. I have never made beanbags for sale, or done much research on ideal materials. This is just a personal hobby for me.

So, I have only very limited experience and knowledge and cannot claim to be an authority on the subject of designing ideal juggling equipment. These instructions are probably not sufficient for professional-grade juggling beanbags, though they may be. However, I am a perfectionist and tend to be obsessive about detail and precision. I put a lot of time, measurement, calculation, and testing into making these instructions as accurate, comprehensive, and well-crafted as I can. I explain the motivations and ideas behind my designs, methods, and material choices and I suggest alternatives. If you know more than I do in some areas, my information can serve merely as a springboard for your ideas.

Back in the mid-1990s I began designing and making my own juggling beanbags. Around 1998 I wrote the first drafts of this document, which was only for personal use so I could record what I had learned. (I thought that I might someday publish it in some way and so wrote it as if for someone other than myself, but I had no immediate plans for publishing it.) At that time I had only developed the four and twelve-panel designs and the latest draft from that period was eight pages long. Over the several years of this phase of the hobby's duration I made a total of 26 beanbags (not including those I only partially finished or discarded as failed designs). Then for a decade or so I moved on to other hobbies.

In August, 2012 I became interested in writing a more formal and detailed instructional document to post on my personal website (which I only give out to people I know). I worked on the formal draft intensively for over a month before submitting it (and continued editing it long after that), and my research for it lead me to develop two more designs: the rounded cube and rounded octahedron. I became fascinated by the math and concepts involved in designing spherical polyhedral beanbags.

I discovered during my research that there seems to be no document like this one anywhere on the web. There was a website called The Internet Juggling Database (www.jugglingdb.com/) which evidently had documents like mine judging by the references and links to that site I have found on other sites, but the administrator of that site has closed it down. Some people have offered stitching patterns for juggling bags on other websites, but almost none of them define the shapes so the bags can be altered, and nobody I know of has a generalized or in-depth instructional document. It may be that nobody is enthusiastic enough about making juggling beanbags to care about the kind of detail I have included. But this document was fun to write and I gained a lot of education in the process, so at least I benefited.

My designs require fairly advanced geometry and trigonometry and possibly other mathematical branches to fully understand and since I don't have much knowledge of advanced math, I cannot fully explain some aspects of the designs or prove certain of my assumptions. Some of the more advanced formulas and concepts that I present in this document (such as the arc length formula, trigonometric functions, and Reuleaux polygons) I learned (or relearned) through my research for this document. I provide explanations of how I created my designs so you know where they came from and can improve them if you know more than I do. I include a few notes on the designs within each instructional chapter, but there is a fuller discussion in the "How I Developed the Designs" chapter.

As a note of interest, except for the beach ball design, the designs in this document constitute the middle three "Platonic solids". There are five Platonic solids which are the tetrahedron, cube, octahedron, dodecahedron, and icosahedron. For a definition, images, and in-depth information on Platonic solids, look them up in Wikipedia. My beach ball design, aside from being a good design on its own, serves as a basis for my cube and octahedron. This is because I use rounded faces for these designs to produce better spheres, and the beach ball concept enabled me to design the curves. See the "How I Developed the Designs" chapter to learn more about this.

Uses for fabric balls

This document is focused on making juggling balls, but fabric balls can be used for more than juggling. Here are some ideas, some of which involve modifications to my designs:

- Indoor games of catch or general indoor throwing (consider lighter-weight filling for less danger to lamps and other household items)
- Indoor kickballs (when made soccer ball size and stuffed with soft stuffing)
- Footbags, a.k.a. hacky sacks (you'll probably want to make them smaller and fill them loosely)
- Baby toys (best when made larger and stuffed with something soft, with perhaps a bell or rattle inside; you might also consider quilting contrasting figures or letters onto each panel or attaching beads, bells, and miniature stuffed figures by ribbons)
- Unbreakable Christmas ornaments (made with soft stuffing, colorful trim, and a loop for hanging – see the Appendix for an example of this)
- Decorative centerpieces (made with elegant solids or prints, or with leather – see the Appendix for examples of this)
- Pin cushions
- Beanbag chairs (by making them very large)

GENERAL NOTES AND TECHNIQUES

Prerequisites

This document assumes you have some basic experience with hand-sewing (I neither use a sewing machine nor describe any techniques for one). I describe stitching techniques and use terminology in a way that someone with no experience at all may not be able to follow. I also take certain fundamental principles of sewing partly for granted such as the need for both a stitching pattern and a larger cutting pattern to produce a seam allowance between the two, and sewing an item inside out so that when it is turned right side out the stitching and seam allowance fabric are hidden inside. If you have never sewn anything before, I recommend you find a tutorial somewhere and sew some basic projects first like a rectangular pillow, which is what I started with as a child. I describe and use the "back stitch", so that is a stitch you should practice.

The document also assumes you have some experience with higher mathematics, though this is only needed for altering the size of the beanbags. If you only want to make the tennis ball size I give instructions for, you need no high math at all as long as you are familiar with terms like radius, arc, and equilateral. For the alteration instructions, though I do all the advanced calculations for you, I use notations, terminology, and mathematical syntax that may be difficult to understand by someone who hasn't at least some basic experience with algebra and trigonometry.

Depending on the design you choose, you will have to use a compass or a protractor to draw the templates by hand, and I assume that you know how to use these tools. I also provide some information and tips about using a computer application called SketchUp for drawing the templates. This is optional, but I do assume computer literacy and ability to learn a new application as I do not provide full directions for using the application.

Size

My instructions are for tennis ball sized bags (diameter ≈ $2^5/_8$" or 66.675mm) when made with denim and filled tightly. This size fits my large hands the best for a combination of standard catches and "clawing" (catching with palms down), but I include instructions for altering the size. $2^5/_8$" and $^1/_8$" on either side of that are the most common sizes for juggling beanbags from what I have seen at stores.

I do not solely use math to determine the template sizes; I use measurements of the finished bags, which are larger than the templates by 1-8.5% (depending on the design and how tightly I fill it) when I use denim. There are two reasons for this varying increase in size. First, the bags are polyhedrons, some of which are modified to have circular sides, and are distorted into approximate spheres; the resulting spherical circumferences are unpredictable. Second, fabric stretches (some more than others). My instructions give the inflation in size I experienced with each design. Also, some of the designs have different circumferences depending on how they are measured. The beach ball design is 15.9% larger, mathematically, along its seams than between them. This evens out somewhat when the bag is filled and stretched, but the bag is still significantly larger along the seams. I account for this as best I can so the finished size will average out to the target size.

Size will also be affected by the type of fabric, how tightly you fill the bags, and your sewing practices. If you use a material that has a different stretch from denim or do not fill the bags as tightly as I do, your bags may end up being a noticeably different size than I claim they will be and so you may want to adjust the template size a bit. As for sewing, I sew just on the inside of the stitching lines so that I am sewing where the edges of the template were, and I pull the stitches very tight. Loose stitches or stitching on the outside of the lines may increase the size measurably, especially in the dodecahedron design with twelve panels. You may need to do some trial and error to determine the best fit for your hands and juggling practices. It's a good idea in any case to assume your first bag is a practice bag. If you're as much a perfectionist as I am, you'll find that it takes some experience to know how to make these things right.

If you want to measure your bag's finished size, the easiest way (if you don't have a dressmaker's tape measure) is to wrap a strip of paper around it, mark the meeting point, measure the strip up to the mark, and then divide by pi (3.1416) if you want the diameter. I made a reusable paper measuring tape by marking a small range of measurements on a narrow strip of paper.

Roundness and uniformity

Roundness is important so that no matter how the bag lands in your hand, you can grip it the same. Small adjustments of grip that a non-spherical bag requires each time you catch it will make juggling more difficult and error-prone. All of the designs in this document produce approximately spherical bags, but the first two with the lowest panel count are somewhat cubic when made with denim. In contrast to this, I made a Reuleaux tetrahedron bag according to a design I found on the web that produced a rounded pyramid - not at all spherical (see the "Other Designs and Variations" chapter under "Reuleaux tetrahedron"). I also found instructions for a very angular pyramid-shaped beanbag made from two squares. Designing a spherical beanbag requires more skill and knowledge than the classic, flat, corn hole-style beanbag or even a 3-D, angular beanbag, but it will be easier to juggle and feel more pleasant in your hand.

Uniformity, meaning an arrangement of seams and panel faces that feels the same no matter how the bag is gripped, is, to me, important mostly for tactile aesthetics. This is less of an issue with thin or very flexible fabrics because the seams can't be felt as much.

The more panels a bag has, the more spherical it will be and, in most cases, the more uniform it will feel. So, the four-panel beach ball design is the least round and least uniform of the designs in this document. Its seams all run longitudinally and meet at two "poles", and, having only four panels, it will feel slightly cubic around the equator (using a stretchy fabric and filling the bag tightly will improve its roundness). Its advantage is that it is quick to make. A beach ball with more panels would be more spherical, but still not very uniform. Catching it around the equator will feel different from catching it around a pole. The rounded octahedron (eight panels) is much more spherical and feels more uniform because of the greater number of panels and the addition of latitudinal seams around what was the equator of the beach ball. The rounded cube, with six panels and a combination of horizontal and vertical seams, has a roundness and uniformity about in the middle between the beach ball and the octahedron. It is a very good design. The dodecahedron is the most luxuriously spherical and uniform of the four designs, but is also the most difficult and tedious to make. My favorite design is the octahedron. It has nearly the roundness, uniformity, and aesthetic beauty of the dodecahedron, but is easier and quicker to make.

My cube and octahedron designs use rounded faces which improve their roundness over true polyhedral equivalents. The dodecahedron does not use rounded faces, but I don't feel it needs them.

Fabric

I use denim from sacrificed jeans. You can find very colorful jeans in the women's section of a thrift store. (A photo of each of my designs made with this denim is at the beginning of each instructional chapter, and they are all shown together in the "How I Developed the Designs" chapter.) I use this source of denim because the fabric stores I've been to carry a poor selection of the colors I like for juggling bags. Be aware that different jeans use different denims and so if you're not careful, each bag in your set may feel different, be stretchier, or have other differing characteristics. There are probably better fabric choices but I have not done much research or experimentation on that. Denim is soft and durable, has a pleasant texture and an attractive appearance, and is easily available. I encountered some professional corduroy bags recently that looked and felt very nice. The corduroy was similar to the denim I use, but the ridges were more prominent and loosely spaced. I liked the look and feel of it more than the denim. Aside from corduroy, professional manufacturers often use leather, ultrasuede or other leather substitutes, or vinyl-coated elastic fabric. Be warned that leather may be very difficult to sew with a standard needle and thread and may require special tools, thread, and stitching techniques.

Choose a fabric that is long-wearing, has good traction against your hand and a pleasant feel, and can take some abuse without bursting. Denim produces a firm, rugged bag while a stretchy vinyl or an elastic fabric will result in a squashy texture like a stress reliever ball. You might consider using interfacing to reinforce or stiffen an overly thin or stretchy fabric, but I have no experience with interfacing so I can't say much about it. Select colors in accordance with your personal aesthetics and the color arrangements possible in the design you intend to make, and take ease of visibility into account for you and, if applicable, your audience.

Some women whose blogs I've read on the internet enjoy using random scraps of fabric or old garments to make patchwork balls for babies to play with (they make them larger than juggling size – sometimes as large as 9 inches in diameter). They fill the balls with soft stuffing and sometimes put rattles or bells in them. I give examples of these designs in the "Fabric Ball Ideas" chapter.

You should probably consider washing and drying new fabric to preshrink it and remove excess dye before making it into beanbags. I have also seen advice to press the fabric after washing it. I haven't used new fabrics, so these are not issues for me.

Marking the fabric

I recently started using a regular pencil (actually an automatic one with a 0.9mm lead) to transfer the patterns onto the fabric. (I do not pin a paper template to the fabric as some people do.) You can also buy special dressmaker's pens and pencils, but they have drawbacks. First, they only seem to come in light blue, purple, and white, and so if you are going to use a variety of fabric colors you will probably have to buy at least two different colors of pens to get enough contrast against all your fabrics. Graphite pencils are dark enough to work on most colors and graphite is also somewhat shiny which helps with visibility on dark fabrics. Maybe black fabric would defeat it; I haven't tried it. I have tried it on navy blue and it worked well. Graphite fades somewhat during use (including during stitching), and probably even more if you laundered the bags, as the graphite particles disperse and rub off, but it may leave a bit of a stain, so try to hide it within the seams when you sew from the outside. Second, pens dry out over time; pencils do not. Third, the white dressmaker's pencil I tried made such a faint line on the navy blue denim I was using that I could hardly see it (I remember that the white pen I used years ago also made faint lines, but it was better than the pencil). Maybe these pencils work better on other fabrics.

Between my dried pens and the useless dressmaker's pencil I just bought, I gave up using official marking tools and that is how I ended up using a standard pencil. I recommend using a medium or soft lead. Do not use a standard, permanent pen as I did at the beginning years ago because if it peeks out from between the panels that you sew from the outside it will look tacky. It may also bleed through the fabric if you wet it.

To help prevent the pencil lines from showing on the outside, position the front stitching patterns slightly too close to the edges of the fabric and then stitch farther inside them (toward the center of the panels) than usual which will result in the lines being hidden within the seams.

Filler

Many juggling bags are filled with a natural filler such as millet or crushed walnut shell, but these fillers will absorb water, swell, and possibly mold (and grow in the case of seeds) if you get them wet. This means you must neither wash them nor drop them into a puddle. Also, these fillers will probably break down somewhat over time if you use the juggling bags extensively. To avoid these problems, I use an artificial filler called "Poly-Pellets". These are polypropylene (plastic) pellets made for weighted, posed dolls, which I buy in two-pound bags from Jo-Ann Fabrics. Poly-Pellets are smooth and rounded and have a good feel to them in a beanbag. They are about twice the size of millet and so don’t have quite as smooth a texture.

Poly-Pellets produce a somewhat lightweight beanbag compared to the professional beanbags I have encountered. My beanbags at $2^{5}/_{8}$" weigh about 3.4oz (96g) while most of the professional bags I looked up weigh 4.2-4.6oz (120-130g). If you like a heavier bag, you could mix metal BBs with the Poly-Pellets (I recommend *not* using lead!). There may also be heavier plastic pellets available online.

You could also make heavyweight bags for upper-body aerobic workouts or for "joggling" by filling them primarily or purely with metal BBs. Make sure you use durable fabric, thread, and stitching if you use a heavy filler. I have not tried this, so I can't say much about it. If you want to make an exercise ball, it would probably be better to use an actual ball rather than a bag for better durability. You can just cut a small slit in a street hockey ball, tennis ball, racquet ball or similar and fill it with BBs or even pennies and then glue it shut.

I found an idea on the internet, which is to fill the bags with something lightweight so that they can be thrown around in the house by children and not break anything. One woman filled hers with shredded fabric scraps from her sewing projects. You could also use cotton or polyester pillow stuffing (a. k. a., fiberfill). For a more jugglable weight that will still reduce the risk of broken household knick-knacks, you could fill the bags with a mix of fiberfill and pellets. Some women add rattles or bells inside the bags for babies. They often make the fabric balls larger than juggling size – sometimes as large as 9 inches in diameter.

Filling the bags

I use a small funnel to fill the bags (with pellets). After I have filled the bags loosely, I pack more in until they are tightly filled using the following method: I stick my finger down into the full funnel and stop up the spout and then press the funnel down into the bag to stretch it out. Then I remove my finger and slowly raise the funnel while tapping or shaking it gently to help release the pellets so they fill in the void left behind by the funnel's spout. I repeat this until I have a tight, spherical bag. Filling the bags tightly will improve their roundness. They will not remain rock-hard forever, but will stretch out and become softer over time (depending on your fabric choice).

You may want to use a scale to make sure each bag is filled with the same weight of pellets. This depends on whether a consistent firmness or a consistent weight is more important to you (with enough accuracy of sewing and consistency of fabric choice across bags, you should be able to get both). If you use a mix of two types of filler such as plastic and metal, a scale is more important.

If you prefer to under-fill your bags and leave them soft, I recommend that you first over-fill them briefly and kneed them with your hands to fully stretch out and shape the bags, and then remove what you don't want.

Template material

I do not pin a paper template to the fabric and stitch around it as some people do; I make a rigid template and trace around it with a marking tool. You can use a hard cardboard (or even index cards, but they're a bit thin and lack durability) or you can buy template plastic from a craft store. I have used both and I prefer the plastic. The plastic is translucent, flexible, and easy to cut and is much more durable than cardboard for long-term use. I used to be able to get a thick template plastic from Jo-Ann Fabrics, but now they only sell a thin variety. I recommend the thick variety if you can find it because it will be easier to trace around and is less flexible which will help keep it flush with the fabric so the pencil doesn't slip underneath it.

Making templates

Precision in drawing and cutting out the template is important. If there is a significant error, you may find that the stitching lines on two panels you are sewing together don't match up, or there may be some imperfections in the shape of the bag, and the bag may be slightly larger or smaller than you expected (this is especially true for the dodecahedron because its large number of panels compounds any error several times). I recently learned of a computer application called SketchUp, which will allow you to draw and print the templates with perfect precision. I discuss it in the next chapter. I recommend this if you are computer literate. Otherwise, it's ruler, pencil, compass, and protractor.

If you draw the templates by hand, I recommend that you draw them on paper first and then use double-sided adhesive tape to attach them to the template material. That way, if you make a mistake, you don't waste your template material or have to erase. Also, you can position the shape closer to the edge of the template material than you could if you drew it directly on it.

Thread

I used to use a standard thread (I think it was cotton) and I tried to pick a neutral color like grey that would blend as well as possible with the variously colored panels of my beanbags in case it peeked out. This had two drawbacks: 1) It did not blend very well and where the stitches peeked out, they looked a bit tacky; 2) It tended to fray and break during sewing a couple times per beanbag even though it was a high quality thread and I doubled it.

I'm now using a new thread which is called invisible. It is made of a single, transparent filament like fishing line. It is stronger judging by how much harder it is to break, and there is no risk of running my needle through the fibers of the thread when I make a retreating stitch, which frays and weakens it and caused it to break more often in my early bags. Strength is very important to me because I use denim for my beanbags and I pull the stitches very tight. This new thread is indeed invisible except when the light glints off its glossy surface. The only drawback is that it is springy and therefore harder to manage, but it's not too bad.

If you use a standard thread, it might be a good strategy to pick a color that, rather than ineptly camouflaging with the panels, will contrast boldly with them such as black for light fabrics, white for dark fabrics, blue for red fabric, etc. I haven't tried this, but it may look better than a bland grey peeking out.

Stitching techniques

I sew my juggling bags by hand. I neither have nor know how to use a sewing machine, and I doubt that a machine would be able to do a job like this very well anyway.

Accurate, dense, and tightly pulled stitching is important if you want an elegant-looking, perfectly round bag, smooth seams with no bulges or skews, and no stitching visible from the outside. I am not very experienced with sewing and there are many techniques I have not tried. My technique is oriented toward straight, smooth, and elegant seams, but it is also slow. To see how my seams look, look at the photos of my beanbags which are at the beginning of each instructional chapter and are shown together in the "How I Developed the Designs" chapter. I have seen leather beanbags that have a much rougher seam and a sparser stitch. These bags have a very different, but not necessarily inferior, visual and tactile aesthetic. You can see an example of this style in the "Other Designs and Variations" chapter under "Cuboctahedron" (page 39).

Back when I used standard thread, I doubled it to make it stronger (so I could pull the stitches tighter). Doubling the thread also makes it very easy to tie at the beginning; just run the needle through the fabric one way and back the other so both ends are on the same side, and then run the needle through the loop made by the folded thread and pull it tight: instant knot!

For the seams that are sewn inside out, I use the "back stitch". Start with the two layers of fabric placed front to front (so that the bag will be inside out). On one side make a long, advancing stitch. On the other side, make a short, retreating stitch (half the length of the long stitch). Back on the first side, make another long, advancing stitch. Thus, one side of the seam contains long, overlapping, advancing stitches and the other side contains short, adjacent, retreating stitches. This is the best stitch I have found for making the finished seam beautifully straight and not rippled as a simple running stitch will do.

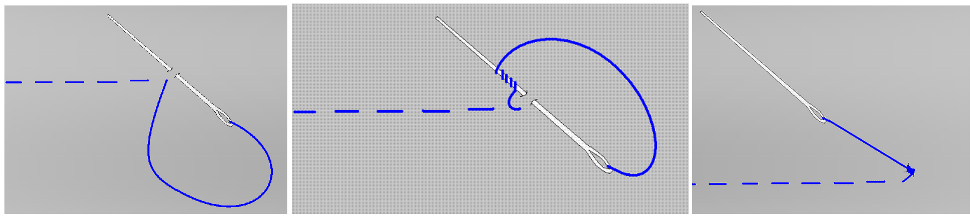
It also has the advantage of locking itself against the fabric so that the stitches do not easily loosen after you have pulled them tight.

For the seams that are sewn from the outside, I use the back stitch again but with the technique modified so it can be performed from the outside. My technique is similar to the "ladder stitch". A ladder stitch is really a running stitch done from the outside. My stitch is done the same way but with a retreating stitch added. Fold the seam fabric into the bag and pin the two edges together if necessary. Pinch the edges together, making sure the stitching lines are aligned, and make a long, advancing stitch in and out one edge or "lip" of fabric, through the "tunnel" made by the folded seam allowance, and then cross straight over to the other lip and make a short, retreating stitch in and out. Cross over again to the first side and make another long, advancing stitch. Pull the stitches tight after a couple have been made which will pull the two lips of fabric more tightly together and hide the stitches between them. This will result in a back stitch as if it was done from the inside and, if you do it right and pull the stitches tight enough, it will be as hidden from view as the inside stitches.

If you are filling the bags with pellets, you can sew the opening entirely closed before filling. All you have to do is leave the last several stitches loose (or loosen them later) and when you're finished, pull them out part way so that a funnel can be pushed between them. After filling the bag, you can pull the stitches tight again. I do this by sticking the tip of my automatic pencil under each stitch in turn starting where the stitches begin to be loose and pulling on it to tighten the stitches behind it. I continue this until I reach the end and then pull the thread itself to tighten the last couple stitches.


Surgeon's knot illustration from Wikipedia

Tying the thread securely and tightly is important so that your stitches don't loosen and peek out or, worse, unravel. Some people advise starting a thread by tying a large knot in the end of it which prevents it from pulling through the fabric. I don't trust the knot never to pull through, so I tie a surgeon's knot (see illustration above) around a few strands of the fabric, or around the thread of a previous stitch if one is available. To tie a surgeon's knot, start with a double overhand knot (the knot you use to begin tying your shoelaces, but with one additional pass) followed by a standard overhand knot. Tying a knot around a bit of fabric will tend to cause the knot to spring open while I'm tying a second knot and I want everything to be very tight. So I always tie it around the thread of a stitch if possible, or around as little fabric as possible. I also usually tie the thread a stitch or two back from where my thread starts/ends or make a couple of overlapping stitches in place first.



http://www.shushanna.com/handsew.html

There is a very good method for ending the thread which I discovered on the internet recently. Above are illustrations for it. Stick the needle under the previous stitch (or through the fabric as the illustration shows) and then back out on the same side of the fabric and leave it there, halfway through. Take the thread opposite the needle where it comes out of the fabric and wrap it around the tip of the needle 3-6 times. Finish pushing the needle through so that it pulls the thread through the thread coil you made and pull it tight. I do this a couple of times to make sure I have a secure knot.

There is no need in my opinion to add whip stitches or other extra stitching at the seam intersections. I did this years ago and I have seen advice on the internet to do it, but as long as you stitch accurately and make the stitching all meet at the corners of your stitching pattern, there is no more possibility of filler leakage at the corners than anywhere else on the bag. I also no longer think that any additional strength is needed at the corners. Whip stitches around the seam allowance can also make the corners look less elegant because of the puckering and bunching up of the fabric they cause.

Technique for extra elegance

This is a technique I invented to make the finished seams flatter and straighter. It is more important the thicker the fabric is. Before turning the bag right side out, separate the two layers of seam allowance fabric at each seam and fold each side out flat. Iron them so they stay that way. This can be done by putting your fingers inside the bag and pressing the seams against the iron (be careful!). I use denim, which protects my fingers from easily getting burned. If you use a thin fabric you may not be able to use this technique, but it is also less important for a thinner fabric. You could also try a water-soluble glue instead of the iron, but you would have to wash the bag when you finish it to remove the glue. Another technique I found, but have not used, is to use a large running stitch on both sides of each seam to keep the seams folded out flat, and then pull the stitches back out after the bag is finished. If you use the ironing method, you will have to readjust the seams when you turn the bag right side out and make sure they are still folded out flat. Flattening out the seams will help prevent the seam allowance fabric from puckering and bending in different directions and causing lumps and ripples in the stitching lines on the outside. Thus, it is probably more important for designs with long, curved seams than for the dodecahedron which has short, straight seams because it is the curvature that compresses the seam and causes it to pucker or bend to one side.

SKETCHUP – AN APPLICATION FOR DRAWING THE TEMPLATES

I just learned of a free computer application called SketchUp. This is a better way to draw and print the panel shapes than using manual tools. I include both manual and SketchUp directions for drawing each panel shape. I will not provide a tutorial for using SketchUp except for a few notes, but there are tutorial videos at http://www.sketchup.com/intl/en/training/videos.html. I got everything I needed within the first two videos (and a few Google searches).

I have been using the project template called "Product Design and Woodworking – Millimeters". My SketchUp directions use millimeter units (I normally use centimeters except when a measurement is less than 1cm). Make sure whatever template you use has the correct units.

To make 1:1 printing work, and to simplify the interface for 2D drawing, set the camera to a standard view other than "Iso" such as "Top" (Camera menu -> Standard Views -> Top) and set the camera view mode to Parallel Projection (Camera menu -> Parallel Projection).

To eliminate the fill color of polygons, go to View menu -> Face Style and select Wireframe. Or, you can erase the background color later.

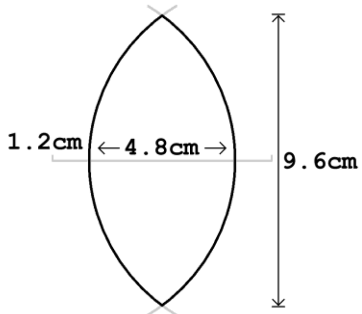
Circles default to 24 sides (they are actually polygons). I set them to a high number like 360 or even 720 (why not?) for extra curve precision. To do this, select the Circle tool (notice that a text box labeled "Sides" appears at the bottom of the window), type the number of sides you want (the value appears in the box) and press Enter. You can also type the number of sides followed by "s" at any time and press Enter. All future circles during that session will have the new number of sides. Incidentally, you do the same to set the number of sides for the polygon tool.

I am not very experienced with using SketchUp, so I found a forum that explained how to print my panel shapes at a precise 1:1 scale. Here are the instructions, which worked for me. If they don't work for you, Google is your friend.

1. First, to save ink, I recommend that you erase any polygon fill color you don't need (if you are using a Face Style that has fill color) and any extraneous lines from your design. For arcs, I find it helpful to leave some excess to help guide the scissors as they enter the panel so I get a more accurately curved cut, but that's up to you. Whatever you crop outside the view pane won't print anyway, so you don't have to delete all of the extra lines. You can erase line segments by clicking on them with the eraser tool, or by drawing a box around them with the Select (arrow) tool and pressing Delete on the keyboard. You can erase background color by right-clicking on it with the eraser tool and selecting Erase from the context menu.
2. Set the Camera view mode to Parallel Projection and the standard view to something other than "Iso" if you haven't already.
3. Zoom in on the relevant portion of the design so that it fills the view pane as well as possible and is not cropped (the print will include everything in the view pane, including empty space, and nothing outside it). This does not affect printing accuracy, but it prevents the empty space, and your design, from being split across multiple pages. With some trial and error you can find the optimal zoom and figure placement that will result in the template fitting at the edge of the page with no extraneous pages printed.
4. Go to File menu -> Print Preview, and in the Print Preview dialog:

a. Uncheck "Fit to page".
b. Uncheck "Use model extents".
c. Enter the same value and units for "In the printout" and "In SketchUp" to set a 1:1 scale between the application and the printout.
d. Look at the print preview screen to see if your figure is positioned on the page as you want it to be. If it isn't, go back and re-zoom and re-position your figure.

4-PANEL BEACH BALL STYLE INSTRUCTIONS



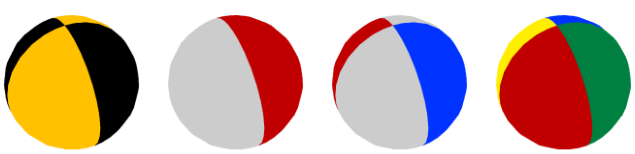
Design Notes

4 seams, 44.5cm (17.5") of stitching.

These are quick to make because of having so few panels, but have the disadvantage of being somewhat cubic around the equator (at least when using the thick, stiff denim I use) and having a non-uniform feel due to the sparse and strictly longitudinal seam arrangement. A stretchy fabric is probably better for this design because it will allow the sides to bulge out for a more spherical shape. For information about how I designed this panel shape, see the "How I Developed the Designs" chapter under "4-Panel beach ball design" (page 43).

You can make a beach ball style bag with more panels to improve its roundness (I describe how to do this and provide some photos in the "Other Designs and Variations" chapter on page 38), but if you're willing to sew more panels, there are designs with better seam uniformity than the beach ball. (The only merit I see in a six or eight-panel beach ball is the attractive stripiness of it.) The beach ball design's value, as I see it, is that you can make a simple, nearly spherical bag with just four panels, which is ideal for someone who isn't concerned about aesthetic elegance or tactile luxury but just wants something to toss around or for kids to play with. You can make a bunch of these in a day or two.

This design is much larger along the seams than between them. Mathematically, using the arc length formula $2r(\sin^{-1}(c/2r))$, where r = radius and c = chord length, it is 15.91% larger. I take this into account in the template size. I tried to make the finished bag average out to the $2^{5}/_{8}$" target size and feel the same size in my hand as the other designs.



I have compiled four good color arrangements for this design. The images above give examples of each arrangement in order.

- **2 colors**
  - Alternating stripes of contrasting colors.
  - One hemisphere (two adjacent panels) of each color (Pokéball style).

- **3 colors**
    - Color A on a pair of opposite panels and colors B and C on the other pair.
- **4 colors**
    - Each panel a different color.

Supplies

- **For the templates (assuming you are drawing them by hand)**
    - Cardboard or Template Plastic, fine-point Pencil, metric Ruler, Compass, Scissors, Paper & Double-Sided Tape (optional).
- **For the bean bag**
    - Fabric, Needle and Thread, Scissors, Pencil or non-permanent Pen for marking the fabric, bean bag Filler, Funnel.
- **For your information**
    - Unless you're experienced with this sort of thing, I recommend that you browse through the "General Notes and Techniques" chapter before you get started. You may find some tips there that will improve your experience and your beanbags.

Panel design for a tennis ball sized bag (diameter ≈ 67mm, $2^5/_8$")

The panel shape is a pointed ellipse or lemon shape whose height is twice its width and has circular sides.

- **Manual directions**

1. Draw a horizontal 4.8cm line, mark each end of it, then continue the line to either side of it and mark compass points 1.2cm out beyond each end of the 4.8cm section.
2. Place a compass on one of the two outer points and extend the arm until it reaches the opposite end of the 4.8cm section (it will extend 6cm). Draw an arc that extends from directly above to directly below the center of the 4.8cm section.
3. Without changing the compass radius, repeat the previous step on the other side. This should result in a lemon shape that is 4.8cm wide and 9.6cm tall. Cut this out and use it as the stitching template.
4. To make a cutting template which includes an 8mm seam allowance, draw everything the same but extend the compass 8mm farther than you did before and then draw the two arcs from the same two points using that new radius.

- **SketchUp directions (using millimeter units)**

1. Draw a 72mm line.
2. Draw circles of radius 60mm centered at each end of this line. The intersection of the circles forms the 48mm x 96mm lemon-shaped panel.

3. To make a cutting template which includes an 8mm seam allowance, start with the same 72mm line, but increase the circle radii by 8mm.

Altering the size of the bag

The circumference of the bag is composed of two panel lengths around the poles and four panel widths around the equator, which is why the length must be exactly twice the width. I define the panel size by its width. Its length is determined by the radius of the curves that form its shape.

Measured between the seams or around the equator, this design will end up **6.5-8.5%** larger (depending on whether I fill it loosely or over-fill it) than a calculation based on the width/length of the template when made with the thick, stiff denim I use. Along the seams it measures 12.5% larger. I target the max inflation minus 1% when sizing my beanbags and for this design I make an adjustment for the larger diagonal circumference, but the calculation I use for that adjustment is too flaky to bother describing. If you feel the need, you can average out the two circumferences and form your own adjustment.

To make a bag of diameter d inches (inch units are more natural for me) with an assumed 7.5% (0.075) inflation, multiply d by 2.54 to convert it to centimeters, multiply by pi to get the circumference, divide by 1+the decimal inflation (plus the adjustment, x, for the larger diagonal) to get the pre-inflated circumference, and then divide by 4 to get the template width. Thus, the formula for the template width (in centimeters) is

$$\mathrm{d} \times 2.54 \times \pi \div (1.075 + \mathrm{x}) \div 4$$

To adjust the bag's finished diameter by $^{1}/_{8}$" (3.175mm), I recommend changing the width of the template by **2.268mm - 2.310mm** (accounting for the 8.5% and 6.5% inflations, respectively). This includes my flaky adjustment for the larger diagonal.

Now you need to know the relationship between the width of the panel and the radius of the curves (which is determined by the outer, compass points). The formula for the curve radius is

$$\mathrm{r} = \mathbf{1.25}\mathrm{w}$$

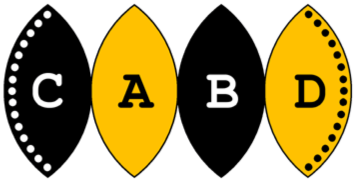
where w = panel width (not the distance between the compass points). This means that the compass points should be marked r − w = **0.25**w beyond each end of the initial line that marks off the panel width, and the total distance between the compass points is 2r − w = **1.5**w. Note that I found the width:radius ratio using experimentation, not geometry, but it worked for both a 2cm width and a 10cm width, and then I later confirmed it by drawing the template in SketchUp and measuring it there.

To make the cutting template, draw everything the same, but extend the compass farther than you did before (increase the circle radii) by the desired seam allowance and then draw the two arcs from the same two points using that new radius. The cutting template will be larger than, but parallel to, the stitching template.

Making the panels

1. Use the larger, cutting template to trace the cutting pattern onto back of the fabric. You will need 4 panels.
2. Use the smaller, stitching template to trace the stitching pattern within each cutting pattern, being sure to center it well.
3. Cut out the panels.

Assembly

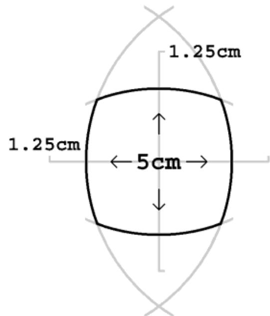


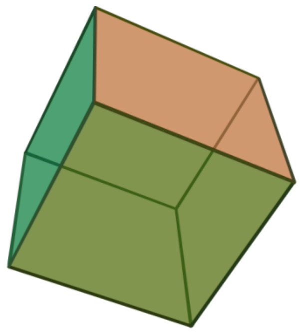
My method below can use as few as 2 threads if you cut them long enough and continue them from one seam to the next. The method consists of sewing A and B together, turning the pair right side out to expose the front faces, and then sewing C onto one side and D onto the other. The final seam between C and D will be closed from the outside along the dotted lines.

1. Lay the panels out as shown in the diagram above (I prefer to place them front face up) and arrange them according to your color pattern. Use the stitching template to draw stitching lines on the *fronts* of the two panel edges shown with dotted lines in the diagram. Be sure to center the template well (it should align as well as possible with the stitching patterns on the backs). My stitching pathway will leave these two edges partially unsewn so the bag can be turned out between them. They will then be sewn from the outside using the front stitching lines.
2. Sew panels A and B front faces together.
3. Turn the A+B pair inside out (actually right side out for the finished bag) so the front faces will be exposed for attaching C and D to them.
4. Add panel C and sew it to A, front faces together, along the edge of C that does not have the front stitching line, that is, orient C so that the edge with the front stitching line is along the seam between A and B.
5. Repeat the previous step on the other side with D and B.
6. Turn the A+B pair inside out so that the entire bag is inside out. You should now have a nearly complete bag with one open seam, and that seam should have front stitching lines on both sides (currently on the inward faces).
7. If you did not leave a hanging thread at one end of the opening, start a new one. Either way, sew a few starter stitches along the final seam to make it easier to continue from the outside. You probably won't need the entire open seam to turn the bag out, so you can continue to sew as much as you don't need.
8. Turn the bag right side out.

9. Pull out one stitch of the final thread so that the thread is on the outside where you can get to it. Continue sewing the opening closed following the front stitching lines. Fill the bag at some point during this final sewing with a funnel.

6-PANEL ROUNDED CUBE INSTRUCTIONS



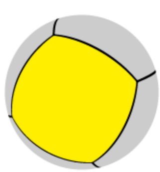
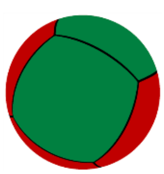
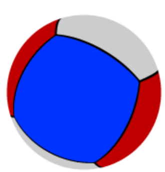


Design Notes

12 seams, 52.0cm (20.5") of stitching.

This design is based in part on the four-panel beach ball design. It can be thought of as that design but with the tops and bottoms of the four panels transformed into two new panels. Around the equator, both bags are the same. As you can see from the panel design diagram above, this panel is simply the middle portion of the lemon curve panel even though, technically, it is based on a square. For a bit more discussion on this design, see my "How I Developed the Designs" chapter under "Rounded cube design" (page 47).

This design feels very spherical (when tightly filled). At the beginning I could feel a hint of cubeness when I caught it, but it rounded out and it now feels and looks almost perfectly spherical – more so than my denim, 4-panel beach balls. If you use a thinner, stretchier fabric than denim, the bag will probably feel more spherical at the beginning. The arrangement of seams is more uniform than on the beach ball design and I like the feel of it better.

There are three good color arrangements for this design (possibly more that I haven't thought of). The images above give examples of each arrangement in order.

- **2 colors**
  - Color A on a pair of opposite panels and a belt of color B around the middle.
  - Color A on three panels in a row and color B on the other three, forming a tennis ball layout.
- **3 colors**
  - Each color on pairs of opposite panels.

Supplies

- **For the templates (assuming you are drawing them by hand)**
    - Cardboard or Template Plastic, fine-point Pencil, metric Ruler, Compass, Scissors, Paper & Double-Sided Tape (optional).
- **For the bean bag**
    - Fabric, Needle and Thread, Scissors, Pencil or non-permanent Pen for marking the fabric, bean bag Filler, Funnel.
- **For your information**
    - Unless you're experienced with this sort of thing, I recommend that you browse through the "General Notes and Techniques" chapter before you get started. You may find some tips there that will improve your experience and your beanbags.

Panel design for a tennis ball sized bag (diameter ≈ 67mm, $2^5/_8$")

The panel shape is a rounded square. The radius of the circle that produces the curves comes from the four-panel beach ball design. I draw this shape in a manner similar to the beach ball panel. Precision is important because if the compass points or the compass radius are not consistent across all four arcs, the sides of the panel will be unequal lengths and the stitching lines will not match up.

- **Manual directions**

1. Draw a horizontal 5cm line (mark each end of it) and then continue the line to either side of it and mark compass points 1.25cm out beyond each end of the 5cm section.
2. Place a compass on one of the two outer points and extend the arm until it reaches the opposite end of the 5cm section (it will extend 6.25cm). Draw an arc that extends from directly above to directly below the center of the 5cm section. Then, keeping the compass radius unchanged, repeat this on the other side. This should result in a pointed ellipse or lemon shape.
3. Use the crossing points of the arcs above and below the center line to draw a vertical line through the center of the horizontal line and mark the same four points on this line as you did on the horizontal line, that is, mark a point 2.5cm above and below the center point to produce a 5cm segment centered on the horizontal line, and then mark compass points 1.25cm farther above and below those points. (Note that, once you have the compass set to the correct radius, you don't need to mark the 5cm portion of the lines. Just draw 7.5cm lines and use their end points as compass points. I left that step in because I think it makes the process more understandable.)
4. Keeping the compass at the same radius as before, place it on each vertical compass point (it should extend to the opposite side of the vertical, 5cm segment) and draw two horizontal arcs that intersect the first pair of arcs. You now have your rounded square between the four arcs. That is the part you want; cut it out.

5. To make a cutting template which includes an 8mm seam allowance, draw everything the same but extend the compass 8mm farther than you did before and then draw the four arcs from the same four points using that new radius.

- **SketchUp directions (using millimeter units)**

1. Draw a horizontal 75mm line.
2. Draw a vertical line of the same length and center it on the horizontal line.
3. Draw circles of radius 62.5mm centered on all four ends of the two lines. The intersection of the four circles forms the rounded square which should be 50mm wide.
4. To make a cutting template with an 8mm seam allowance, draw everything the same but increase the radius of the four circles by 8mm.

Altering the size of the bag

The circumference of this bag is composed of four panel widths. Measured around the widths of the panels (around the diagonals is a larger circumference), this design will end up **2.5-6%** larger (depending on whether I fill it loosely or over-fill it) than a calculation based on the width of the template when made with the thick, stiff denim I use. I target the max inflation minus 1% when sizing my beanbags.

To make a bag of diameter d inches (inch units are more natural for me) with an assumed 5% (0.05) inflation, multiply d by 2.54 to convert it to centimeters, multiply by pi to get the circumference, divide by 1+the decimal inflation to get the pre-inflated circumference, and then divide by 4 to get the template width. Thus, the formula for the template width (in centimeters) is

$$d \times 2.54 \times \pi \div 1.05 \div 4$$

Based on this formula, to adjust the bag's finished diameter by $^{1}/_{8}$" (3.175mm), change the width of the template by **2.352mm - 2.433mm** (accounting for the 6% and 2.5% inflations, respectively).

Now you need to know the relationship between the width of the panel and the radius of the curves that form its shape (which is determined by the outer, compass points). I use the same curve radius as for the four-panel beach ball panel. The formula is

$$r = \mathbf{1.25}w$$

where w = panel width (not the distance between the compass points). This means that the compass points should be marked r − w = **0.25**w beyond each end of the initial line that marks off the panel width, and the total distance between the compass points is 2r − w = **1.5**w.

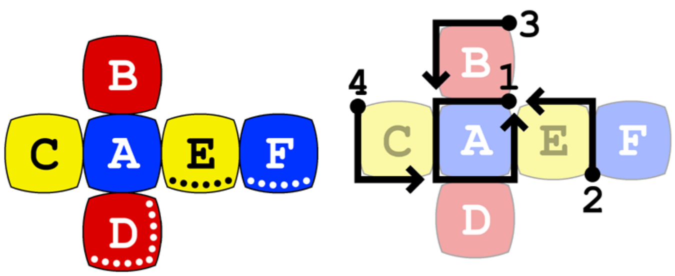
(For my own reference, the diagonal of the 5cm panel is 6.02cm and the chord of the arcs is 4.25cm. This makes the diagonal circumference greater than the parallel circumference by 3.56%.)

To make the cutting template, draw everything the same, but extend the compass farther than you did before (increase the circle radii) by the desired seam allowance and then draw the four arcs from the same four points using that new radius. The cutting template will be larger than, but parallel to, the stitching template.

Making the panels

1. Use the larger, cutting template to trace the cutting pattern onto the back of the fabric. You will need 6 panels.
2. Use the smaller, stitching template to trace the stitching pattern within each cutting pattern, being sure to center it well.
3. Cut out the panels.

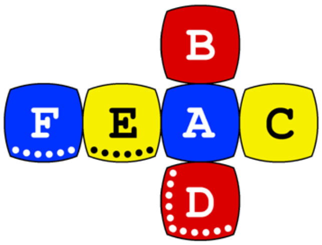
Assembly

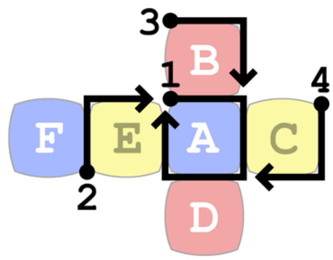


Below is my current favorite method of assembly which uses 5 threads (this bag can be assembled with 4, but that is a little more complicated). The diagrams above depict my method. Each numbered path in the stitching map is a new thread. The final two seams will be sewn from the outside along the dotted lines in the layout diagram. I am right-handed and so the diagrams are oriented for stitching toward the left. In case you are left-handed, I included left-handed versions below.

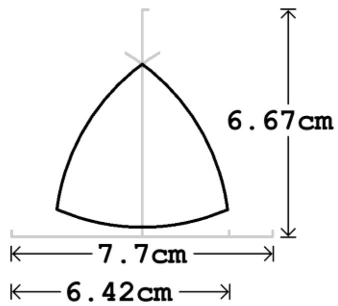
1. Lay the panels out as shown in the diagram above (I prefer to place them front face up) and arrange them according to your color pattern. Use the stitching template to draw stitching lines on the *fronts* of the four panel edges shown with dotted lines in the diagram. Be sure to center the template well (it should align as well as possible with the stitching patterns on the backs). My stitching pathway will leave these four edges partially unsewn so the bag can be turned out between them. They will then be sewn from the outside using the front stitching lines.
2. Start with panel A and sew side panels B-E to each of its edges. Sew them with their front faces together so the bag will be inside out.
3. Add panel F and sew it to E, stitching toward B, and then continue down the side of E, attaching it to B. Tie and trim the thread.
4. Sew B to F, stitching toward C, and then continue down the side of B, attaching it to C. Tie and trim the thread.
5. Proceeding around the cube in the same manner as the previous two steps, sew C to F and then continue down the side of C, attaching it to D. Tie and trim the thread. You should now have just two open seams (four edges) and they should be the ones with the front stitching lines.
6. Start a new thread at either end of the final pair of adjacent seams and sew a few starter stitches to make it easier to continue from the outside. You probably won't need the entire two open seams to turn the bag out, so you can continue to sew as much as you don't need.
7. Turn the bag right side out.

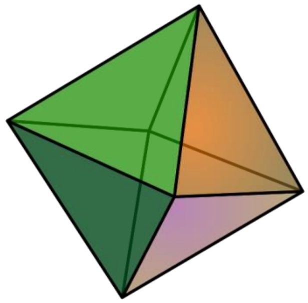
8. Pull out one stitch of the final thread so that the thread is on the outside where you can get to it. Continue sewing the opening closed following the front stitching lines. Fill the bag at some point during this final sewing with a funnel.





8-PANEL ROUNDED OCTAHEDRON INSTRUCTIONS

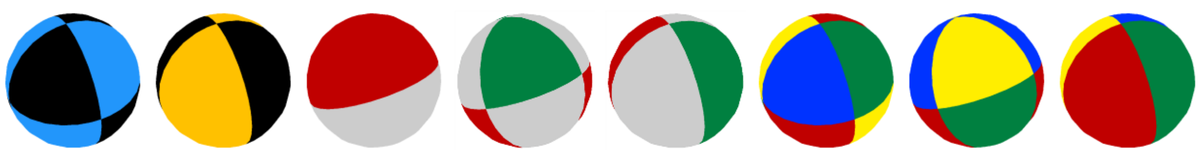




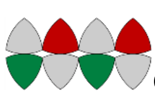
Design Notes

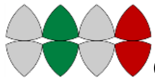
12 seams, 62.2cm (24.5") of stitching.

This design is based conceptually on dividing each panel of the four-panel beach ball in half widthwise, and the radius for the curved sides comes from the beach ball design. The design is technically based on the regular octahedron, which is composed of eight equilateral triangular faces, but the use of circular triangles rounds out the otherwise sharp corners. For a fuller discussion on this design, see the "How I Developed the Designs" chapter under "Rounded octahedron design" (page 46). This design is a good balance between the uniform seam arrangement, roundness, and aesthetic elegance of the dodecahedron, and the ease of construction of the four-panel beach ball.



There are many color arrangement possibilities in this design. Following are all of the reasonable arrangements I could think of. To take full advantage of the aesthetics of the panel structure, choose an arrangement that has no neighboring panels of the same color. The checkering effect this produces is very striking. The images above give examples of each arrangement in order.

- **2 colors**
  - Checkered pattern of contrasting colors.
  - Alternating two-panel stripes of contrasting colors (beach ball style).
  - One hemisphere (four adjacent panels) of each color (Pokéball style).
- **3 colors**
  - Colors A and B alternate on one hemisphere and A and C alternate on the other with no panel having a neighbor of the same color. There will be four panels of color A, which can be thought of as a background color for B and C which are checkered throughout it.

- Color A on a pair of opposite, two-panel stripes and colors B and C on the other pair of two-panel stripes (beach ball style).

- **4 colors**
  - Each color on pairs of opposite panels. No panel has a neighbor of the same color and all four colors are visible at any angle.
  - Two colors alternate on one hemisphere and the other two alternate on the other hemisphere. Each hemisphere (when viewed from the "pole") has a distinct color personality. Around the equator between them all four colors are visible.
  - Two-panel stripes of each color (beach ball style).

Supplies

- **For the templates (assuming you are drawing them by hand)**
  - Cardboard or Template Plastic, fine-point Pencil, metric Ruler, Compass, Scissors, Paper & Double-Sided Tape (optional).
- **For the bean bag**
  - Fabric, Needle and Thread, Scissors, Pencil or non-permanent Pen for marking the fabric, bean bag Filler, Funnel.
- **For your information**
  - Unless you're experienced with this sort of thing, I recommend that you browse through the "General Notes and Techniques" chapter before you get started. You may find some tips there that will improve your experience and your beanbags.

Panel design for a tennis ball sized bag (diameter ≈ 67mm, $2^{5}/_{8}$")

The panel shape is based on a regular (equilateral) triangle but with rounded sides to produce a more spherical juggling bag. I draw this shape by constructing an equilateral triangle (actually just enough of it to get the three corners), setting the compass radius to 83.33% of the length of the triangle's sides, and then, using the three corners as compass points, inscribing the rounded triangle inside the regular triangle. Precision is important because if the compass points or the compass radius are not consistent across all three arcs, the sides of the panel will be unequal lengths and the stitching lines will not match up.

For more information on this design, see below under "Altering the size of the bag".

- **Manual directions**
    1. Draw a 7.7cm horizontal line and mark each end of it. This is the base of an imaginary guide triangle. Mark another point 6.42cm in from one end which will be used to extend the compass to the correct radius.
    2. Extend the compass to the 6.42cm radius, place it on each of the end points of the initial line, and draw arcs that extend from the line up to directly above the center. Draw partial arcs below the line as well to form an X that marks the horizontal center below the line.
    3. Draw a vertical line through the two arc intersections and mark a point 6.67cm above the horizontal line (the top end of this line is the third corner of the imaginary equilateral triangle).
    4. Place the compass on this new point (keeping its radius unchanged) and draw a third arc between the first two, completing the rounded triangle. The rounded triangle should be 4.88cm from corner to opposite side.
    5. To make a cutting template which includes an 8mm seam allowance, draw everything the same but extend the compass 8mm farther than you did before and then draw the three arcs from the same three points using that new radius.
- **SketchUp directions (with greater precision and using millimeter units)**
    1. Draw a 66.682mm vertical line.
    2. Draw a 77mm horizontal line and position its midpoint at the bottom point of the vertical line. The outer endpoints of these two lines form the three corners of an imaginary equilateral triangle.
    3. Draw circles of radius 64.164mm centered on the three endpoints of the skeleton. (See the alteration section below for where this radius comes from.) The intersection of the circles forms the rounded triangle panel shape, which should be 48.811mm from corner to opposite side.
    4. To make a cutting template which includes an 8mm seam allowance, draw everything the same but increase the circle radii by 8mm.

Altering the size of the bag

This panel is formed using the same curve as the four-panel beach ball. If the initial line used by that design is thought of as the base of the triangle, with the two outer, compass points being two of the triangle's corners, this makes the ratio of the compass radius to the length of the guide triangle's sides 5:6, that is,

$$\mathrm{r} = \mathbf{0.8333}\mathrm{s}$$

The height of the triangle (the location of the third corner or compass point) can be found using the following formula (h = height and s = side length):

$$\mathrm{h} = \sqrt{3}\mathrm{s}/2 = \mathbf{0.8660}\mathrm{s}$$

The circumference of the bag (measured between the seams) is 4 × panel height, not the outer triangle height. So, to alter the size of the bag, you need a relationship between the length of the imaginary triangle's sides and the height of the rounded triangle inside it. I do not know a formula for this, but by using SketchUp to draw this template and measure it I have the following, non-mathematical formula (s_t = side of the triangle and h_p = height of the panel):

$$s_t = \mathbf{1.5775} h_p$$

(For my own interest, I originally calculated the ratio above using a hand-drawn template with 14.4cm triangle sides and measured it with a ruler. The ratio I calculated using that method was 1.5824, which is only off by 0.0049!)

This design has a more uniform circumference than the previous two when it is tightly filled and can be measured between the seams or along them. By my measurements it ends up **6-8.5%** larger (depending on whether I fill it loosely or over-fill it) than a calculation based on the height (corner to side) of the template when made with the thick, stiff denim I use. I target the max inflation minus 1% when sizing my beanbags.

To make a bag of diameter d inches (inch units are more natural for me) with an assumed 7.5% (0.075) inflation, multiply d by 2.54 to convert it to centimeters, multiply by pi to get the circumference, divide by 1+the decimal inflation to get the pre-inflated circumference, divide by 4 to get the template height, and multiply by 1.5775 to get the guide triangle's side length. Thus, the formula for the guide triangle's side length (in centimeters) is

$$d \times 2.54 \times \pi \div 1.075 \div 4 \times 1.5775$$

Based on this formula, to adjust the bag's finished diameter by $^1/_8$" (3.175mm), change the length of the triangle's sides by **3.626mm - 3.711mm** (accounting for the 8.5% and 6% inflations, respectively).

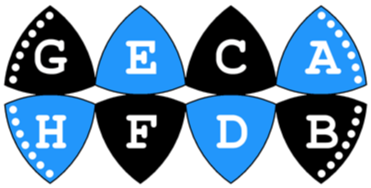
For my own reference, the chord of the 4.88cm high panel is 5.04cm. This makes the seam circumference greater than the panel height circumference by 6.14%.

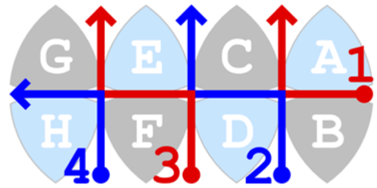
To make the cutting template, draw everything the same, but extend the compass farther than you did before (increase the circle radii) by the desired seam allowance and then draw the three arcs from the same three points using that new radius. The cutting template will be larger than, but parallel to, the stitching template.

Making the panels

1. Use the larger, cutting template to trace the cutting pattern onto back of the fabric. You will need 8 panels.
2. Use the smaller, stitching template to trace the stitching pattern within each cutting pattern, being sure to center it well.
3. Cut out the panels.

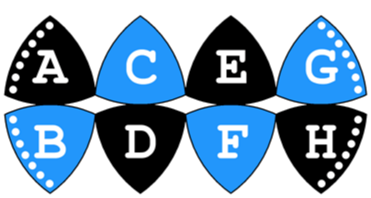
Assembly

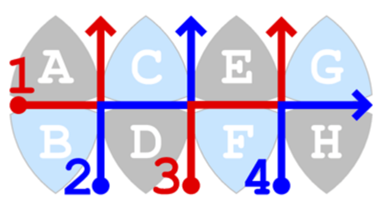




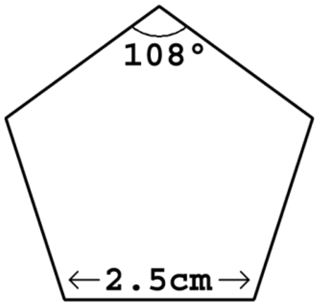
Below is my current favorite method of assembly which uses 5 threads. The diagrams above depict my method. Each numbered path in the stitching map is a new thread. The final two seams will be sewn from the outside along the dotted lines in the layout diagram. I am right-handed and so the diagrams are oriented for stitching toward the left. In case you are left-handed or find this orientation confusing, I included left-handed versions below.

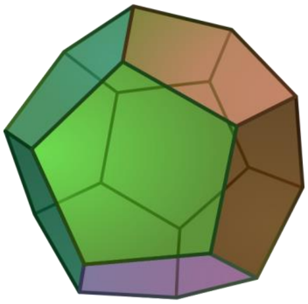
1. Lay the panels out as shown in the diagram above (I prefer to place them front face up) and arrange them according to your color pattern. Use the stitching template to draw stitching lines on *fronts* of the four panel edges shown with dotted lines in the diagram. Be sure to center the template well (it should align as well as possible with the stitching patterns on the backs). My stitching pathway leaves these four edges partially unsewn so the bag can be turned out between them. They will then be sewn from the outside using the front stitching lines.
2. Place panels A and B front faces together and sew them together toward C and D, then add C and sew it to A. Tie off the thread at the end and trim it.
3. Continue adding panels in alphabetical order as in the previous step and sewing them according to the pathways depicted above. Sew them front faces together so the bag will be inside out. Tie and trim the threads when you reach the end of each path. At the end you should have an inside-out bag with two adjacent, parallel seams open (four panel edges), and these should have stitching lines on the fronts.
4. Start a new thread at either end of the final pair of adjacent seams and sew a few starter stitches to make it easier to continue from the outside. You probably won't need the entire two open seams to turn the bag out, so you can continue to sew as much as you don't need.
5. Turn the bag right side out through the opening.
6. Pull out one stitch of the thread you started so that the thread is on the outside where you can get to it. Continue sewing the opening closed following the front stitching lines. Fill the bag at some point during this final sewing with a funnel.





12-PANEL DODECAHEDRON INSTRUCTIONS





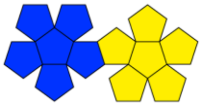
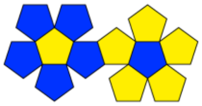

Design Notes

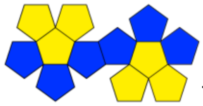
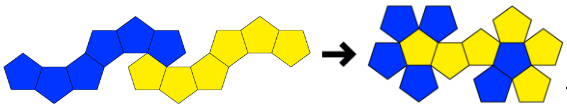
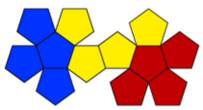
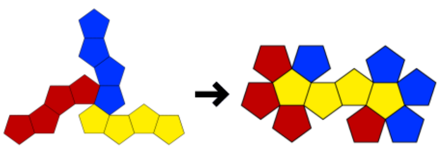
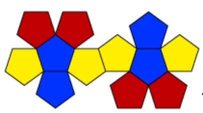
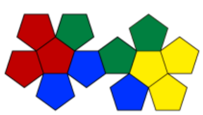
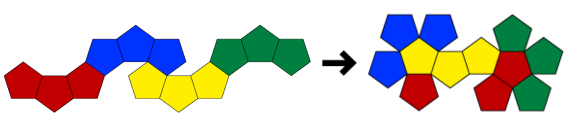
30 seams, 75cm (29.5") of stitching.

Unlike the previous two designs (the rounded octahedron and rounded cube), this design is not at all related to the beach ball design. There are no curves here and the panel layout is very different. I discuss the possibility of using a rounded pentagon for the panel shape in the "Other Designs and Variations" chapter under "Circular-sided dodecahedron" (page 40). In short, it would add unnecessary complication since this design is already very round and smooth.

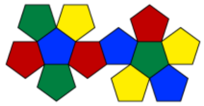
This design has the advantage of feeling wonderfully spherical and uniform in the hand and looking the most elegant, but the disadvantage of being tedious and difficult to make because of having so many panels. This design also has an advantage in the number of color arrangements that are possible. 12 is divisible by 1, 2, 3, 4, 6, and 12 which means you can use 6 different numbers of colors in many different patterns and still have a balanced look.

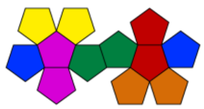
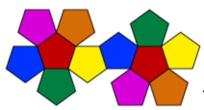
Following are twelve color arrangement ideas of which half are mine and half are from http://www.pjb.com.au/jug/leatherballs.html (the latter are the ones with the British spelling of "colour"). To help me design the more complicated of my arrangements and create the diagrams, I stuck colored thumbtacks into each panel of an old, solid beige, dodecahedron beanbag. This worked very well and I highly recommend it if you want to have some fun playing around with color arrangements.

- **2 colors**
  -  Each color covering an entire hemisphere.
  -  Five-fold rotational symmetry arises with one face of colour A at the top surrounded by five faces of colour B, and one face of colour B at the bottom surrounded by five faces of colour A.

  -  Three-fold rotational symmetry arises with a patch of three faces of colour A at the top, a patch of three faces of colour A at the bottom, and a belt of six faces of colour B around the equator.
  -  Two intertwining spirals [I would describe them as S-curves, though they can also be flipped into "2"-curves] of six faces can give a pattern with chirality, either left-handed or right-handed. This can be derived from the four-colour pattern by pairing the colours, and so the four-colour patterns also have chirality, though this is not obvious at first glance. [The above diagrams assume that the fronts of the panels are facing up. To reverse the direction of the "spirals" (to turn the S-curves into 2-curves), simply lay the panels out in the same pattern with their backs facing up.]
- **3 colors**
  -  Each color on a diamond-shaped patch of four panels.
  -  Each color on an S-shaped row of four panels. The above diagrams assume that the fronts of the panels are facing up. To reverse the orientation of the curves (to turn the S-curves into 2-curves), simply lay the panels out in the same pattern with their backs facing up.
  -  Three separate two-fold reflectional symmetries arise if each colour is arranged in two opposite patches of two faces each.
- **4 colors**
  -  Each color on a patch of three panels that share a corner.
  -  Each color on a row of three panels. By using the two-color intertwining spiral arrangement to form this arrangement, you will get a pattern as follows (using abbreviations for the diagram colors): colors in the pairs B&Y and R&G curve into each other and each pair is opposite and perpendicular to the other while colors in the pairs B&G and Y&R curve away from each other and these pairs are also opposite and perpendicular to each other. There are other, random ways to position 4 three-panel curves, but this method produces a balanced pattern. The above

diagrams assume that the fronts of the panels are facing up. To reverse the direction of the bi-color "spirals" (to turn the S-curves into 2-curves), simply lay the panels out in the same pattern with their backs facing up.

-  Four different colours arranged so that no side has a neighbour of the same colour. This would illustrate the Four-Colour theorem except that the Four-Colour theorem applies to maps on a plane rather than maps on a sphere; still, it refers to it. The four-colour patterns have chirality, though it is not obvious at first glance; it can be seen by pairing the four colours into two groups of two.

- **6 colors**
  -  Patches of two panels of each color such that each pair is perpendicular to the pairs that surround it. That is, the end of one pair fits into the side of another like the panel layout of a traditional volleyball. This is similar to the third three-color arrangement.
  -  The two faces of each colour are placed opposite each other, and whichever angle the ball is viewed from, all six colours are visible.

Supplies

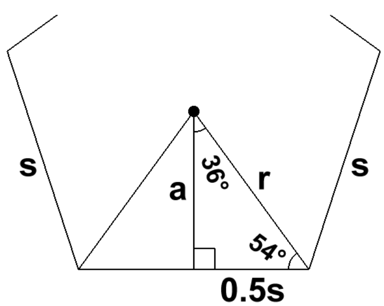
- **For the templates (assuming you are drawing them by hand)**
  - Cardboard or Template Plastic, fine-point Pencil, metric Ruler, Protractor, X-acto Knife or Scissors, Paper & Double-Sided Tape (optional).
- **For the bean bag**
  - Fabric, Needle and Thread, Scissors, Pencil or non-permanent Pen for marking the fabric, bean bag Filler, Funnel.
- **For your information**
  - Unless you're experienced with this sort of thing, I recommend that you browse through the "General Notes and Techniques" chapter before you get started. You may find some tips there that will improve your experience and your beanbags.

Panel design for a tennis ball sized bag (diameter ≈ 67mm, $2^5/_8$")

The panel shape is a regular (equilateral) pentagon with 2.5cm sides and 108° corners. There is more than one way to draw a regular pentagon. Following is how I do it. Precision is important because if the sides or angles of the panel are not equal the stitching lines will not match up and you may get a visibly skewed bag.

- **Manual directions**
  1. Draw a 2.5cm line, mark each end of it, and then continue the line to either side of it for several centimeters to aid in accurately aligning the protractor along it.
  2. Place a protractor on the line, center it on each of the endpoint marks in turn, and mark points that form 108° angles with the origin points (or 72°, depending on which side of the point is your reference).
  3. Draw a line through each pair of points, forming the bottom half of the pentagon. Mark a point 2.5cm out from each origin point and continue the line on both ends as in the first step.
  4. Place the protractor on each new line, centered on the new endpoints, and mark points at 108° as in step 2.
  5. Draw lines through the two new pairs of points, completing the pentagon. Make sure the lines meet exactly 2.5cm from their origin points. If they do, you drew a perfect pentagon. Any error you make will be compounded several times in the juggling bag, so be as precise as you can.
  6. Take advantage of the straight sides and cut the template out with an X-acto knife and straight edge.
  7. To make a cutting template which includes an 8mm seam allowance, draw a pentagon with 3.7cm sides. (Why? See the cutting template alteration instructions below.)
- **SketchUp directions (using millimeter units)**
  1. Use the polygon tool (View -> Toolbars -> Drawing) set to 5 sides and draw a pentagon with radius = 21.266mm, which will result in a pentagon with 25mm sides. The radius is explained in the alteration section below.
  2. To make a cutting template which includes an 8mm seam allowance, draw a pentagon with 31.157mm radius. (Why? See the cutting template alteration instructions below.)

Altering the size of the bag



The circumference of the bag is composed of 4 panel heights and 2 side lengths. To alter the size of the bag, you need the relationship between the pentagon's side length and its height. Following is a series of formulas that reference the figure above and will give you this height as well as the diameter and circumference of the dodecahedron. In the formulas, s = side length. (To draw a pentagon in SketchUp, which uses a defined radius instead of a defined side length, you will need the first formula.)

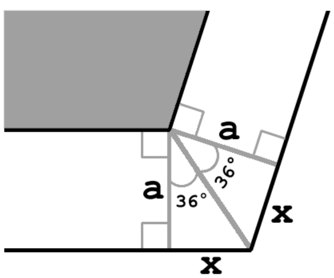
- **Pentagon radius** (center to corner) $= 0.5\text{s}/\sin 36° = 0.5\text{s}/\cos 54° = \mathbf{0.8507}\text{s}$
- **Pentagon apothem** (center to midpoint of side) $= 0.5\text{s}/\tan 36° = 0.5\text{s}(\tan 54°) = \mathbf{0.6882}\text{s}$
- **Pentagon height** (sum of above) $= \mathbf{1.5388}\text{s}$
- **Dodecahedron circumference** $= 4(1.5388\text{s}) + 2\text{s} = \mathbf{8.1552}\text{s}$
- **Dodecahedron diameter** $\approx 8.1552\text{s}/\pi = \mathbf{2.5959}\text{s}$

This bag has a completely uniform circumference and can be measured in any orientation. By my measurements it will end up **1-4%** larger (depending on whether I fill it loosely or over-fill it) than a calculation based on the above formulas when made with the thick, stiff denim I use. I target the max inflation minus 1% when sizing my beanbags.

To make a bag of diameter d inches (inch units are more natural for me) with an assumed 3% (0.03) inflation, multiply d by 2.54 to convert it to centimeters, multiply by pi to get the circumference, divide by 1+the decimal inflation to get the pre-inflated circumference, and divide by 8.1552 to get the template's side length. Thus, the formula for the template's side length (in centimeters) is

$$\text{d} \times 2.54 \times \pi \div 1.03 \div 8.1552$$

Based on this formula, to adjust the bag's diameter by $^1/_8$" (3.175mm), change the length of the template's sides by **1.176mm - 1.211mm** (accounting for the 4% and 1% inflations, respectively).



To make a cutting template you need to use more trigonometry. In the diagram above, x is the amount to extend one end of each of the two sides shown to get a seam allowance "a" on those two sides. You would have to double that to get the full amount by which to extend each side for the cutting template. Thus, the formula for the cutting template extension is

$$2\text{x} = 2(\tan 36°)\text{a} = \mathbf{1.4531}\text{a}$$

In the case of the 2.5cm pentagon and 0.8cm seam allowance, you would want the sides of the cutting template to be

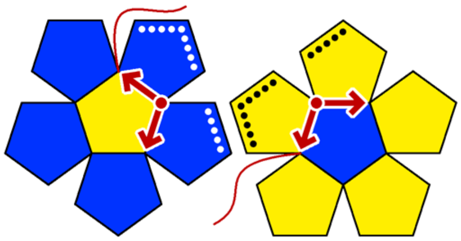
$$2.5 + 2(\tan 36°)0.8 = 3.6625\text{cm}.$$

Making the panels

1. Use the larger, cutting template to trace the cutting pattern onto back of the fabric. You will need 12 panels.
2. Use the smaller, stitching template to trace the stitching pattern within each cutting pattern, being sure to center it well.

3. Cut out the panels.

Assembly



Following is my current favorite method of assembling the panels. It uses 10 threads. The diagram above is a depiction of my method. You will be forming two separate hemispheres and then sewing them together around the equator. Your stitching for each hemisphere will begin at the corner between the arrows. You will be sewing 3 equatorial seams from the outside (indicated by the dotted lines) using a thread that continues from one of the side seams (indicated by the red threads hanging out). Only one of these threads needs to be long enough to sew the three front seams, but either hemisphere can have that long thread.

If you use multiple colors, be careful to assemble the panels correctly to form your chosen pattern, especially when you join the two hemispheres. With so many panels and seams, it's easy to make a mistake and misalign the panels. I found it helpful to make a cardboard dodecahedron with colored panels or labels to help me visualize the pattern and keep track of what I was doing. I recommend building a model anyway to help you visualize the stitching sequence unless you can see it from my description. Index cards or something of similar thickness work well for this. Just use your stitching template and cut several layers at a time to produce the panels faster.

1. Lay the panels out as shown in the diagram above and arrange them according to your color pattern. Note that my color arrangement diagrams assume that the fronts face upward (the orientation makes a difference in some of the arrangements, and this is noted in the descriptions of those arrangements). Use the stitching template to draw stitching lines on *fronts* of the six panel edges shown with dotted lines to form 3 seams in a row around the equator that will be stitched from the outside. Be sure to center the template well (it should align as well as possible with the stitching patterns on the backs).
2. Cut a thread that is long enough to sew 6 seams.
3. Start with a center panel and sew a panel to each of its sides beginning at the corner between the two arrows in the diagram and proceeding in either direction. Sew the panels with their front faces together so the bag will be inside out. Note that the starting point, which will also be the ending point, is at the corner where the front stitching path splits across two panels.
4. When you have attached all five panels and the thread has reached the starting point, sew the adjacent sides of the two outer panels together, which connects the two segments of the front stitching lines into one, and then tie off the thread and trim it.
5. Using one new thread per seam, sew together the remaining adjacent sides of the outer panels starting at the center panel and sewing outward. Leave the threads hanging at the ends of their seams (there is no need to tie them off, but you may). The thread at the position of the red thread in the diagram must be long enough to also sew the three equatorial seams that have the

front stitching lines, and the rest will have to be long enough to sew one more seam. At the end of this step you should have a half sphere composed of six panels with four threads hanging from its rim.

6. Join the other six panels in the same way, except that you won't need that extra long, 4-seam thread unless you didn't cut the first one long enough for all three front-sewn seams.
7. Before starting the next step, make sure you know how you are going to align the two hemispheres when you join them. They must be joined in such a way as to form your intended color pattern and to make the three front stitching lines on each half meet each other.
8. Sew the two hemispheres together using the hanging, short threads in the following manner: Start with a thread that is adjacent to the long, 4-seam thread and sew toward it, ending at the long thread's corner. Tie that thread and trim it. Now, start sewing one seam per short thread always ending where the last thread began and tying and trimming the threads when you reach the ends of the seams. Stop when only 3 open seams in a row remain. The long thread should be at one end of those three seams.
9. Start the long thread with a few stitches to make it easier to continue from the outside.
10. Turn the bag right side out through the opening.
11. Pull out one of the long thread's stitches so that the thread is on the outside where you can get to it. Continue sewing that seam and proceed to the second and third, following the front stitching lines. Fill the bag at some point during this final sewing with a funnel.

OTHER DESIGNS AND VARIATIONS

Other beach ball style panel multiples

Images from http://www.dube.com/beanbag/dube-pro-beanbags.php and http://www.higginsbrothers.com/html/juggling_balls.html

You can use the beach ball concept to make juggling bags with any number of panels greater than or equal to four (I doubt that a two or three-panel bag would be very spherical). The above examples use six and eight-panel arrangements, and I read a blog in which a guy said he made one with five panels. To design the panel, all you have to do (if, like me, you don't know the math involved) is to draw a stick skeleton (two perpendicular lines that cross at their centers) whose height is half the desired circumference and whose width is equal to the circumference divided by the number of panels, and then find through trial and error a radius that produces a curve that meets the top, middle, and bottom points of this skeleton. Draw that curve on both sides of the skeleton and you have your beach ball panel.

I did this for six and eight panels using a compass to find the approximate radii and SketchUp to find the precise radii and then I calculated the ratios (r = curve radius, w = panel width):

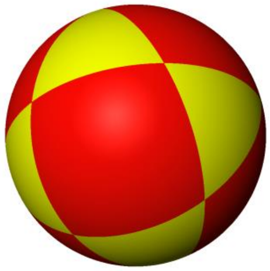
For 6 panels:

- $\mathrm{r} = \mathbf{2.5}\mathrm{w}$
- Distance between compass points $= 2\mathrm{r} - \mathrm{w} = \mathbf{4}\mathrm{w}$
- I would use 3.2780cm wide panels for a $2^5/_8$" diameter (accounting for an estimated 6.5% inflation when the bag is filled).

For 8 panels:

- $\mathrm{r} = \mathbf{4.25}\mathrm{w}$
- Distance between compass points $= 2\mathrm{r} - \mathrm{w} = \mathbf{7.5}\mathrm{w}$
- I would use 2.4818cm wide panels for a $2^5/_8$" diameter (accounting for an estimated 5.5% inflation when the bag is filled).

Cuboctahedron




CG cuboctahedron from Wikipedia. Beanbag photo from http://www.higginsbrothers.com/html/juggling_balls.html.

There are popular polyhedron-based bag designs that use more than one shape for the faces. One that I have seen frequently and show above is the cuboctahedron which has eight equilateral triangles and six squares. I have also seen truncated icosahedron (soccer ball style) bags which use hexagons and pentagons. The beanbag example above appears to use a modified cuboctahedron design in which the triangles have flat corners and so are actually irregular hexagons. This probably helps to reduce the prominence of the vertices on the finished bag.

To figure out the correct size of the panels for a cuboctahedron, you first need to know how its circumference is formed. There are two ways to measure the circumference. One is across the width of the squares in which case the circumference is 2(square width) + 4(triangle height). The other is across the squares' diagonals in which case the circumference is 4(square diagonal). By my calculations, the latter is 3.5265% greater than the former.

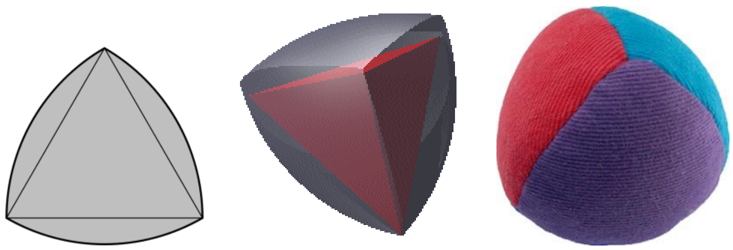
A square's diagonal is calculated using the Pythagorean Theorem which, in the case of a 1x1 square, means that the diagonal (which would be the hypotenuse if you divided the square into two right triangles) is $\sqrt{1^2 + 1^2}$ = 1.4142. An equilateral triangle's height is calculated using the same theorem and dividing a 1-unit triangle in half to form two right triangles whose hypotenuse is 1 and whose short side is 0.5. The formula is $\sqrt{1^2 - 0.5^2}$ = 0.8660. You can also use the formula $\sqrt{3}/2$. Those two values, 1.4142 and 0.8660, can be multiplied by the side length of any square or equilateral triangle, respectively, to obtain the circumference of the bag you're designing.

I prefer to use the squares to determine the panel size, and so I will add $3.5265\%/2$ to the target template circumference to average out the two measurement methods. I will assume the inflation increase of the dodecahedron and I'll target the max inflation minus 1%. So, if I want $2^5/_8$" finished diameter, which is a 20.9466cm finished circumference, I will first reduce it by (3% − $3.5265\%/2$) = 1.2368% which gives me a template circumference of 20.6907cm. Now I divide that by 4 to get the diagonal of each square: 5.1727cm. Then I divide that by 1.4142 to get the side of each square: 3.6577cm. I would round that up to 3.7cm if I drew the panel shapes manually and use it as the length for the sides of my squares and triangles (the two shapes must have matching side lengths, of course).

You could curve the sides of the panels to enhance the roundness of the bag, which I suspect is more important in this design than in the dodecahedron because the corners are sharper by 24°. I don't know what curve radius to use, but since a square and two triangles can be thought of as a beach ball panel (their length is half the circumference), I would theorize that the beach ball concept could be used to find the curve radius. In this case, the width:height ratio of the beach ball panel is 1:2.732. Using the beach ball method to find the curve radius that fits the stick skeleton, I arrived at a radius of 2.116w.

Using this curve to form the square is easy. Using it for the triangle is more complicated. I haven't fully designed that shape yet.

Reuleaux tetrahedron



Reuleaux triangle and tetrahedron from Wikipedia. Beanbag photo by me.

I found some juggling bag patterns by someone named Peter Billam on the web which produced polyhedrons with faces having bulged sides (http://www.pjb.com.au/jug/leatherballs.html). From this website I got the idea for my rounded cube and rounded octahedron (I designed the curves myself, though). I also designed a rounded tetrahedron from the same inspiration using four Reuleaux triangular faces. (Through measurement and experimentation I determined that the panels used by Peter Billam are Reuleaux triangles, or close enough as not to matter.) However, the Reuleaux tetrahedron was so non-spherical that I did not deem it worth including formal instructions for in this document. As Billam puts it, "it has a shape a bit like an egg with four ends." It was a cool-looking beanbag, but not as easy to catch as a spherical bag. My tetrahedron bag is pictured above.

If you want to make one, draw a Reuleaux triangle for the panel. This is done by drawing an equilateral triangle and then using a compass placed on the corners with a radius equal to the length of the sides to draw curves from corner to corner around the triangle. For the cutting template, use the same sized triangle and compass positions, but increase the compass radius by the desired seam allowance. I used a triangle with 6.5cm sides to draw the Reuleaux triangle and that resulted in a bag with a 20.1cm circumference (64mm, $2^{1}/_{2}$" diameter). This is smaller than I like, but I'm not going to try again. You might get a somewhat more spherical bag by using an even steeper curve than is used for the Reuleaux, which means you'd have to place the compass closer in toward the triangle's center rather than on its corners. I do not think, however, that even this would make the bag spherical enough to compete with the other designs in this document.

Rounded dodecahedron

Peter Billam's juggling bag patterns (http://www.pjb.com.au/jug/leatherballs.html) included a dodecahedron with bulged sides. I have not tried rounding the faces of my dodecahedron (that would be rather a pain to design and I do not know what the curve radius should be). I don't feel that it needs it as the dodecahedron bag is wonderfully round and smooth already. Billam does not provide definitions for the panel shapes and so I do not know what curve radius he used to round the polygons. (Actually, the bulges used by most of his panel shapes are not circular but angular for some reason.) The curves of a Reuleaux pentagon look much steeper than Billam's curves, so he is using something shallower than that.

4-panel spiral cube

I found a very interesting four-panel cube design here: http://juggleballs.amielmartin.com/. In this design, four 2"x5" rectangles are sewn together in a pinwheel fashion to form a spiral-sided cube.

Tennis ball style

Some juggling stores sell a tennis ball style bag made with two panels that wrap around the bag in perpendicular directions. I do not know how to design this panel shape.

Volleyball style


Images from http://www.jugglingstore.com/volley-bag-737.html

The above design is sold at various online juggling stores. It is called the "Volley Bag". I tried and failed to figure out how to design the panel. I do not know enough geometry to tessellate the surface of a sphere in this manner. I very much want to make a bag like this.

Original Concepts by Me

CG billiard ball from http://fadigeorge.wordpress.com/2011/04/11/3-d-billiard-ball-tutorial/. Still from *The Court Jester* from http://hollywood-elsewhere.com/2011/04/the_short_list.php.

I came up with three design concepts that I have not yet finished. The first is a twelve-panel bag based on a striped billiard ball. It would be composed of four rounded triangles around each pole and four rounded rectangles around the equator. The panels could be designed, in part, by considering that each pole-to-pole set of three panels (triangle-rectangle-triangle) is similar to the lemon panel of the beach ball. If you cut a lemon panel into three slices widthwise and round the new edges using the right curve radius, you would have your panel shapes.

The second concept is inspired by the traditional jester's uniform such as the one worn by Danny Kaye in the movie *The Court Jester* which has a checkered pattern of contrasting diamonds. I would transform the diamonds into rounded forms which would result in lemon panels similar to those used in the beach ball. This bag would have eighteen panels. It would be composed of six lemon panels around each pole whose outer tips meet at the equator, and six lemon panels around the equator between the polar panels. The panel lengths would be one quarter of the ball's circumference, and the widths would be based on the circumference at the point where their middles meet. The panels around the equator would have a width = circumference/6 and the panels around each pole would need to be narrower (I think) using a formula for the circumference at a particular latitude. According to my research, this formula is (equatorial circumference) × cos(latitude angle). The middles of the polar panels will meet half way between the equator and the pole, which would be a 45° angle from the equator. I would use a 3-color arrangement for this design such that colors A and B alternate around each pole and alternate end to end where they meet at the equator, and color C is used around the equator between the polar panels.

The third design, which I actually invented before the jester design, is a simplified form of the jester design in which there are two rows of lemon panels rather than three. It would be composed of eight or twelve lemon panels arranged with four or six around each pole. The panels would overlap around the equator such that the tip of each panel reaches the middles of the opposing panels on either side of it. The length of the panels would be $^1/_3$ of the ball's circumference and the width would be calculated by the latitude circumference formula at a 60° angle from the equator. I would use four colors for this design with A and B alternating around one pole and C and D alternating around the other.

If anyone reading this knows how to design the panel shapes (including the curve radii) for these concepts using math to prove the correctness of the designs, I would love to be taught. A combination of a lack of knowledge and a mind hampered by chronic depression prevents me from following through on these concepts and being sure they are designed correctly.

HOW I DEVELOPED MY DESIGNS

My denim, proof-of-concept bags

US to metric conversion note

I originally made my four and twelve-panel juggling bags (which are my earliest designs) using inch measurements because I live in the USA and I think in terms of inches. The stories for those two designs below, which I first drafted around that same time, use those measurements. But during the process of writing these formal instructions in August, 2012, I decided that using metric measurements is much easier since I often have to make calculations on a calculator which results in decimal values which I can't easily translate onto a ruler with multiples-of-two fractional increments ($^1/_2$, $^1/_4$, $^1/_8$, $^1/_{16}$). Also, millimeters give me smaller increments to work with than sixteenths of an inch. So, I converted all the measurements into the metric system. Over the course of writing the instructions and doing further calculations and more accurate measurements, I adjusted the sizes a bit so that all the bags, when inflated, will be as close to $2^5/_8$" (66.675mm) in diameter as possible.

If you need to convert the measurements back to inches, remember that 1 centimeter = 0.3937 inches and 1 inch = 2.54 centimeters.

4-panel beach ball design

Round Bean Bags $12.00 per set of 3

These balls are made from corduroy in solid four-panel construction. Each is approximately the size of a tennis ball. Assorted colors. Weight, 6 oz.

Rubber Juggling Balls $12.00 per set of 3

Good bouncers, these balls are solid rubber with a smooth, glossy, but high-tack finish. Each set contains one red, one blue and one yellow ball. Available only in sets of three.

Original advertisement from "The Flying Apparatus Catalogue" by Klutz, scanned by me

This is my first design which I developed in the mid-1990s when I was in my mid-teens. I got the idea for it from some corduroy, four-panel juggling bags sold in the Klutz Flying Apparatus catalog. I badly wanted them but I didn't want to cough up the dough ($12 for three). I decided to do what I often do in this kind of situation: make them myself. Unfortunately, I had no idea how to draw the panel shape. I had to use the small, black and white images of the bean bags from the catalog (shown above) to figure out the shape and proportions of the panels. It took me weeks of experimentation and failed attempts to realize how simple it is to draw the panel shape.

My initial idea was that the ratio of the width to the length should be 1:2, which was correct since the circumference of the finished bag would be composed of four panel widths in one direction but only two panel lengths in the other. So I drew a stick skeleton of the panel with a vertical line forming the length and a horizontal line, half the length of the vertical, centered on it to form the width. Then all I had to do was draw the panel shape around this frame. I drew the curved sides of the panel free-hand because I had no better idea how to do it. I had rarely ever used a compass in my life and that is probably why it did not immediately occur to me to use one. I had also not studied whatever advanced mathematics would be required to understand that a circularly curved lemon shape, when wrapped around a sphere, would produce the correct, circumscribing curve I needed. I still do not really understand this; I just know it works. Despite all of this, I should have known that drawing an arbitrary curve had to be the wrong approach, but though I had subtle misgivings about it, I tried it anyway.[1]

In my first attempt, I drew the curves too steep at the ends which resulted in the panels and, consequently, the bean bag to be too narrow at the ends. When I saw the finished, slightly lemon-shaped bag, I unfortunately decided that my perfectly logical width:length ratio must be flawed. This began a series of varying attempts at different styles of curves and width:length ratios. I even tried drawing the ends of each side as a straight line to form a right angle at each end and only curving them in the middle. I hoped this would flatten those deformed ends.

After I had assembled or partially assembled over a dozen different bags and even bought a styrofoam ball around which I fitted paper models, I finally realized that the arbitrary curve was the problem. The only way to make a non-arbitrary curve was by using a compass. (After I began drawing the sides with a compass, I marked the templates “All Round” to differentiate them from the ones with partially straight sides.) It took a little longer and the assembly of a collection of four or five identical bean bags (which are now at my grandparents' house for kids to play with) to come full circle with the realization that, with the now truly circular curves, my original width:length ratio would work (by this time I had settled on a different ratio with a shorter length). After that, there was just the matter of deciding on the best size of the bean bag. I wanted the bag to be as close as possible to the size of a tennis ball because I liked that size for juggling (I also do not like to choose things arbitrarily and the tennis ball gave me a nice, non-arbitrary choice). So, I would make a bag and spend a few days tossing it around and comparing it to a tennis ball, trying to decide if I would like it better if it was a different size.

I decided on a panel size of $1^7/_8$" by $3^3/_4$" inches and made nine bean bags of this size. When I began making twelve-panel bags, I decided on a panel size that made the new bags a little larger than the four-panel bags. When I compared these new bags to my old, smaller bags, I found that I liked the larger size better. Conveniently, the panel size that would make the four-panel bags the desired size happened to be a nice, simple, 2 by 4 inches.

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[1] When writing the formal version of these instructions, I felt foolish for having tried to draw a free-hand curve rather than thinking of using a compass, and I didn't like admitting to it. But I have found through my web research that this is actually the most common way to draw this panel shape (by amateurs). I have seen PDF patterns that are obviously drawn free-hand, and I have seen written instructions that explicitly say to draw the curve by hand and give no formula for a curve radius so a compass can be used. So far I have never seen anyone provide a mathematical definition of the beach ball panel or any explicit instruction to use a circular curve.

My stitching technique went through its own evolution. I started out with a basic running stitch (that and the whip stitch were the only stitches I knew as I had only a rudimentary knowledge of sewing), but I found that this resulted in the finished seams looking rippled. I figured this was because each stitch pulled the fabric toward itself and had no opposing stitch to pull the fabric back. So I tried using a double running stitch: I stitched from one end to the other and then back the other direction so that each new stitch was on the opposite side from the first stitch. This helped, but not enough to satisfy my perfectionist nature. Finally, after much thought, I came up with a stitch that worked. The method for this new stitch is described under the "Stitching Techniques" heading (page 11) in the "General Notes and Techniques" chapter. This new stitch made the seams perfectly straight and, because half the stitches are backwards, they sort of locked themselves against the fabric and stayed tighter.

When I decided to write the first, informal draft of these instructions (which I wrote during that time), I wanted to give a name as well as a description to my stitch. My mom had several sewing books from her younger years, and I figured that since a stitch as good as mine probably didn't originate with me, I might find it (or something similar to it) in one of those books. Well, it was in one of the books and it was called a "Back Stitch". I think that's pretty cool: *I invented the back stitch!* (I just wasn't the first to do so.)

Eventually, I also figured out a shortcut to drawing the panels that did not involve drawing two perpendicular lines (one marking off the width and the other the length of the panel) as guides for the curves. I now only need one line (the width) because I know how to place the compass so that the arc formed therewith will be exactly the right vertical distance from the center of the panel.

Dodecahedron design

This is my second design. In the 1998 or 1999 Bright, Indiana volunteer fire department festival, I played one of the games and won an el cheapo, green, vinyl-coated hacky sack with the BP (British Petroleum) logo on it (BP had just built a gas station in Bright and was trying to make itself known). I liked the way it was designed: twelve pentagonal panels sewn together to make an almost perfect sphere. It was much rounder and more stylish-looking than my four-panel design, so I decided to try to emulate this new design in my juggling bags.

As I discovered later, aside from looking more elegant, this design also has the benefit of allowing me to be much more creative with color arrangements than the four-panel design. Four is only divisible by 1, 2 and 4 and so I can only use these multiples of colors and still make the bag look balanced (actually, I came up with a very attractive 3-color pattern during the writing of these formal instructions). I also have very few color arrangement options. Twelve, on the other hand, is divisible by 1, 2, 3, 4, 6, and 12 and this design also offers numerous ways to arrange the colors on the bag.

However, before I could make a twelve-panel bean bag, I had to figure out how to draw a perfect, regular pentagon (a pentagon having all sides and angles equal). I tried and failed to come up with a simpler shape (triangle, square, etc) around which a pentagon could be formed using the sides and corners of the inner shape as guides (similar to my stick skeleton idea for the four-panel design). I then knew that I would have to draw the pentagon without any guide. This meant that I would have to draw a side, measure out the angles at its ends with a protractor, and then draw the adjacent sides (there are ways to draw a pentagon using circles, but I did not know about these and I still do not understand them well enough to use them). So I had to figure out what the measure of the angles should be. I first thought that since a square has a total of 360°, a pentagon might as well. I actually had to draw the first angle before I realized that $^1/_5$ of 360° is an acute angle which a regular pentagon cannot have. I then tried 360° + 180° = 540°. This made each angle 108° which was perfect.

The second problem was what size to make the panels so that the finished bag was the most comfortable size for juggling. This was more difficult than in the case of the four-panel design because there were many more panel sides and heights to add up to get the circumference. Also, I did not know, nor could I come up with, any formula to tell me the distance from a side to the opposite corner (i.e., the height) of a pentagon. I had to figure it out the crude way: by drawing the pentagon (or at least half of it) and then measuring it with a ruler. After arriving at a reasonable circumference I would make a bean bag and, as with the four-panel bags, decide over a period of days or weeks if it fit my hands just right. I started out making panels with $^3/_4$ inch sides. This made the bag a little too small, so I moved up to $^7/_8$ inch. I was almost satisfied with this size, but I decided to try 1 inch. This size, at last, seemed perfect and I loved the fact that the sides were such a simple length

I later learned from Dr. Lee Sanders at Miami University's Hamilton, OH campus that the shape of this bag is called a dodecahedron. It has 12 faces, 30 edges and 20 corners. I learned during the research for this document that the dodecahedron is one of five Platonic solids (see http://en.wikipedia.org/wiki/Platonic_solids for more information).

Rounded octahedron design

I developed this design in August, 2012 while writing the formal version of this document. I found a website (http://www.pjb.com.au/jug/leatherballs.html) that had patterns for several rounded polyhedral beanbag designs by someone named Peter Billam, and these included an octahedron. The website did not give the definitions for the shapes, however, and so I did not know what curve radius they used (I later measured the octahedron panel template with a ruler and tested it with a compass and it appeared to be a Reuleaux triangle). But I saw that, roughly, a rounded octahedron is my four-panel beach ball design with each panel cut in half widthwise and the two new edges rounded to match the adjacent edges. The panel would have to be based on an equilateral triangle because that is what forms the faces of a regular octahedron, and its sides should be rounded using the same curve radius as the beach ball's lemon curve panel because the octahedral bag has the same seam pattern of two perpendicular, circumscribing seams but with one additional, identical seam around the equator. So I set about constructing an equilateral triangle with sides curved using that radius.

It took some time to figure out what size I needed the guide triangle to be and how to best use it to form the rounded triangle. I initially experienced some lapses of logic and designed the panel by using a radius equal to the height of the triangle which resulted in the rounded triangle meeting each of the guide triangle's sides. I later realized my mistake and redesigned it so that the ratio of the curve radius to the guide triangle's sides is the same as the ratio of the beach ball's curve radius to the distance between the two outer, compass points. My thinking was that the two compass points of the beach ball panel design could be thought of as two corners of a triangle, and then I can create a third corner and draw the same curve from that as from the first two. This makes the ratio of the compass radius to the length of the guide triangle's sides 5:6, that is, the radius is 83.33% of the length of the side.

Rounded cube design

I developed this design a few days after the octahedron and got the idea from the same website (http://www.pjb.com.au/jug/leatherballs.html) by Peter Billam. I had made a true cubic beanbag years ago as a novelty, but I had little interest in making another or in including it in these instructions. The idea of a rounded cube seemed worth trying, however. It turned out so well that I consider it as worthwhile a design as any of the others and so I wrote out the formal instructions for it.

I designed the cube using the same assumption about the curve radius of the panel sides as I did for the octahedron bag; that is, I assumed it should match the curve of the four-panel beach ball design. At first this assumption was based mainly on intuition – the sphere was the same circumference and the panels form a quarter of the bag's circumference as they do in the beach ball, so the panel curve should have the same radius.

After I made the proof of concept bag, I began to think that I might have made an incorrect assumption about the curve. Perhaps a curve radius equal to the width of the panel (as opposed to width × 1.25) would make more sense and also be simpler to construct.

I almost got to the point of constructing a bag according to this design, and then began to understand my initial assumptions more clearly and realized that they did make more sense because the cube is basically the same as the four-panel beach ball design except that the four tips of the panels at each pole have been exchanged for a new panel that matches the equatorial portions. Around the equator both bags are the same. My rounded square is really just the middle portion of the lemon curve panel and is arranged on the bag in the same positions as the lemon curve panels. I also thought that if the prominence of the corners was reduced any more, the corners of the cube would become slightly concave.

A few days later I began designing a rounded tetrahedron using curves based on a three-panel beach ball instead of those for the Reuleaux triangle as Peter Billam calls for (according to my measurements of his panel shape), and the thought process involved in designing a three-panel beach ball caused me to realize that I had designed my rounded octahedron incorrectly (the difference was not great) and had made some very foolish assumptions. This caused me to rethink all of my designs, including the cube. I measured Peter Billam's rounded cube panel shape and found that the curve happened to have (whether by design or not I do not know) a radius whose ratio to the square's width is the same as the ratio of a square's width to its diagonal (0.7071). This was a much steeper curve than I used (my radius was width × 1.25). I thought that a steeper curve might, after all, work better because, while the curves of my beach ball meet at a four-point rounded corner and the seams join to form continuing, circumscribing seams, the curves of the cube, after a short span, meet at a three-point corner and each seam runs into the face of a flat panel rather than another curved seam. A steeper curve might join more smoothly with the flat panel and reduce the prominence of the corners. I decided that I could not be happy until I had determined whether my curve or this much steeper curve made a better bag.

I constructed templates and sewed a bag using 5cm squares rounded using a 3.5cm curve radius. The result confirmed what I had assumed before: The corners were pulled inward too far and were not only concave, but caused sharp puckering of each edge of the cube between them so that the bag had twelve "corners" instead of eight. It was a cool shape (sort of a stubby, polyhedral star), but not at all what I wanted. I don't know how Peter Billam made this design work, but it does not work for me. With this experience, and looking again at the first cube I made, I think I can see that the same effect is nearly beginning to happen to it. It is not happening, but I think, as I did before, that if the corners were pulled inward any farther, it would be too much. Perhaps with a thin and stretchy fabric the steeper curves would work, but I want a more universally viable design.

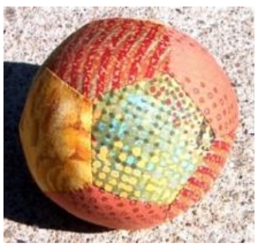
APPENDIX

Fabric ball design ideas

There are many ideas for fabric ball projects on the web. Following are some of the best ideas and patterns I have found. Most or all of these can be found by Googling "fabric ball". These balls are stuffed with soft stuffing, not pellets. Some of these balls use types of polyhedra I do not provide definitions for in this document, but they can be found on the internet.

Child-safe Christmas ornaments by The Shishi Girl from http://shishigirl.blogspot.com/2008/12/basic-geometry-ii.html. This is a six-panel beach ball design. I provide instructions for a four-panel beach ball and an explanation for using other panel multiples (See the "4-Panel Beach Ball Style Instructions" chapter and the "Other Designs and Variations" chapter for more information).

Icosahedron



Dodecahedron

Truncated Octahedron

Icosidodecahedron

Rhombicuboctahedron

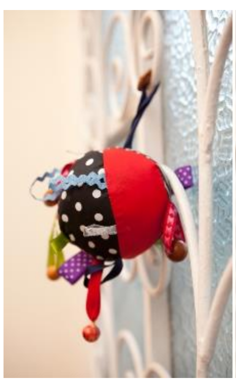
Rhombic triacontahedron

Patchwork Puzzle Balls by Jinny Beyer from http://www.personal.psu.edu/axd2/quilt/qlt36.html. These balls are all three to five inches in diameter and I assume they are intended to be used as children's toys or perhaps as decorative centerpieces. I define the dodecahedron in this document, but not the others.

Appendix

Large, corduroy toy balls by Crafty Panties (inspired by Jinny Beyer's Puzzle Balls) from http://www.craftster.org/forum/index.php?topic=269264.0. These are Icosidodecahedrons, which I do not provide instructions for.

Random fabric scrap balls by Jennifer Murphy from http://andothersillythings.blogspot.com/2011/05/pentagon-colorful-fabric-balls-tutorial.html. These are dodecahedrons, which I define in this document.



Activity Ball for babies by ElmaRi from http://www.lovilee.co.za/2012/diy-activity-ball/.

Appendix

Here are some very unusual polyhedron or geometry-based fabric ball designs.