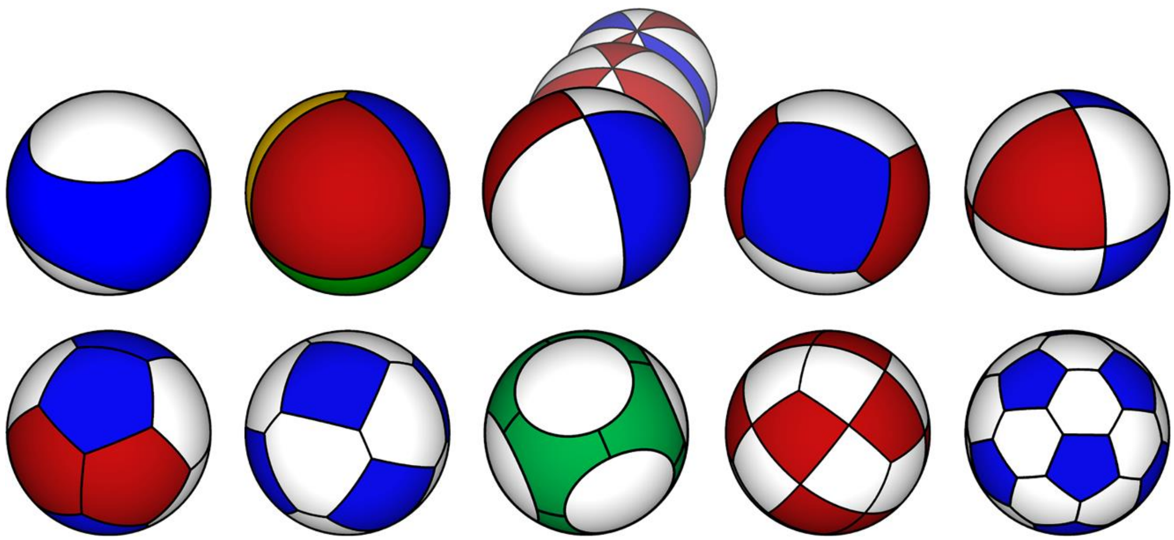
How to Make Spherical, Paneled, Juggling Beanbags (and Footbags)



Technical Instructions and information for the detail-oriented, mathematically-inclined, perfectionist, DIY juggler

By Joshua Clifton

[First Edition, latest draft. First Edition originally published late 2012.] Last edited 9/9/2018

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INTRODUCTION

About me and this document

The primary purpose of this document is to provide definitions, not mere patterns, for spherical beanbag designs so you can create your own patterns in any size you want. (I do include ready-to-print patterns in *Appendix V* in case that's all you want.)

I am not a serious or very skilled juggler, but I have been casually juggling to some extent for as long as I can remember (I am 34). I can juggle up to four balls and I can juggle several two and three-ball variations. I have not learned to juggle any other objects proficiently.

I also have no expertise in sewing, but I learned the basics when I was young and I have a natural propensity for craft and DIY projects. The stitching and assembly techniques I describe are mostly my own, though I have done some research to improve them. I have never made beanbags for sale or done much research on ideal materials. This is just a casual, personal hobby for me. Also, my interest lies more in designing the beanbags than in making them. I find the tracing, cutting, and sewing very tedious, though the anticipation of the product I made myself lends some pleasure to the process.

So, I have only very limited experience and knowledge and do not claim to be an authority on the subject of designing and constructing ideal juggling equipment. However, I am a perfectionist and tend to be obsessive about detail and precision, and I have a fairly good foundation in mathematics and an aptitude for it. My aim in designing my panel shapes is not to find a "good enough" design, but to find the mathematically correct (or optimal) design. That is not to say I have necessarily achieved this; I think I have, but a better mathematician/juggler than me must judge (I define and explain my designs thoroughly in the *How I Developed My Designs* chapter). As for the rest of the document, I put a lot of research, calculation, measurement, and testing into making the instructions as accurate, comprehensive, and well-crafted as I can. I explain the motivations and sources behind my designs, methods, and material choices and I suggest alternatives. If you know more than I do in some areas, my information can serve merely to supplement yours.

Back in the mid-1990s I began designing and making my own juggling beanbags. My original motivation was saving money, but it soon became a hobby (see the *How I Developed My Designs* chapter under 4-panel beach ball). Around 1998 I wrote the first drafts of this document, which was only for personal use so I could record what I had learned. At that time I had only developed the four-panel beach ball and dodecahedron designs, and the latest draft from that period is eight pages long. Over the several years of this phase of the hobby I made a total of twenty-six beanbags (not including those I only partially finished or discarded as failed designs). Then for a decade or so I moved on to other hobbies.

In August, 2012 I became interested in writing a more formal and detailed instructional document to post on my personal website (which I only give out to people I know). I worked on the formal draft intensively for over a month before publishing it, and I continued editing it long after that, adding many new chapters. My research for it led me to develop the rest of the designs in this document – the cube and octahedron in that first month and the others as much as a year and a half later. I became fascinated by the math and concepts involved in designing fabric spheres.

I found during my research that there seems to be no document like this one anywhere on the web or in book form. Some people offer stitching patterns for spherical juggling bags and a few give some information on materials and techniques and even step-by-step instructions for assembling them, but none of them (except The Shishi Girl, but her fabric balls are not for juggling - See *Appendix I*) define the shapes so the bags can be altered or the designs improved, and nobody I know of has a generalized and in-depth instructional document.

Since writing this document I have found several tutorials that are pretty good. I have listed them below. But they still don't define the panel shapes beyond Llama Nerds saying on the second page of his tutorial that the octahedron uses equilateral triangles with rounded corners (which is technically incorrect as it is the edges, not the corners, that are rounded). The closest to this kind of article I knew of during the original writing of this document was Peter Billam's article on leather balls which has some useful information and gave me a lot of ideas, but is pretty brief. It may be that nobody is enthusiastic enough about making juggling beanbags to care about the kind of detail I have included. Frankly, I don't know why I am myself. But this document was fun to write and I gained a lot of education in the process, and it has given me a hobby when I very much needed one, so at least I benefited.

Since writing the above paragraph I have found that there is a lot of enthusiasm for making footbags and there are forum departments, websites, and YouTube videos dedicated to the craft. (This motivated me to add what little I know about footbags to this document to make it useful to a wider audience.) It is still hard to find any definitions of the designs, though. There are two online 32-panel pattern generators that I have found[1], one of which is no longer available, but no formulas or geometry. I have seen several people asking in forums how to design the panels for a footbag or how to figure out the pattern size needed to produce the bag size they want, and nobody seems to have good answers for these questions. I believe my math and design ideas are what they need. It may be, though, that not many would be able to make use of my formulas and would need something automated like the pattern generators. My tables of pre-calculated pattern measurements will hopefully be of use to those people.

Note: The biographical details I include throughout the document are for my own purposes as this document doubles as a personal journal of my experience.

Spherical beanbag tutorials I have found

- "**Basic Geometry II**" by the Shishi Girl -- http://shishigirl.blogspot.com/2008/12/basic-geometry-ii.html
- "**How to Make Leather Juggling Balls**" by Peter Billam -- http://www.pjb.com.au/jug/leatherballs.html
- "**Custom 'Octahedral' Juggling Bags (or hacky sacks)**" by "Llama Nerds" -- http://www.instructables.com/id/Custom-Octohedral-Juggling-Bags-or-hacky-sacks/
- "**Barnsey bags how-to**" by Dave Barnes -- http://www.2diabolo.net/index.html%3Fpage=14.html
- "**Making Round Beanbags**" from the Coulee Region Jugglers and Unicyclists -- http://www.jugglingpoet.com/crju/beanbag.html
- "**Sew Your Own Footbags**" by Daniel Botkin(?) -- http://valinet.com/~dbotkin/sew.html

[1] http://patterns.mhansen.org (no longer available) and http://commo.xtreemhost.com/

- "**Umbrella Footbags**" by John Beckerman -- http://umbrellabags.wordpress.com/stitching-tutorial/
- "**How to Stitch a 32 Panel Footbag**" by Gary Gargan -- https://www.youtube.com/watch?v=elZXAJcJDFw

Juggling bags versus footbags

This document is focused on making juggling beanbags as I am not much into footbagging, but all of the panel structures can be used for footbags (a.k.a., Hacky Sacks). Since the primary goal of my designs is to make the bags as spherical as possible, they are in that respect optimized for accurate kicking. All the sizing formulas also apply to footbags, though if you gather the seams as seems to be the usual practice, you will have to figure out how much to increase the template size so as to get your desired finished size. For a method of doing this and for information on sizing and measuring your beanbags, read the *General Notes and Techniques* chapter under "Calculating your pattern size".

I have only done a moderate amount of research on the craft of making paneled footbags, but from what I have learned, the differences between footbags and juggling bags are that footbags are usually smaller (typically 1.5 - 2" in diameter according to my research), are under-filled for easier stalling (catching and holding on part of the body) and probably to make them more forgiving of bad kicks and uneven shoe surfaces, often have more panels to make them rounder and give them a denser, more supportive seam structure (32 panels appears to be the most common and is the freestyling standard according to Wikipedia[2]), and often (maybe always) have gathered seams which I assume makes them more springy.

If you want to use my designs to make footbags and want an enthusiast-grade result, I recommend that you research the craft first to learn how to choose your size, shell material, panel count, stitching technique, filler type, amount of fill, weight, and any other important attributes. Wikipedia is one good place to start: http://en.wikipedia.org/wiki/Footbag, and here is a blog about footbags: http://hackysackblog.com/. There are also many tutorials (text and video) and forums on the craft.

If all you want is a small beanbag to kick, my techniques and materials will probably work perfectly well. You'll just want to make the bag smaller (my Sipa Sipa crocheted footbags are about 2" in diameter), under-fill the bag (35 - 75% full according to my research) to make it stallable and more forgiving, and I recommend using one of the higher panel count structures for better roundness. The octahedron (8 panels) and dodecahedron (12 panels) are good choices unless you are ambitious enough to go all the way to 32 panels. From what I have read, more panels yield a truer and more responsive kick due to being more spherical and rigid, but these qualities also make the bag harder to stall as it rolls more easily. Using a thin, flexible fabric (Ultrasuede Light is the most popular material) and a heavy filler should help make the bag floppy enough to stall. See the *General Notes and Techniques* chapter under Filler and beanbag weight for filler ideas.

My step-by-step instructions for drawing the panel shapes result in patterns for tennis ball-sized bags, but I provide formulas in each chapter for calculating the pattern size needed to produce any beanbag diameter, and I provide tables of pattern measurements for sizes from 2" to 3". Again, if you gather the seams, you'll have to use the formulas and substitute your own adjustment factor.

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[2] http://en.wikipedia.org/wiki/Footbag

Importance of a spherical beanbag

A spherical shape is important in any juggling object so that no matter how it lands in your hand you can grip it the same. Small adjustments of grip that a non-spherical shape requires each time you catch it will make juggling more difficult and error-prone. This is even more important for footbags because you need to be able to kick them accurately. All of the primary designs in this document (those having instructional chapters devoted to them) produce approximately spherical bags, but the beach ball, with only four panels, has a hint of a cubic feel when made with something stiff like denim (its seams are circular, though, and there are no corners). The baseball can also be a bit angular in places and the spherical cube may give a subtle impression of corners and a cubic feel before it’s broken in, but my denim cube became spherical after a while. Filling the bags loosely with a heavy filler, which is usually done for footbags, will greatly reduce the effects of the angularity and will probably make even the beach ball work decently well for footbagging.

A spherical beanbag requires more skill and knowledge to design and, in most cases, more time and effort to make than the classic, flat, corn hole-style beanbag or even a simple polyhedral beanbag such as a cube or pyramid, but it will be easier to juggle or kick, feel more pleasant in your hand, and look more elegant. Below are examples of popular, non-spherical designs for beanbags. The first design uses one rectangular panel folded in half or two squares, the pyramids are made with two squares, and the cubes use two rectangular panels (cubes are also often made with one long rectangle and two squares). Very simple designs, but not as good for juggling and almost useless for footbagging.

Images from (respectively): http://www.instructables.com/id/Beginner-Juggle-Bags/, http://www.things-to-make-and-do.co.uk/fabric-and-sewing/how-to-make-a-juggling-bean-bag/how-to-make-a-juggling-bean-bag.html, http://www.etsy.com/listing/119719278/juggling-balls?ref=market

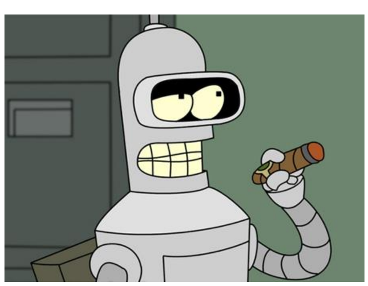
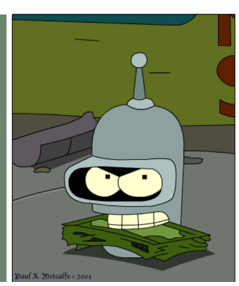
Juggling with beanbags versus balls

The benefits of juggling with beanbags over rigid balls, aside from the fact that you can make them yourself and customize them any way you want, are that they do not bounce away when you drop them, they roll much less than a ball, they are easier to catch, and they can be caught more easily on other parts of the body similarly to a footbag. They are recommended for beginners because of the relative ease of catching them. Russian balls have the dead fall and anti-roll characteristics of beanbags, but because they are partially filled with heavy filler, they will wobble in the air if you give them any spin. There are various other types of rigid balls with differing characteristics. Wikipedia has information on some of them and their relative benefits (http://en.wikipedia.org/wiki/Juggling_ball). Your juggling practices, skill level, and personality will, of course, determine which types you prefer and I do not claim that beanbags are superior to other type of balls. They just happen to be the type in which I am most interested.

Other uses for fabric balls

Fabric balls can be used for more than juggling or footbagging. Here are some ideas:

- Indoor games of catch or general indoor throwing (consider a lighter-weight filling for less danger to lamps and other household items)
- Swimming pool splash balls. Fill any fabric bag with an absorbent, sponge material and you have a watery, splattery, harmless ball for throwing in a pool.
- Grip strengthening or stress reliever balls. If you want the ball to squash, use a stretchy fabric and a small, smooth, fluid filler such as millet or something similar.
- Indoor kickballs (when made soccer ball size and stuffed with soft stuffing)
- Ye olde dunking booth ammunition. The Maryland Renaissance Festival's dunking booth uses softball-sized, leather, sand-filled, cube beanbags.
- Baby toys (best when made larger and stuffed with something soft, with perhaps a bell or rattle inside; you might also consider quilting contrasting figures or letters onto each panel or attaching beads, bells, and miniature stuffed figures by ribbons). See *Appendix III* for an example of this.
- Unbreakable Christmas ornaments (made with soft stuffing, colorful trim, and a loop for hanging – see *Appendix III* for an example of this)
- Decorative centerpieces (made with elegant solids or prints, or with leather – see Appendix III for examples of this)
- Throw-pillows (made large and filled with fiberfill)
- Beanbag chairs (by making them very large)
- The top of Bender's head (see below), or any other spherical or hemispherical shape such as a hat. I once wanted to build a costume of Bender the robot from Futurama using mainly grey vinyl and foam rubber. I realized that I could form the domed top of his head out of the vinyl using either beach ball panels laid horizontally or four circular triangular panels to form a hemisphere and then stuff it with fiberfill to round it out. The rest of his head would be a cylinder with the facial features cut into it. I have not yet made the costume, but it'll be great if I do. It will have a working chest cabinet that holds two bottles of beer and a cigar (if my belly is lean enough to make room for them).

The designs: Discussion and comparison

I designed all the panel shapes in this document myself (except for the regular polygons, of course), but the panel structures are not my own. I based my designs on panel structures commonly used by manufacturers. I picked designs I liked and figured out how to emulate them. In the *How I Developed My Designs* chapter I provide in-depth explanations of the math, methods, and motivations behind my panel shapes so you know where they came from and can improve them if you know more than I do.

All but the second design pictured at the beginning of this document, the spherical tetrahedron, (and the additional beach ball panel multiples) are what I call "primary" designs, meaning that I wrote full instructional chapters for them. I provide the mathematical definition of the spherical tetrahedron as well as a lot of information about it, but not full instructions for making it. It is described in the *Other Designs and Variations* chapter. I would have given it its own chapter except that it is inferior to the beach ball design in roundness, symmetry, and color arrangement options, but has the same number of panels. I consider it to be merely a novelty. An explanation of how to design beach balls with other panel multiples is in the first section of the *Other Designs and Variations* chapter.

The first four of the five Platonic solids are represented in this document. The five Platonic solids are the tetrahedron, cube, octahedron, dodecahedron, and icosahedron. The cuboctahedron and icosidodecahedron, which are the bases of my 14 and 32-panel designs, are Archimedean solids. My deltoidal icositetrahedron (24 panels) is based on a Catalan solid (a dual of an Archimedean solid). For information on Platonic, Archimedean, and Catalan solids, look them up in Wikipedia. They and the relationships between them are fascinating if you enjoy that sort of thing.

My beach ball design, aside from being a good design on its own, serves as a foundation for my tetrahedron, cube, and octahedron designs. This is because I use circularly curved faces for those designs to produce better spheres and the beach ball concept enabled me to design the curves. See the *How I Developed My Designs* chapter to learn more about this. Incidentally, all designs in this document that have curves use circular curves (as opposed to elliptical or function based) and so can be easily drawn with a compass or a computer application such as SketchUp.

Following is a comparison of the designs to help you decide which one to make. In the *General Notes and Techniques* chapter there is section on Roundness and uniformity which may also be helpful. Each instructional chapter also begins with a brief discussion of the design and color arrangement ideas. One thing to consider is that the beach ball and the octahedron (and, to a lesser extent, the icositetrahedron) are simpler to sew and require fewer threads than the other designs because they have 4-way vertices as opposed to 3-way. What I mean is that four seams come together at each intersection which means that when you sew into a corner you can continue away from it on one of the three remaining seams, circle around a panel, return to the same corner on the third seam, and then sew the remaining seam and proceed to further panels. You could theoretically sew an entire bag with one (very long) thread without overlapping any stitching. At a 3-way corner, if you circle back to complete the corner, you run into a dead end. You've now sewn all three seams. You either have to re-stitch a seam to proceed around the ball, or you have to tie off the thread and start a new one.

(The length of stitching given for each design applies to the tennis ball size that my unaltered patterns produce.)

2-Panel Baseball - 40.5cm (15.9") of stitching

Best for those who like the baseball or yin-yang look. It has the second least spherical shape after the beach ball. It's mostly spherical and feels a bit better in the hand than the beach ball because of the compound curvature and greater uniformity of the seam, but at the profile shown on the bottom right (and only at that profile) it is a bit squarish, at least before being broken in. Consider using a flexible, stretchy fabric or filling it loosely for better sphericity (I used denim which is stiff and nearly non-stretch). The panel design is significantly more complicated and the bag more difficult to make than the beach ball, so I don't recommend it for beginners.

4-Panel Beach Ball - 44.5cm (17.5") of stitching

Best for those who want a decently spherical beanbag with minimal time and effort. It is the least round and elegant of the designs, but very simple and quick to make. It feels a bit more cubic in my hand than the baseball because of the strictly longitudinal seams. As with the baseball, consider using a stretchy fabric or filling it loosely for better sphericity. Remember that you can get the aesthetics of the beach ball with more roundness by making it with more panels. See the *Other Designs and Variations* chapter under [How to design beach balls with any number of panels].

6-Panel Spherical Cube - 52.0cm (20.5") of stitching

With certain color arrangements this design is reminiscent (to me) of those cubic, wooden alphabet blocks for children and therefore has a playful, childish aesthetic. Aside from that, it is good for those who enjoy the simplicity and smoothness of minimal seams but with good roundness and uniformity, or the visual aesthetic of the cube. I like the look and feel of this design a lot. It is rounder and more uniform than the beach ball and feels better in my hand.

8-Panel Spherical Octahedron - 62.2cm (24.5") of stitching

Best for those who want a high degree of roundness, seam uniformity, and elegance, and a wide variety of color arrangement possibilities, all with minimal effort. This design supports all the color arrangements of the 4-panel beach ball as well as many unique arrangements, and is fairly easy to make.

12-Panel Dodecahedron - 75cm (29.5") of stitching plus 5cm (2") of overlap if you use my assembly method

Best for those who value an even higher degree of roundness and seam uniformity and don't mind putting in the extra work. This is arguably the smoothest of the high panel count designs as it is reasonably easy to iron the seams, making them flatter, and has short and straight seams. The cuboctahedron, icositetrahedron, and especially the icosidodecahedron are harder to iron because they have so many and such crowded seams. The octahedron has longer and circular seams which tend to make the seam allowances pucker a bit (at least with denim) because of their curvature around the bag, causing the seams to be less smoothly round. It's not at all severe, at least not if you iron the seams flat as I recommend, and it will probably be less of a problem with a thinner or more flexible fabric, but if you like a very smooth surface on your bag, you are likely to prefer the dodecahedron. The dodecahedron also has many interesting color arrangement possibilities.

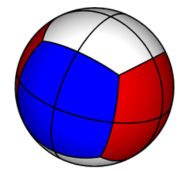
14-Panel Equidistant Cuboctahedron - 84cm (33.1") of stitching plus 3.3cm (1.3") of overlap if you use my assembly method

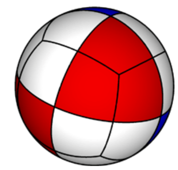
Roughly tied with the dodecahedron for roundness and uniformity, but with a very different, and more complex, visual aesthetic. Best for those who value dazzling visual beauty or enjoy a checkered or soccer ball-like color pattern. This design is more complicated and difficult to make than the dodecahedron due to having two different panel shapes, two different edge lengths, and two more panels.

14-Panel Polka Dot Bag

This is just a variation of the equidistant cuboctahedron. It is described in that chapter. I like its resemblance to a Super Mario Bros. mushroom. The large, soft, round pads separated by firmer, narrow, slightly recessed grooves give it a fun tactile nature and a good grip. It is much more difficult and tedious to assemble, though, because of the convex and concave curves connecting to each other. The tight curvature of the seams also made my denim bag very lumpy until I worked the seam allowances into a smooth shape, which took a long time. In the section on this design I offer a suggestion for an easier version of this concept.

24-Panel Deltoidal Icositetrahedron - 113.1cm (44.5") of stitching if you don't overlap any seams





One benefit of this design is that it is essentially a cube or an octahedron (or a 4-panel beach ball) but with more panels as shown on the right. If you like the look of those designs but want a denser, more uniform seam structure or more roundness, this is the design for you. It also supports many unique arrangements that look very attractive. While this design has a relatively large number of panels, it has an advantage over the 14 and 32-panel designs in that it uses only one panel shape.

32-Panel Equidistant Icosidodecahedron - 127.9cm (50.4") of stitching if you don't overlap

The main benefit of this design is that it supports many creative color patterns. People have made this design look like an eyeball, a Poké Ball, a ladybug, an ice cream sundae, a spiral rainbow, tessellations of Shuriken, and other objects, characters, and patterns (see the instructional chapter for examples). I also like the tactile texture of the dense network of 90 seams. I did not iron the seams, so they retained a prominent texture.

For an under-filled bag, this design will be somewhat more spherical than the others due to the large number of roundish panels and its dense, supportive seam structure. When tightly filled, the 12 through 32-panel designs are all about equally spherical due to fabric's ability to stretch and distort.

Like the cuboctahedron, this design uses two different panel shapes, but it is more complicated and difficult due to having a pentagon in place of the square and over twice the panels, so it is an ambitious project. I recommend using a fairly thin fabric that does not fray such as

Ultrasuede. I made mine with denim and a 4mm seam allowance and it was a bit of a frizzy mess to sew.

US to metric conversion note

I originally made my four and twelve-panel juggling bags (which are my earliest designs) using inch measurements because I live in the USA and I think in terms of inches. The essays for those two designs in the *How I Developed My Designs* chapter, which I first drafted around that time, use those measurements, and I still prefer to define beanbag sizes in terms of inches. But during the process of writing these formal instructions in August, 2012, while I was still drawing my patterns manually, I realized that using metric measurements for the patterns would be much easier since I often have to make calculations on a calculator resulting in decimal values which I can't easily measure out using a ruler with powers-of-two increments ($^1/_2$, $^1/_4$, $^1/_8$, $^1/_{16}$). Also, millimeters give me smaller increments to work with than sixteenths of an inch. So, I converted all the measurements into the metric system. Over the course of writing the instructions and doing further calculations and more accurate measurements, I adjusted the sizes a bit so that all the bags, when inflated, will be as close to $2^5/_8$" (66.675mm) in diameter as possible.

If you need to convert the measurements back to inches, remember that 1 inch = 2.54 centimeters.

GENERAL NOTES AND TECHNIQUES

Prerequisites and fundamentals

Sewing skills

This document assumes you have at least some basic experience with sewing, and hand-sewing in particular (I neither use a sewing machine nor describe any techniques for one). I describe stitching techniques and use terminology in a way that someone with no experience at all may not be able to follow. I leave some techniques, such as how best to join a third panel to two you're sewing together, up to the reader to know or figure out. If you have never sewn anything before, I recommend you find a tutorial and sew some simpler projects first. I describe and use the "backstitch" because of its strength and the smooth seams it produces, so that is a stitch you should practice unless you prefer a different one.

Note that I disregard the concept of bias in woven fabrics. I have little knowledge of how to make use of different ways of aligning my patterns with the fabric weave and what affect they might have on the shape and stretch of the beanbags. This may be something worth looking into, but it is probably is not very important for this application.

Mathematics and tools

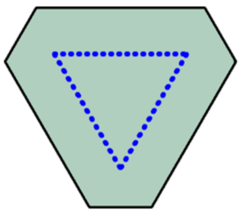
There is a lot of math in this document and while I try to accommodate readers who lack an aptitude for it, I may not have achieved this fully. However, you will only have to do math if you alter the size of the beanbags beyond the 2-3" diameter range I provide in my tables of pre-calculated template measurements in each alteration section, or want to use a different adjustment factor than I use. If you only want to make the tennis ball size I give instructions for, you need no math at all as long as you are familiar with terms like radius, arc, and equilateral. In the alteration instructions, particularly in the background explanation portions, though I do all the advanced calculations for you, I use notations, terminology, and mathematical syntax that may be difficult to understand by someone who hasn't at least some basic experience with algebra and trigonometry.

Depending on the design you choose, you will have to use a compass or a protractor to draw the templates by hand, and I assume that you know how to use these tools. I also provide some information and tips about using a computer application called SketchUp for drawing the templates. This is optional, but I do assume computer literacy and ability to learn a new application as I do not provide full directions for using the application.

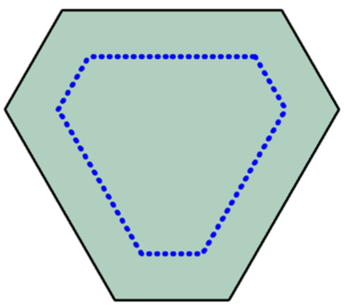
Stitching patterns versus cutting patterns

It is important to understand that, unless I say otherwise, when I talk about patterns in this document I am referring to *stitching* patterns as opposed to *cutting* patterns. If you like to use only a cutting pattern and then stitch just inside the edges of the panels rather than transfer a separate stitching pattern, make sure you understand the difference and why the distinction is important, and have accounted for it in your pattern calculations. My instructions explain how to draw the cutting patterns, which sometimes have different proportions from the stitching patterns in order to maintain a constant width between the two and result in a stitching pattern having the correct shape. Be sure to use those directions so that the shape of your stitching pathway will be what the stitching pattern would have been.

The stitching pattern is what determines the size and shape of the bag and so it is crucial to draw it correctly. The cutting pattern can be anything that provides enough seam allowance and it has no affect on the size or shape of the bag. It doesn't even have to be the same shape as the stitching pattern (though it is helpful for it to be); it could be just a blob. In the end the seam allowance will be merely excess fabric hidden within the bag.



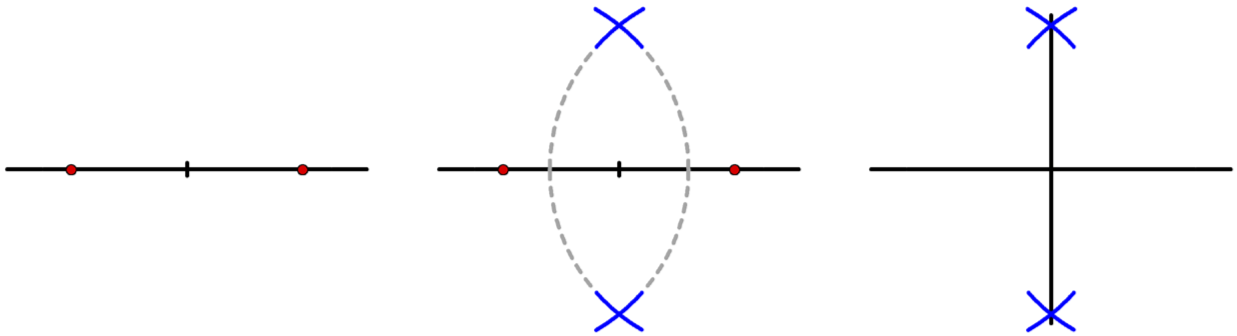
Creating a stitching pattern and using it as a cutting pattern will not only result in a smaller than expected bag size, but can change the bag's shape and appearance. This is because by subtracting an equal seam allowance from all sides of the panel, you change the shape and proportions of the panel (in some cases). The image on the left gives an exaggerated illustration of this from the hexagonal panel of the 14-panel bag. Because the stitching (the blue, dotted line) is inside the hexagonal pattern instead of along it, the panel has effectively been transformed into a triangle.



My cutting pattern calculations are not only larger than the stitching patterns, but change the proportions of the panel (when necessary) so that the stitching pattern, which is parallel to it, will be the correct shape. In the case of the hex panel, the cutting pattern (the outer border in the illustration on the left) would have short sides that are longer in relation to the long sides. The short sides on the stitching pattern are 36.6% the length of the long sides which produces the bag shape I defined, but in the cutting pattern (in my example) they are 44%.

How to draw perpendicular lines without a protractor

If you don't have a protractor to produce right angles, you can use a compass instead. Below are illustrations for this method. Simply mark two points (the red dots in the illustrations) at equal distances on either side of the point at which you want your perpendicular line, extend the compass to any radius greater than that distance, place it at each point in turn, and draw partial arcs to form two Xs that mark perpendicular points above and below the center. Then draw your second line through the centers of the Xs. You'll get greater precision the farther out you mark the compass points and the larger you make the radius as this will move the Xs farther apart. Also, try to use a radius that makes the resultant arcs of the Xs approximately 90° to each other as in the illustration. This will give you the most precisely visible crossing point.



Beanbag size

My pattern drawing instructions for each design are for tennis ball sized bags (diameter ≈ 2 $^{5}/_{8}$" or 66.675mm) when made with denim and filled firmly. This size fits my large hands the best for juggling up to four bags (the maximum I can juggle) with a combination of standard catches and "clawing" (catching with palms down, which is easier with smaller objects). Each chapter also includes instructions for altering the size. 2 $^{5}/_{8}$" and $^{1}/_{8}$" on either side of that are the most common sizes for juggling beanbags from what I have seen at stores, but some people prefer much smaller sizes for juggling large numbers of balls or for a better fit in small hands, and some like to use extra large sizes for stage performances. The Juggling Store's website says that their research has determined 2.3" to be "ideal for those with smaller hands and a great choice for numbers juggling"[3]. For reference, a racquet ball is 2.25".

I made a set of three 2" bags using the octahedron design to carry in my pocket when I feel the desire. I filled them loosely and increased their weight to 75g using metal BBs. This size and weight is very jugglable and yet fits in my jeans pocket. Filled more loosely still, it would also work well as a footbag. I describe how to make this size in the octahedron chapter in the section on altering the size of the bag.

As for footbags, most of Freedom Footbags' products are 1.5" in diameter while most of Flying Clipper's bags are 1.75" or 2". Bomb Footbags says that "most footbags are about 2.25 inches"[4]. My Sipa-Sipa crocheted footbags are 2". For reference, a golf ball is about 1.68".

Size will be affected by the type of fabric, how firmly you fill the bags, and your sewing practices. If you use a material that has a different stretch from denim or do not fill the bags as firmly as I do, your bags may end up being a noticeably different size than I claim they will be and so you may want to adjust the template size a bit. As for sewing, if you use a thick marker for the pattern, it is important to sew on the side of the stitching lines where the edges of the template were. Also, I pull stitches very tight. Loose stitches or stitching too far from the template edges may change the size measurably, especially in the designs with high panel counts. Gathering the seams, which is common for footbags, will (I think) reduce the size drastically.

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[3] http://www.jugglingstore.com/fusion-beanbagfusion-beanbag-782.html

[4] http://bombfootbags.com/picking_a_footbag.html

You may need to do some trial and error to determine the best fit for your hands and juggling or footbagging practices. It's a good idea in any case to assume your first bag is a practice bag. If you are as much a perfectionist as I am, you'll find that it takes some experience to know how to make these things right.

Calculating your pattern size

Important: Remember that I am talking about *stitching* patterns as opposed to *cutting* patterns. If you do not know why this distinction is important, see the "Prerequisites" section at the beginning of this chapter.

To figure out the pattern size needed to produce the bag size you want, first construct a mathematical formula that will allow you to plug in a desired bag size and will return one of the dimensions of your pattern. This is very simple for some designs and very complicated for others. Each of my instructional chapters has such a formula (sometimes several) and a full explanation of it so you can easily calculate your pattern size.

Second, you need an adjustment factor. The formula by itself only expresses the polyhedron size as if you were going to place the stitching patterns edge to edge and construct a rigid model. The actual stitched and filled fabric bag will probably be significantly larger or smaller than this, especially if you gather the seams. My denim bags are larger than the template calculations by 0.8-9.5% (depending on the design and how firmly I fill it), except for the 24 and 32-panel designs which end up *smaller* by 2.24-4% unless I fill them tightly. There are three reasons that I can see for this varying change in size. First, denim stretches somewhat. Second, the bags are polyhedrons, some of which are modified to have circular edges, and are distorted into approximate spheres when they are filled; the resulting spherical circumferences are unpredictable (at least by me). Third, denim is thick and so the folding at the seams will consume some of the panels' width. This is probably why the 24 and 32-panel bags become smaller: they have a lot of seams.

My instructions give the inflation (or deflation) in size I experienced with each design using my denim and stitching practices. The inflation/deflation is important to account for in a juggling bag because a mere 5% amounts to one full centimeter (0.4") of circumference in a tennis ball sized bag, which makes a significant difference in how well your hand can wrap around it.

To figure out your adjustment factor, first make your best guess at the adjustment based on the one I provide for the design you're making and substitute it for mine in the sizing formula (or just use mine). Then create your templates and calculate the size of the polyhedron they would produce (I explain how to do this in each chapter). Make the bag, measure it, and calculate the percent change in circumference between the pattern calculation and the actual bag. This is your adjustment factor. From then on, replace your adjustment factor guess with the real one and you should get a fairly predictable finished size for all future bags of any size.

Take, for instance, the octahedron design (8 panels). This pattern is drawn using a guide triangle and the formula for calculating the triangle's side length is as follows (d is the target bag diameter; the octahedron chapter has a full explanation of this formula and the purpose of each number). The value in red is my adjustment factor (my measurements indicate that my denim bags inflate by 7.25% when I fill them which is 1.0725× the polyhedron size).

Guide Triangle Side Length $= \mathrm{d} \times \pi \div 4 \times 1.5775 \div \mathbf{1.0725}$

Replace the adjustment factor with your own guess. An inflation factor is greater than 1 and a deflation factor is less than 1. When you have done this and drawn your pattern, you need to calculate the polyhedron circumference based on that pattern. I state in the octahedron chapter that, “The circumference of the bag (measured between the seams) is 4 × panel height.” So, measure or calculate your panel height (corner to side) and multiply that by 4 to get the polyhedron size. When you have made your bag and measured it, divide the bag size by the polyhedron size to get your adjustment factor.

The easiest way to measure your bag’s finished size (if you don’t have a dressmaker’s tape measure) is to wrap a strip of paper around it, mark the meeting point, measure the strip up to the mark, and then divide by pi (3.1416) if you want the diameter. I made a reusable paper measuring tape by marking a small range of measurements on a narrow strip of paper. To help ensure a correct measurement, measure the bag a few times in different directions and calculate an average circumference. Note that the lower panel count structures have different circumferences depending on how they are measured. The beach ball design is 15.9% larger, mathematically, along its seams than between them. This evens out a lot when the bag is filled and stretched, but my bag still measured 3.5% larger along the seams. I ignore this in sizing my beanbags and I measure only between the seams, but you might want to adjust for the larger seam circumference.

Filler and beanbag weight

Peter Billam, in his web article *How to Make Leather Juggling Balls*[5] says the following about juggling bag filler:

> For filling, birdseed (unhulled) millet is commonly used, though the seeds slowly crumble making the ball gradually softer; also, some people are allergic to millet. Sesame seeds are longer-lasting, but linseed is the best of the seeds. Crushed walnut shell is still more hard-wearing and moisture-resistant; it is sold in pet stores as "bedding" for birds and hamsters, and is also used in place of sand as an abrasive for sand-blasting. Rice is too heavy and is not durable. Plastic pellets are completely moisture-resistant, but I prefer juggling with the feel of the natural materials.

I had a set of juggling bags filled with crushed walnut shell and one of them unintentionally went through the washer. It came out permanently swollen. Natural fillers are also susceptible to mold, and seeds may grow if they get wet. This means you must neither wash them nor drop them into a puddle. Natural fillers do have a pleasant smell, however. Crushed walnut shell is rough and gritty and less fluid than rounded fillers and so produces a stiffer, crunchier feel in a beanbag.

[5] http://www.pjb.com.au/jug/leatherballs.html

To avoid the problems of water damage and crumbling with use, I use an artificial filler called "Poly-Pellets". These are polypropylene (plastic) pellets made for weighted, posed dolls, which I buy in two-pound bags from Jo-Ann Fabrics. Poly-Pellets are smooth, rounded, inexpensive, and have a good feel to them in a beanbag. They are larger than millet (3-4mm at their largest dimension) and so don't have quite as smooth a texture. You can order them in bulk at http://www.qualitypolypellets.com/.

Poly-Pellets produce a somewhat lightweight beanbag compared to the professional beanbags I have encountered. My beanbags at $2^5/_8$" tightly filled weigh, on average, 3.4oz (96g) while the high-end professional bags of the same size I looked up weigh 4.1 - 4.8oz (115 - 136g). Weight is important for any juggling object because it will help the object to settle into your hand when you try to catch it rather than bounce off.

For a slightly heavier bag, you can order high-density polyethylene (HDPE) pellets at http://www.craftpellets.com/. CraftPellets.com reports that their pellets weigh 5.12oz. per cup. My Poly-Pellets weigh 4.87oz. (settled by tapping the cup with my fingers; unsettled they weigh 4.48oz.). This makes the HDPE 5.13% heavier. This agrees with the research I've done on the density of plastics[6]. So, HDPE won't increase your beanbag weight by much.

I liked the extra weight of a set of professional bags I once borrowed. They landed more solidly and securely into my hands and felt less toy-like. Yesterday (5/4/2013) I decided to experiment with a heavier weight in my own bags. I replaced some of the plastic filler in two of my old bags with copper-coated steel air gun BBs I found in a closet, raising their average weight from 99.5g to 133g. This felt a little too heavy, but I juggled with them for a day to see if I would acclimate to them. I didn't, so I lowered the weight of one to 125g and the other to 120g. I tossed each one around for a few minutes and decided that 125g felt slightly too heavy and 120g felt slightly too light. So I adjusted both to 122.5g (4.3oz) and this feels perfect. Remember that I juggle only up to three (and sometimes four) balls. For numbers juggling a lighter and smaller bag would probably be preferable. The Juggling Store's website says their research has determined that 80g for a 2.3" bag is the ideal weight and size for numbers

[6] See http://www.stelray.com/density_val.htm for one of my sources

juggling[7]. I made a set of pocket-sized, 2" bags and weighted them to 75g which makes this small size very jugglable. 80g also worked well, but I didn't feel the need for quite that much weight.

If you like a heavier bag, mixing metal and plastic is one way to get it. Just make sure the metal is evenly distributed so the bag is balanced. My 2" beanbags wobble in the air when I spin them in certain orientations and I don't know if it is because I failed to distribute the BBs well enough, or because they gathered together on their own. Also, I recommend *not* using lead (due to the toxic dust that might leak out) or anything that will rust. Try searching eBay or Amazon for "stainless steel shot" or "stainless steel tumbler media". Pick a type that is very small and has only rounded shot (some tumbler media have a mix of rounded and various angular or pointy shapes). Some footbag stitchers obtain their metal filler by emptying out shotgun shells. One footbag forum contributor recommended cut wire shot as a metal filler and provided a link to a supplier: http://www.pelletsllc.com/CutWireShot/CutWireShotSizes. Another recommended using small electronics screws (for those who like to disassemble devices and have accumulated screws for which they have no use). There may also be heavier plastic pellets available online, but I have been unable to find any so far that are heavy enough.

If you don't mind using organic fillers, seeds and crushed walnut shell may be heavy enough without metallic supplements. I have some crushed walnut shell and it weighs 6.32oz (179.28g) per cup, settled (unsettled it weighed 5.68oz). This is 29.77% heavier than Poly-Pellets which would make my beanbags 4.41oz (125.02g), only slightly over the weight I found to be ideal for me.

Footbags usually weigh 40 – 65g according to Wikipedia[8] and my own research, and the official rules of footbag sports states that the weight must be in the range 20 – 70g[9]. You will probably need metal, sand, or something heavy as poly-pellets will not provide much weight for such a small and under-filled bag. Footbags are commonly filled with sand.

You could make heavyweight juggling bags for upper-body aerobic workouts, perhaps as part of a "joggling" routine, by filling them primarily or purely with metal shot or sand. Make sure you use durable fabric, thread, and stitching if you use a heavy filler. I have not tried this, so I can't say much about it. If you want to make an exercise ball, consider using an actual ball rather than a bag for better durability. You can just cut a small slit in a street hockey ball, tennis ball, racquet ball or similar and fill it with metal shot, sand, pebbles, or even pennies and then glue it shut. To add color and traction to a tennis ball, I saw advice to stretch a balloon over it.

I found an idea on the internet which is to fill the bags with something lightweight so that they can be thrown around in the house by children and not break anything. One woman filled hers with shredded fabric scraps from her sewing projects. You could also use cotton or polyester pillow stuffing (a. k. a., fiberfill). For a more jugglable weight that will still reduce the risk of overturned lamps, you could fill the bags with a mix of fiberfill and pellets. Some women add rattles or bells inside the bags for babies. They often make the fabric balls larger than juggling size – sometimes as large as 9 inches in diameter.

Flying Clipper, "The world's leading designer of high-quality handmade footbags & juggle balls", uses two isolated fillers for their juggling bags: a heavy filler (crushed rock) between two layers of shell material in each panel and a lighter filler (plastic pellets) within the bag itself. According to Flying Clipper, concentrating the weight in the outside of the ball yields more accurate flight[10]. Filling a bag

[7] http://www.jugglingstore.com/fusion-beanbagfusion-beanbag-782.html
[8] http://en.wikipedia.org/wiki/Footbag
[9] http://www.footbag.org/rules/chapter/100
[10] http://www.flyingclipper.com/home/fly/cpage_129/what_is_a_hybrid.html

with fiber fill but using a heavy filler in the shell is another way to make a ball soft for indoor throwing by children and yet heavy enough to juggle. It would take a lot of extra work, though. I recommend using interfacing material or something else thin and inexpensive for the inside layer.

Roundness and uniformity

As I discussed in the Introduction, the roundness of a beanbag is important for making juggling easier and more successful. All of the primary designs in this document (those having instructional chapters devoted to them) produce approximately spherical bags, but the beach ball, having only four simple panels, has a hint of a cubic feel when made with something thick and stiff like denim. The spherical cube may feel a bit cubic at first, but mine rounded out nicely when I broke it in. The baseball (2 panels) also feels a bit non-spherical in places. While this slight angularity is not enough to significantly affect jugglability, for me it makes the four-panel beach ball design in particular less appealing because I like the look and feel of a perfect sphere. This is something to consider when choosing a design to make.

Uniformity, meaning a dense, consistently-spaced network of seams and panel faces that makes the surface of the bag look and feel the same from any angle, is, to me, important mostly for tactile aesthetics and somewhat for visual. This is less of an issue with thin or very flexible fabrics because the seams can't be felt as much. I use denim which makes the seams very prominent.

The more panels a bag has, the more spherical it will be and, in most cases, the more uniform it will feel. So, the two-panel baseball and the four-panel beach ball designs are the least round and least uniform of the primary designs in this document. The beach ball's seams all run in the same direction and meet at two "poles", and, having only four panels, it will feel a bit angular around the equator (using a stretchy fabric and filling the bag firmly will probably improve its roundness, and so will filling it loosely so that your grip around the bag reshapes it into a rounded shape). Its advantage is that it is quick and simple to make. A beach ball with more panels is more spherical, but still not very uniform. It looks and feels different at the poles than around the equator. The baseball is also a bit angular, but the compound curvature of the seam conceals this somewhat and gives a better impression of a sphere. The spherical octahedron (eight panels) is much more spherical and feels more uniform because of the greater number of panels and the addition of a latitudinal seam around what was the equator of the beach ball. The spherical cube, with six panels and a combination of horizontal and vertical seams, has a roundness and uniformity about in the middle between the beach ball and the octahedron. It is a very good design and I like the feel and look of it. The high panel count structures have dense networks of seams and are more spherical and uniform, but they are also more difficult and tedious to make. My overall favorite design is the octahedron. It has a high degree of roundness, uniformity, and beauty, but is fairly quick and easy to make.

My cube and octahedron designs (and tetrahedron, but that is a secondary design) use faces having circularly curved edges which improve their roundness over true polyhedral equivalents. The rest of the polyhedral designs do not use circular faces. This is partly because I do not know how to design the curves, but also because I don't think they really need curves since their panels are so small and their seams so short, and the beanbags already feel very round as they are (at least when firmly filled).

Note that a tightly filled bag can be very round with only a few panels due to stretching while an under-filled bag relies exclusively on the panel layout for its roundness, especially if you're using a stiff fabric. So if you use a stiff fabric and under-fill your bags and are very picky about roundness, choose a design with a high panel count, or perhaps the octahedron whose roundness is aided by its circular edges.

Fabric

I use denim from sacrificed jeans. You can find very colorful jeans in the women's section of a thrift store. (A photo of each of my designs made with this denim is at the beginning of each instructional chapter, and they are all shown together in the *Introduction* chapter under "Discussion and comparison of the designs".) I use this source of denim because the fabric stores I've been to carry a poor selection of the colors I like for juggling bags. Be aware that different jeans use different denims (and some use an elastic variety) and so if you're not careful, each bag in your set, or even each panel, may feel different, be stretchier, or have other differing characteristics. There are probably better fabric choices but I have not done much research or experimentation on that. Denim is soft and durable, has a pleasant texture and an attractive appearance, and is easily and cheaply attainable.

I encountered some professional corduroy juggling bags recently that I liked a lot. The corduroy was similar to the denim I use, but the ridges were more prominent and loosely spaced. I liked the look and feel of it more than the denim. Aside from corduroy, professional manufacturers often use suede leather, Ultrasuede (a synthetic leather), or rubbery-coated elastic fabric (for the cheapies). Footbag stitchers usually use Ultrasuede.

If you use real leather be warned that it may, depending on its thickness, be very difficult to sew with a standard needle and thread and may even require special tools, thread, and stitching techniques. I have not sewn leather, but I suspect that it is tougher than denim to get a needle through. The article *How to Make Leather Juggling Balls* by Peter Billam[11] has patterns that include stitch dots that act as a template for an awl so the stitch holes can be punched out before sewing. The article also recommends waxed thread and leather that is 1 - 2mm thick.

As far as I can find out, Ultrasuede, which is quite expensive ($37-$90/yd. depending on the type as of 2013), can only be purchased on the internet. Stores like Jo-Ann Fabrics and Hancock Fabrics do not sell it. There may be high-end specialty stores that sell it, but I haven't found them yet in my area. A few footbag stitchers have recommended Field's Fabrics as a supplier of Ultrasuede and Ultraleather: http://www.fieldsfabricsonline.com/Ultrasuede-Ultraleather_c_8.html. This store appears to have brick-and-mortar locations only in Michigan, but their web store is very extensive.

Choose a fabric that is long-wearing, has the amount of stretch and thickness you want, has good traction against your hand and a pleasant feel (if it's for juggling), and can take some abuse without bursting or tearing. Also, consider washability of the fabric if that is important to you. Denim produces a firm, rugged bag while an elastic fabric will produce a squashy texture like a stress reliever ball. Consider using interfacing to reinforce or stiffen an overly thin or stretchy fabric. I have no experience with interfacing so I can't say much about it. When selecting colors, take into account the color arrangements possible in the design you intend to make, and ease of visibility for you and, if applicable, your audience.

Some women whose blogs I've read enjoy using random scraps of fabric or old garments to make patchwork balls for babies to play with (they make them larger than juggling size – sometimes as large as nine inches in diameter). I give examples of these designs in *Appendix III*.

Depending on the type of fabric, it may be good to wash and dry new fabric to preshrink it and remove excess dye before making it into beanbags. I have also seen advice to press the fabric after washing it.

[11] http://www.pjb.com.au/jug/leatherballs.html

Some of my colored beanbag panels bled into the adjacent white panels when I wet them and so I washed the denim to (hopefully) prevent that on future bags.

Seam allowance

Seam allowance is the excess fabric beyond the stitching pattern. I use 8mm seam allowances for denim because it frays a lot, and also because I like to iron the seam allowances flat and this is easier with a wider allowance. (Edit: I used a 4mm allowance for the 32-panel bag because I did not intend to iron the seams and a wider allowance would have been more difficult because of the small size and large number of the panels. 4mm was enough allowance to handle the fraying, but I would not want to go any narrower for denim.) If you use a fabric that does not fray, you probably won't need nearly as much seam allowance. For very narrow seam allowances, some people do not use two templates as I do (one for the stitching pattern and another for the cutting pattern). They use only a cutting pattern and then stitch just inside that pattern, judging by eye where the stitching should be. Make sure you use a wide enough seam allowance to prevent the stitches from popping through the edge of the fabric when the bag gets treated roughly.

Filling the bags

I use a small funnel to fill the bags (with pellets). After I have filled the bags loosely, I pack more in until they are tightly filled using the following method: I stick my finger down into the full funnel and stop up the spout and then press the funnel down into the bag to stretch it out. Then I remove my finger and slowly raise the funnel while tapping it or prodding the pellets through the spout with my finger so they fill in the void left behind by the funnel's spout. I repeat this until I have a firm, spherical bag (I don't stretch the fabric to its utter limit, but just enough to make the bag firm). Filling the bags tightly will usually improve their roundness. They will not remain quite as firm forever, but will stretch out and become softer over time (depending on your fabric choice and how tightly you stretch it).

If you are unsure how firm you like your beanbags or how much they will soften over time, you can allow correction by not tying the thread at the end but instead making several stitches along the edge of the last panel to keep the final seam tight and then running the needle straight through the bag and pulling it out the other side, leaving the end of the thread behind inside the bag (I make three passes through the bag for extra length before trimming the thread). This way you can use the bag for a while and, if you decide to add or remove some filling, you can pull the thread back out of the bag, undo the last few stitches, and then open the bag back up. Leaving yourself several inches of thread allows you to open and reclose the bag with ease, and finish it when you have made your final decision on firmness.

To remove filler, knead the bag rapidly with the opening downward. If the filler is reluctant or the seam closes, stick a slender instrument like a pencil tip into the bag and swirl it around upside down which will simultaneously widen the opening and jostle the pellets out. To remove large quantities of filler, insert a funnel between the stitches into the bag, flip it and the bag upside down, and thrust the funnel in and out while kneading the bag.

If you are filling the bags with pellets, seeds, or something similar, you can sew the opening entirely closed before filling. All you have to do is leave the last several stitches loose (or loosen them later) and when you're finished, pull them out part way so that a funnel can be pushed in between them. After filling the bag, you can pull the stitches tight again. I do this by sticking the tip of my automatic pencil or a thick needle under each stitch in turn starting where the stitches begin to be loose and pulling on it to tighten the stitches behind it. I continue this until I reach the end and then pull the thread itself to tighten the last couple of stitches.

You may want to use a scale to ensure that each bag is filled with the same weight of pellets. This depends on whether a consistent weight or a consistent firmness is more important to you (with enough accuracy of sewing and consistency of fabric choice across bags, you should be able to get both). If you use a mix of two types of filler such as plastic and metal, a scale is more important.

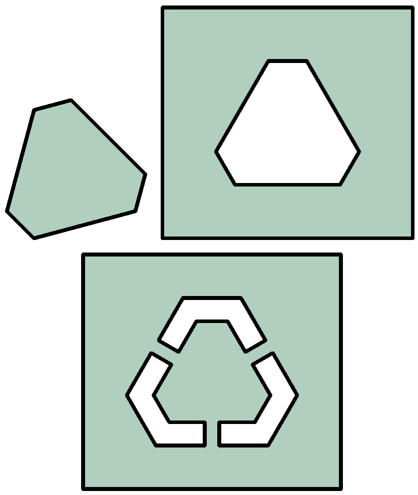
Template material

I do not pin a paper template to the fabric and stitch around it as some people do; I make a rigid template and trace around it with a marking tool. Translucent materials are best because otherwise you must be careful which pattern, cutting or stitching, you trace first so that the second template doesn't hide the lines of the first and prevent you from aligning the two. You can buy template plastic from a craft store. This plastic is translucent, textured for easy marking, flexible, and easy to cut. I used to be able to get a thick template plastic from Jo-Ann Fabrics, but now they only sell a thin variety. I recommend the thick variety if you can find it because it will be easier to trace around and is less flexible which will help keep it flush with the fabric so the pencil doesn't slip underneath it. If you're not concerned about translucency, you can use a hard cardboard (or even index cards or file folders, but they're a bit thin and lack durability). Cardboard is not as durable as plastic for long-term use, though.

Making templates

Precision in drawing and cutting out the template is important. If there is a significant error, you may find that the stitching lines on two panels you are sewing together don't match up, or there may be some imperfections in the shape of the bag, and the bag may be slightly larger or smaller than you expected (this is especially true for the designs with high panel counts because any error is compounded several times). I recently learned of a free computer application called SketchUp which will allow you to draw and print the templates with perfect precision. I discuss it in the next chapter. I recommend this if you are computer literate. Otherwise, it's ruler, pencil, compass, and protractor.

If you draw the templates by hand, I recommend that you draw them on paper first and then use adhesive tape (double-sided tape works best) to attach them to the template material. That way, if you make a mistake, you don't waste your template material or have to erase. Also, you can position the shape closer to the edge of the template material than you could if you drew the pattern directly on it.



There are three types of templates you can make: the type you trace around, the type you trace within, and a combination of the two, which I'll call the exterior, interior, and combo types. The combo type is only needed if you use both a stitching and a cutting template. If you can stitch or cut accurately without a guide, you don't need it. The combo type needs joiners between the two templates as shown in the illustration and so will result in breaks in the traced pattern.

The interior template is easier to trace around and easier than the exterior type to hold down without your fingers getting in the way of the pencil. For panel shapes with straight edges, you can cut out the interior type fairly easily with an X-acto knife and ruler. Curved edges will be much more difficult because they require scissors. I would recommend just using the exterior type for those designs.

Even though it is not necessary for the panel edges to be as precise as the stitching pattern within them, I find it very helpful for them to be. When I am trying to align the stitching lines I'm sewing together, I can merely align the edges of the panels and know that this will align the patterns. Also, when I draw the front stitching lines (used when closing the final opening from outside the bag), it is easier to align the template with the pattern on the back of the panel if I can simply align both with the cutting pattern.

Marking the fabric

To transfer the stitching patterns onto the fabric, you need a way to mark the fabric that is highly visible, won't rub off or disperse as chalk will do, and that either washes out or is permanent and will not bleed but stay hidden within the seam allowance. Do not use a standard ballpoint pen as I did at the beginning years ago because it will likely bleed when wet and make a smudgy stain through the fabric. Remember that if you use a thick marking tool and the size of the bag matters to you, it is important, especially for the high panel count designs, to sew on the edge of the line where the template edge was, not in the middle, as this may noticeably affect the size of the bag.

To help prevent the pencil or permanent marker lines from showing on the outside of the bag, stitch on the inside edge of the stitching lines. For the front stitching pattern (which you will sew from outside the bag) position the stitching patterns slightly too close to the edges of the fabric and then stitch slightly inside them (toward the center of the panels). These techniques will result in the lines being hidden from view within the seams, even if they bleed through the fabric.

The best ways I have found to mark light-colored fabrics is with a fine point, permanent marker (e.g., a Sharpie) or a regular pencil (I use an automatic one with a 0.9mm lead). The Sharpie brand marker I use works well even for medium dark fabrics and does not bleed when moistened or scrubbed with soap (I tried it), but it does soak through the fabric pretty easily while I am drawing with it because it is such a wet ink, so for light colored or thin fabrics, be careful to hide it within the seam. Graphite pencils are dark enough to work on most colors and graphite is also somewhat shiny which helps with visibility on darker fabrics. I recommend using a medium or soft lead if you use a pencil. Pencils work well on firm fabrics like denim, but a soft fabric would tend to fray under the abrasion required to make a good pencil mark. Be aware that while graphite fades somewhat during use, including during stitching (and probably even more if you laundered the bags), as the graphite particles disperse and rub off, it may leave a bit of a stain, so try to hide it within the seams when you sew from the outside.

To mark dark fabric, I found that a correction pen is very effective, and so has one other stitcher I encountered on the web. A correction pen is a ballpoint white-out pen, and the liquid paper sits on top of the fabric and is very visible and does not disperse or fade as I sew. I do not know how it will respond to washing. I also saw advice to use old slivers of worn-down bar soap. That has the advantage of washing out. I have not used that method, however.

You can also buy special dressmaker's pens and pencils, but in my limited experience they do not work very well. My water-erasable pens made very faint lines, at least on denim. They also dried out sooner than I would have expected. The white dressmaker's pencil I tried made such a faint line on the navy blue denim I was using that I could hardly see it (the white water-erasable pen I used years ago was hardly better). Maybe these pencils work better on other types of fabric.

Thread

In the early days of my hobby I used a standard thread (I think it was cotton) and I tried to pick a neutral color like grey that would blend as well as possible with the variously colored panels of my beanbags in case it peeked out. This had two drawbacks: 1) It did not blend very well and where the stitches peeked out, they looked a bit tacky; 2) It tended to fray and break during sewing a couple times per beanbag even though it was a high quality thread and I doubled it.

I'm now using a new thread which is called invisible. It is made of a single, transparent filament like fishing line. It is stronger judging by how much harder it is to break, and there is no risk of running my needle through the fibers of the thread when I make a retreating stitch (I use the backstitch), which frays and weakens it and sometimes caused it to break in my early bags. Strength is very important to me because I use denim for my beanbags and I pull the stitches very tight. This new thread is indeed invisible except when the light glints off its glossy surface. One drawback is that it is springy and therefore harder to manage, but it's not too bad. Being nearly invisible, it is also difficult to see my previous stitches and make my backstitches pass through the same points. However, its slick surface makes it easier to pull out stitches when needed, though this also means stitches will more easily loosen on their own.

Upholstery thread or shoe thread may be a good choice for strength and aesthetics. One manufacturer stated on his website that he uses a thread made for sewing parachutes. Peter Billam in his article *How to Make Leather Juggling Balls*[12] recommends waxed thread for leather, but I don't know why, having no experience with leather myself. Whatever you choose, make sure it is durable enough to withstand your sewing practices as well as the wear and impact that juggling bags sustain.

If you use an opaque thread, it might be a good strategy to pick a color that, rather than ineptly camouflaging with the panels, will contrast boldly and attractively with them such as black for light fabrics, orange for dark fabrics, blue for red fabric, etc. I haven't tried this, but it may look better than a bland grey peeking out.

[12] http://www.pjb.com.au/jug/leatherballs.html

Stitching techniques

I sew my juggling bags by hand. I neither have nor know how to use a sewing machine, and I doubt that a machine would be able to do a job like this very well anyway.

I am not very experienced with sewing and there are many techniques I have not tried. My technique is oriented toward smooth and elegant seams like the Gballz in the left-hand image below, but it is also slow (photos of my own beanbags are at the beginning of each instructional chapter and they are all shown together in the *Introduction* chapter under "Discussion and comparison of the designs"). Some leather beanbags such as the Renegade brand suede bean bags shown on the right have an attractively rippled seam which I assume uses a sparser stitch and would therefore be much quicker to sew. I don't know what type of stitch they use, but it is probably a running stitch. You can see two more examples of this style in the *How I Developed My Designs* chapter under "Equidistant cuboctahedron".

Smooth-seamed Gballz from http://www.gballz.com/ and wavy-seamed Renegade suede balls from http://www.renegadejuggling.com/8-Panel-Suede-Ball-p166.html

For either seam style, accurate and tightly pulled stitching is important if you want an elegant, perfectly round bag no bulges or skews, and with no stitching visible from the outside. If you pull the stitches so tight that they crinkle or warp the fabric, wriggle it straight again so that the finished seams are not distorted (unless you like the puffiness of gathered seams, but it must be done right). The denser the stitches (the more closely spaced), the smoother the seams will be, though the backstitch I use does not need to be extremely dense to produce a smooth seam. I make about three stitches per centimeter (about $^{1}/_{8}$" per short stitch) for my bags and that may be overkill; two may be enough.

Make sure your stitching comes together at the corners of each panel (on the stitching pattern) so that you have tightly closed, leak-proof vertices on the bag, and a more elegant appearance. When you add a new panel to a pair you finished sewing together, make sure the needle enters the new panel at the same point it exited the panel to which it will be joined so the corners will line up. Sometimes this means you will have to enter the new panel a short distance away from the corner and begin with a retreating stitch before proceeding down the edge so that the stitching still meets the corner.

For the seams that are sewn inside out, I use the backstitch. Start with the two panels placed front to front. On one side make a long, advancing stitch. On the other side, make a short, retreating stitch (half the length of the long stitch). Back on the first side, make another long, advancing stitch. Thus, one side of the seam contains long, overlapping, advancing stitches and the other side contains short, adjacent, retreating stitches. This is the best stitch I have found for making the finished seam beautifully smooth and not rippled as a simple running stitch will do. It also has the advantage of locking itself against the fabric so that the stitches do not easily loosen after you have pulled them tight.

For the seams that are sewn from the outside, I use the backstitch again but with the technique modified so it can be performed from the outside. My technique is similar to the "ladder stitch". A ladder stitch is really a running stitch performed from the outside. My stitch is done the same way but with a retreating stitch added. Fold the seam allowance of the two adjacent panel edges into the bag along the stitching lines and pinch them together, forming two "lips" of fabric with the stitching lines along the tops. Pin the two lips together if necessary to keep them aligned and folded along the stitching lines. Pinch the two lips together just ahead of where you're stitching. Make a long, advancing stitch in and out one lip of fabric (following the exposed stitching line) through the "tunnel" made by the folded seam allowance, and then cross straight over to the other lip and make a short, retreating stitch in and out. Cross over again to the first side and make another long, advancing stitch. Pull the stitches tight periodically which will pull the two lips of fabric more tightly together and hide the stitches between them. This will result in a backstitch as if it was done from the inside and, if you do it right and pull the stitches tight enough, it will be as hidden from view as the inside stitches.

There is no need in my opinion to add whip stitches or other extra stitching at the seam intersections. I did this years ago and I have seen advice on the internet to do it, but as long as you stitch accurately and make the stitching all meet at the corners of your stitching patterns, there is no more possibility of filler leakage at the corners than anywhere else on the bag. (The exception is a design like the eight-panel beach ball where many panels come together at the intersections. I had to take special steps to make those intersections close tightly together. For more information on this, see the *Other Designs and Variations* chapter under "How to design beach balls with any number of panels".) I also no longer think that any additional strength is needed at the corners. Whip stitches around the seam allowance can also make the corners look less elegant because of the puckering and bunching up of the fabric they cause.

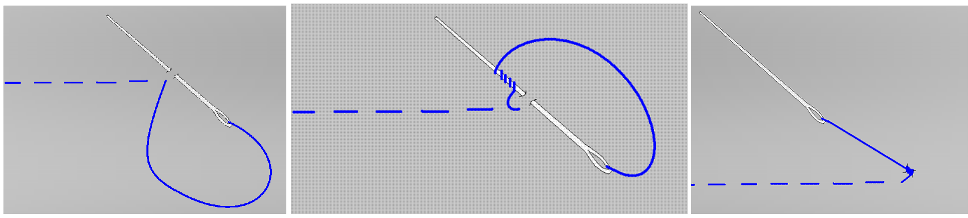
Knotting techniques

Surgeon's knot illustration from Wikipedia

Tying the thread securely and tightly is important so that your stitches do not loosen and peek out or, worse, unravel. Some people advise starting a thread by tying a large ball knot in the end of it which prevents it from pulling through the fabric. I don't trust the knot never to pull through, so I tie a surgeon's knot (left) around a small amount of fabric, or around the thread of a previous stitch if one is available. To tie a surgeon's knot, start with a double overhand knot (the knot you use to begin tying your shoelaces, but with one additional pass) followed by a standard overhand knot. I usually tie the thread a stitch or two back into my previous stitching (if there is any) or make an overlapping stitch in place over the knot to make sure there isn't a loose point in the stitching.

Back when I used standard thread, I doubled it to make it stronger (so I could pull the stitches tighter). Doubling the thread also makes it very easy to tie at the beginning; just run the needle through the fabric one way and back the other so both ends are on the same side, and then run the needle through the loop made by the folded thread and pull it tight: instant knot!

There is a very good method for ending the thread which I discovered on the internet recently. Below are illustrations for it. Stick the needle through a small amount of fabric and then back out on the same side and leave it there, halfway through. Take the thread opposite the needle where it comes out of the fabric and wrap it around the tip of the needle 3-6 times. Finish pushing the needle through so that it pulls the thread through the coil you made and pull it tight. I do this a couple of times to make sure I have a secure knot.



Ending knot illustration from http://www.shushanna.com/handsew.html

Better seams by ironing

This is a technique I invented to make the finished seams look more elegant by pressing the seam allowances flat. The motivation behind this is that the curvature of the seams causes the seam allowances to pucker and bend in different directions resulting in lumps and ripples in the seams. Pressing them so they sit flat against their respective panels will give you flat, straight, professional-looking seams.

Note: I use 8mm seam allowances because denim frays a lot. If you use a fabric that does not fray, you do not need much seam allowance and so you may not need this technique at all. With a very narrow seam allowance, you don’t really have a choice because there isn’t enough fabric to fold back and iron.

Before turning the bag right side out, separate the two layers of seam allowance fabric at each seam and fold each side out flat. Iron them so they stay that way. This can be done by putting your fingers inside the bag and pressing the seams against the iron (be careful!). It may help to dampen the seams first. I use denim, which protects my fingers from easily getting burned. If you use a thin fabric you may not be able to use this technique, but it is also less important for a thin fabric because the puckering won’t cause as much distortion in the seams. You will have to readjust the seams when you turn the bag right side out and make sure they are still folded out flat. You could also try a water-soluble glue instead of the iron, but you would have to wash the bag when you finish it to remove the glue. Depending on the fabric, it may also work just to dampen the seams, flatten them out with your fingers, and then let them dry that way or fill the bag right away and let the filler hold them flat.

Another technique I found, but have not used, is to use a large running stitch on both sides of each seam to keep the seams folded out flat, and then pull the stitches back out after the bag is finished. This sounds overly tedious and time-consuming to me, though.

SKETCHUP – AN APPLICATION FOR DRAWING THE TEMPLATES

I just learned of a free computer application called SketchUp by Trimble. This is a better way to draw and print the panel shapes than using manual tools. I include both manual and SketchUp directions for drawing each panel shape. I will not provide a tutorial for using SketchUp except for a few notes, but there are tutorial videos at http://www.sketchup.com/intl/en/training/videos.html. I got everything I needed within the first two videos (and a few Google searches).

I have been using the project template called "Product Design and Woodworking – Millimeters". My SketchUp directions use millimeter units (I normally use centimeters). Make sure whatever template you use has the correct units (or remember to do the conversions).

To make 1:1 printing work, and to simplify the interface for 2D drawing, set the camera to a standard view other than "Iso" such as "Top" (**Camera menu -> Standard Views -> Top**) and set the camera view mode to Parallel Projection (**Camera menu -> Parallel Projection**). You can also hide the axes to clean up the interface further (**View menu ->** uncheck **Axes**).

To eliminate the fill color of polygons, go to **View menu -> Face Style** and select **Wireframe**. Or, you can erase the background color selectively.

Circles default to 24 sides (they are actually polygons). I set them to a high number like 360 for extra curve precision. To do this, select the Circle tool (notice that a text box labeled "Sides" appears at the bottom of the window), type the number of sides you want (the value appears in the box) and press Enter. You can also type the number of sides followed by "s" at any time and press Enter. All future circles during that session will have the new number of sides. Incidentally, you do the same to set the number of sides for the polygon tool (in the **Draw** menu), though you can use the circle tool as a polygon tool. They are the same thing as far as I can tell.

Before printing, I recommend that to save ink and speed up printing you erase any polygon fill color you don't need (if you are using a Face Style that has fill color) and any extraneous lines from your design. Whatever you crop outside the view pane won't print anyway, so you don't have to delete all of the extra lines. You can erase line segments by clicking on them with the eraser tool, or by drawing a box around them with the Select (arrow) tool and pressing Delete on the keyboard. You can erase background color by right-clicking on it with the Eraser tool or the Select tool and selecting Erase from the context menu.

For arcs, I find it helpful to leave some excess to help guide the scissors as they enter the panel so I get a more accurately curved cut, but that's up to you. To do this, draw a line through the arcs a short distance away from the intersections, separating the arcs into segments that can be erased separately. Then erase the portion of the circles beyond the lines, and then the lines themselves.

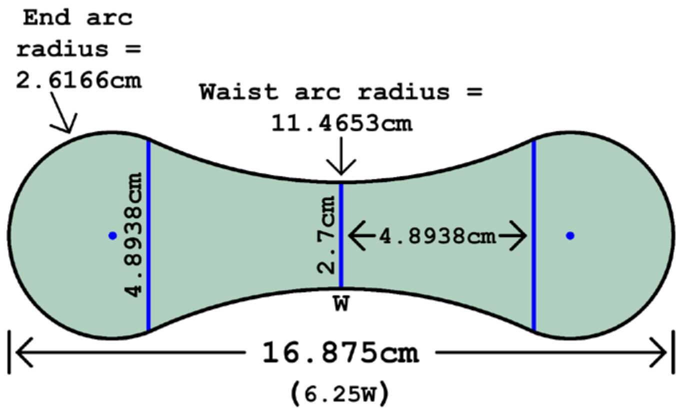
I am not very experienced with using SketchUp, so I found a forum that explained how to print my panel shapes at a precise 1:1 scale. Here are the instructions, rewritten by me. If they don't work for you, Google is your friend.

How to print at a 1:1 scale:

1. Set the Camera view mode to Parallel Projection and the standard view to something other than "Iso" if you haven't already.

2. Zoom in on the relevant portion of the design so that it fills the view pane as well as possible and is not cropped (the print will include everything in the view pane, including empty space, and nothing outside it). This step does not affect printing accuracy, but it prevents the empty space, and your design, from being split across multiple pages. With some trial and error you can find the optimal zoom and figure placement that will result in the template fitting at the edge of the page with no extraneous pages printed.
3. Go to **File menu -> Print Preview**, and in the Print Preview dialog:
    a. Uncheck "Fit to page".
    b. Uncheck "Use model extents".
    c. Under the "Scale" heading, enter the same value (e.g. "1") and unit type for "In the printout" and "In SketchUp" to set a 1:1 scale between the application and the printout.
    d. Look at the print preview screen to see if your figure is positioned on the page as you want it to be. If it isn't, go back and re-zoom and re-position your figure. If it is, it's ready to print.
    e. OPTIONAL: The Print Quality drop-down changes the line quality. "High Definition" and "Ultrahigh Definition" make the lines thinner and less pixilated and therefore more accurate.

2-PANEL BASEBALL INSTRUCTIONS



Stitching pattern dimensions for a 2⁵/₈" ball

Design notes

1 seam, 40.5cm (15.9") of stitching.

This is the panel structure of the original Hacky Sack by Wham-O from the 1980s. Here are a couple of articles giving a history of footbags in case you're interested:

- Brand Story of Hacky Sack: http://www.wham-o.com/brands/hacky_sack.html
- The 1980's Hacky Sack Footbag Fad: http://www.mortaljourney.com/2010/11/1980-trends/hacky-sack-footbags

This design is arguably superior to the 4-panel beach ball (in the next chapter) because the compound curvature of the seam and its greater uniformity make the bag feel somewhat more spherical. I also prefer the look of it. The panel shape is much more complicated to draw, however, and the bag is more difficult to assemble due to having concave and convex curves connecting to each other. You cannot simply lay the panels flat together and sew along the seam as you can with the other designs in this document; you must continually adjust the panels as you sew to keep the curves aligned at the point you're sewing, and this makes it more difficult to sew them correctly and prevent distortions in the seam. I spent a long time and used small and extremely precise stitches to make mine look as good as it does (I also ironed the seam allowances flat).

My panel shape is not exactly the same as the one used for baseballs, but it is close and does form a good ball. I think the baseball panel shape requires a function-based curve whereas mine uses circular curves. My design can be thought of as a simplified version of the baseball shape that is much easier to define and draw. For a thorough explanation of how I designed this panel shape and verified that it fit together into a good sphere, and for photos of other perspectives of the bag, see the *How I Developed My Designs* chapter under "Baseball".

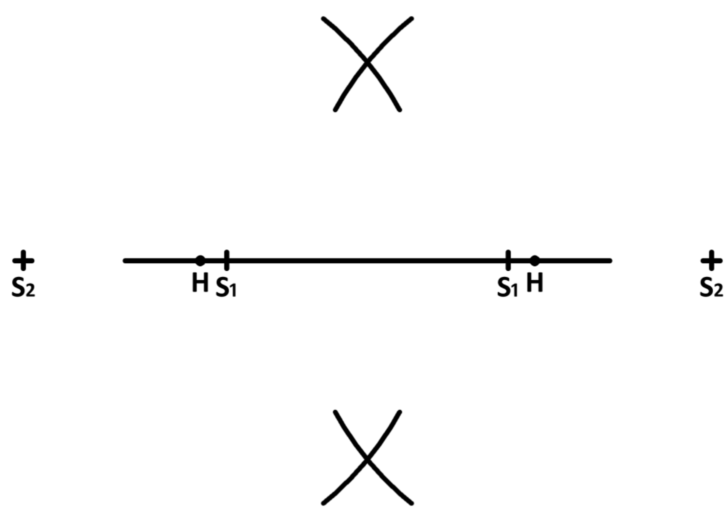
This design may feel somewhat squarish at the profile shown on the right (at least when made with the thick, stiff denim I use). A flexible, stretchy fabric tightly filled should enable it to bulge out into a better sphere, and for stiff, non-stretch fabrics, filling the bag loosely helps it feel rounder. For the bag in the photos I used denims that had a small amount of stretchiness to them (but not much).

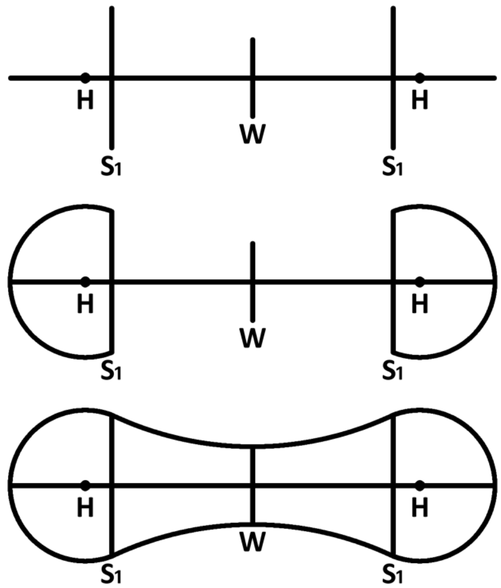
Supplies

- **For the templates (if you draw them by hand)**
  - Cardboard or Template Plastic, fine-point Pencil, metric Ruler, Compass, Scissors, Paper & Adhesive Tape (optional).
- **For the beanbag**
  - Fabric, Needle and Thread, Scissors, Fabric Marker or Pencil, bean bag Filler, Funnel, Safety Pins (optional).
- **For your information**
  - Unless you're experienced with this sort of thing, I recommend that you browse through the *General Notes and Techniques* chapter before you get started. You may find some tips there that will improve your experience and your beanbags.

How to draw the panel shape for a tennis ball sized bag (diameter ≈ 67mm, 2⁵/₈")

The panel shape is composed of a pair of convex curves at the ends and a pair of concave curves at the waist, both of which are circular. Below is a pictorial summary of the procedure for drawing the shape by hand which is referenced in the written directions. Not all of the steps are shown.





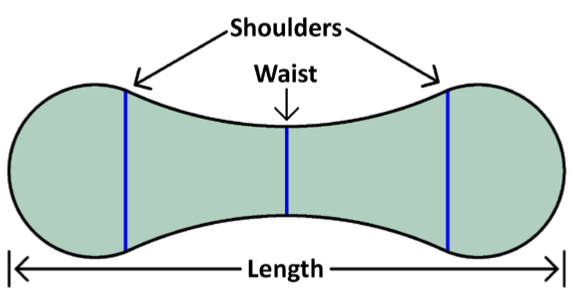
- **Manual directions**

1. Draw a horizontal 16.9cm line (the panel's length), mark the center, and mark compass points that are 2.62cm in from each end (labeled H for "Head" in the illustrations). OPTIONAL (see step 4): Mark another pair of points (S_1 for "Shoulder") that are 4.89cm out from either side of the center, and then a third pair that are 4.89cm further beyond each of those points (S_2). These shoulder points are not necessary for drawing the panel but will produce reference points to help you keep the panels aligned when you sew them together.
2. Use the compass placed at each end of the main line in turn and, using a large radius (around 11cm, ideally), draw small arcs above and below the center of the line to form an X above and below the line as shown in the first illustration. These mark the vertical centers to aid in drawing the vertical waist line (W).
3. Use the X points from the previous step to draw a vertical 2.7cm line centered on the horizontal line (labeled W for "Waist" in the illustrations). Then mark compass points that are 11.47cm beyond each end of this line (not labeled).
4. If you want the shoulder reference points, perform similar steps to draw vertical lines through points S_1. Place the compass on points S_2 and W in turn to form an X above and below S_1 and use those Xs to draw the vertical lines (the lines only need to be 4.89cm long, but their length will not be used for anything but markers at the edges of the panel, so they can be longer).
5. Place the compass on each of the four compass points you marked (H and the unlabeled points above and below W), set its radius to match the distances of those marks from their reference points (2.62cm for H and 11.47cm for W) and draw arcs that meet each other at the shoulder endpoints and form the panel shape. (If you didn't draw the shoulder lines, just draw a bit extra for each curve and erase the excess after you draw all four.)
6. To make a cutting template, draw everything the same but increase the radius of the end curves by the desired seam allowance (I use 8mm) and decrease the radius of the waist curves by the same amount.

- **SketchUp directions (using millimeter units and greater precision)**

  1. Draw a horizontal 168.75mm line (the Length of the panel). Mark points that are 26.166mm in from each end of the line (labeled H for "Head" in the illustrations) and another pair of points that are 48.938mm out from each side of the center (S_1 for "Shoulder"). S_2 is not needed.
  2. Draw a vertical 27mm line and center it at the middle of the horizontal line (labeled W for "Waist" in the illustrations).
  3. Mark compass points that are 114.653mm above and below each end of this line (not labeled).
  4. Draw vertical 48.938mm shoulder lines centered on points S_1.
  5. Draw circles centered on each compass point (H and the unlabeled points above and below W). The circles centered at H extend to the ends of the horizontal line (26.166mm radius). The circles above and below W extend to the ends of W (114.653mm radius). The arcs should cross each other at the shoulder endpoints and produce the panel shape. You can also use the arc tool to create the arcs in which case you don't have to mark the four circle centers.
  6. To make a cutting template, draw everything the same but increase the radius of the end circles by the desired seam allowance (I use 8mm) and decrease the radius of the waist circles by the same amount.

Altering the size of the bag



The circumference of the bag is measured in two ways: Length + Waist and 4 × Shoulder width. The shoulder points will be aligned when the panels are sewn together. The width of the shoulders is equal to their distance from the center which is one quarter of the bag circumference. Note that the shoulders are not located at the apexes of the head curve, but slightly closer to the center. The convex curves at the ends and the concave curves at the waist are circular.

I define the panel size by its waist and define the other measurements in terms of the waist. The panel's length is 6.25 × Waist, which makes the circumference of the bag equal to 7.25 × Waist. The shoulder width is equal to Circumference ÷ 4 = 1.8125 × Waist. For more information about this design, see the *How I Developed My Designs* chapter under "Baseball". If you don't care about the rest of the mathematical explanation of the panel sizing formulas, just skip to where I provide sizing formulas and a table of pre-calculated measurements.

I also define the curve radii by the waist size. The formula to calculate the radius of a circular arc (found on Wikipedia[13]) is as follows (C = chord or width of the arc, H = height of the arc from chord to apex):

$$R = \frac{C^2}{8H} + \frac{H}{2}$$

Let w be the Waist width. The chord of the head curves is the shoulder width, or 1.1825w. The height of the head curves is calculated below.

$$\text{Head curve Height} = \frac{Panel\ Length}{2} - Shoulder\ Distance \quad = \quad 3.125w - 1.8125w \quad = \quad 1.3125w$$

So the radius formula is

$$\textbf{Head curve Radius} = \frac{(1.8125w)^2}{8(1.3125w)} + \frac{1.3125w}{2} \quad = \quad \mathbf{0.9691w}$$

The height of the waist curve is shoulder minus waist divided by 2. Defined in terms of the waist it is

$$\text{Waist curve Height} = \frac{Shoulder - w}{2} \quad = \quad \frac{1.8125w - w}{2} \quad = \quad 0.4063w$$

The chord is simply 2 × Shoulder, and the shoulder has already been defined in terms of the waist. So the radius formula is

$$\textbf{Waist curve Radius} = \frac{(3.625w)^2}{8(0.4063w)} + \frac{0.4063w}{2} \quad = \quad \mathbf{4.2464w}$$

This design will end up **5-9.5%** larger (depending on whether I fill it loosely or over-fill it) than a calculation based on the template when made with the thick, slightly stretchy denim I used and with my stitching practices. I target halfway between the min and max inflations when sizing my patterns. Some circumferences are greater than others due to the slight angularity of the bag and so I calculated an average of four measurement directions.

Template sizing formulas

To make a bag of diameter d with an assumed 7.25% (0.0725) inflation, multiply d by pi (3.1416) to get the circumference, divide by 7.25 to get the waist size, and finally divide by 1+the decimal inflation (in red) to get the pre-inflated size. (If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may have to substitute a different inflation or deflation factor in the formula. For a method of figuring out your adjustment factor, see the *General Notes and Techniques* chapter under "Calculating your pattern size".) The remaining template measurements are calculated from the waist. **(Don't forget to multiply the final result by 2.54 if you need to convert inches to centimeters.)** Thus, the formulas are as follows (w is the width of the waist):

Waist = d × π ÷ 7.25 ÷ 1.0725

Length = 6.25w

Shoulder = Circumference ÷ 4 = 1.8125w

[13] http://en.wikipedia.org/wiki/Arc_%28geometry%29

Head Curve Radius $= 0.9691\text{w}$

Waist Curve Radius $= 4.2464\text{w}$

Based on these formulas, to adjust the bag's finished diameter by $^1/_8$" (3.175mm), change the waist of the template by **1.256mm - 1.310mm** (accounting for the 9.5% and 5% inflations, respectively).

Table of pre-calculated template measurements

Below is a table of template measurements for each $^1/_8$" diameter increment from 2" to 3" using the 7.25% adjustment factor.

Finished Diameter	**Waist** (cm)	**Length** (cm)	**Shoulder** (cm)	**Head Curve Radius** (cm)	**Waist Curve Radius** (cm)
2" (5.08cm)	2.0525	12.8280	3.7201	1.9891	8.7157
2$^1/_8$" (5.40cm)	2.1808	13.6298	3.9526	2.1134	9.2604
2$^1/_4$" (5.72cm)	2.3090	14.4315	4.1851	2.2377	9.8051
2$^3/_8$" (6.03cm)	2.4373	15.2333	4.4176	2.3620	10.3498
2$^1/_2$" (6.35cm)	2.5656	16.0350	4.6502	2.4863	10.8946
2$^5/_8$" (6.67cm)	2.6939	16.8368	4.8827	2.6106	11.4393
2$^3/_4$" (6.99cm)	2.8222	17.6385	5.1152	2.7350	11.9840
2$^7/_8$" (7.30cm)	2.9504	18.4403	5.3477	2.8593	12.5288
3" (7.62cm)	3.0787	19.2420	5.5802	2.9836	13.0735

Cutting pattern adjustments

To make the cutting template, draw everything the same, but extend the compass farther than you did before (increase the circle radii) for the convex curves (at the ends) by the desired seam allowance and *decrease* the radii of the concave curves (at the waist) by the same amount and center the four arcs at the same four points. The cutting template will be larger than, but parallel to, the stitching template.

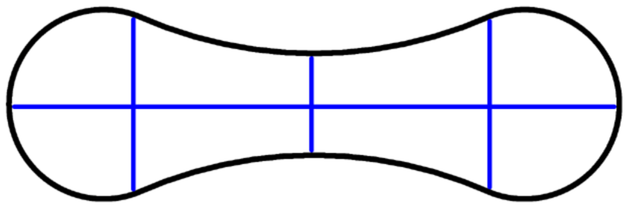
Making the panels

Depending on the type of template you're using and whether it is translucent or not, you must be careful which pattern, cutting or stitching, you trace first so that the second template doesn't hide the lines of the first and prevent you from aligning the two. Do not necessarily use them in the sequence below.

1. Use the larger, cutting template to trace the cutting pattern onto back of the fabric. You will need 2 panels. It may be helpful to make a mark at each endpoint of the pattern framework (shown in blue in the illustration below) to aid in aligning the stitching pattern to the cutting pattern.
2. Use the smaller, stitching template to trace the stitching pattern within each cutting pattern, being sure to center it well and align it with the framework marks if you made them. I recommend adding framework marks in the seam allowance at least for the stitching pattern to aid in keeping the continuous curves of the two panels aligned as you sew.
3. Cut out the panels.

Assembly

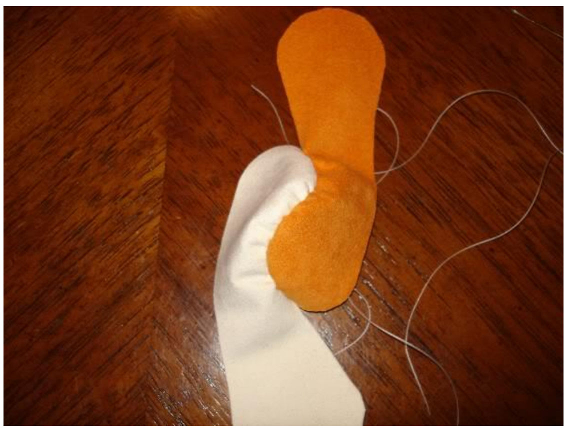
I found a forum in which someone asked for assembly instructions for this design[14]. The photos from the response follow my written instructions. Ideally you should have 8 markers along the edge of each panel to help you match the curves to each other as you sew. Because of the continual adjustment of the panels needed as you sew, it would be difficult without markers to proceed at an equal rate on each curve and avoid distortions in the seam. The markers are at the endpoints of the blue framework in the diagram below which you will get if you follow my instructions for drawing the panel shape. The seam length between all the marks is equal, so the marks of each panel should meet each other in the assembled bag.

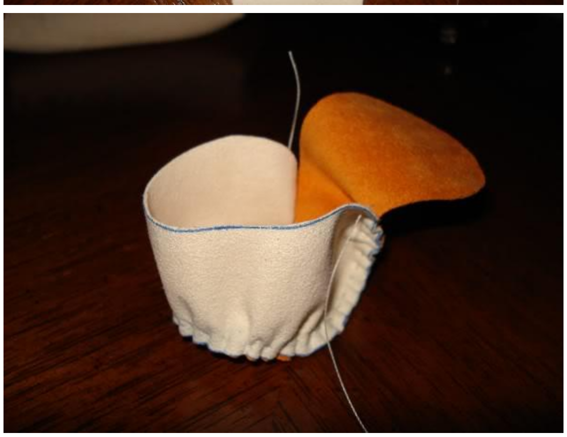


1. First, decide where you're going to start stitching on both panels and in which direction and then use the stitching template to draw a couple inches (the distance between two of the markers or a bit more) of stitching line on the *fronts* of the two panels behind that point. Be sure to center the template well (it should align as well as possible with the stitching patterns on the backs; you can use a needle poked through the panel at a specific point on the stitching line to match the position of the pattern on the front to the one on the back). You will use these lines to help you sew the final opening closed from the outside.
2. Aside from that, assemble the panels as shown in the photos below, ignoring the gather applied to the seams (unless this is what you want). Make sure the fronts of the panels are together and remember that the center of each convex curve will align with the center of a concave curve. You will have to continually adjust the panels as you sew to keep the patterns aligned at the point you're stitching.
3. When all that remains is a small opening having the front stitching lines along it, turn the bag right side out. A good method for this is to use a pen or other slender tool to push the fabric through the opening from the opposite side and then gently invert the bag around the tool. Consider pressing the seams; see the *General Notes and Techniques* chapter under "Better seams by ironing". For this design the seams can be pressed fairly easily *after* turning the bag out by sticking your finger inside the bag, separating the seam allowances, and pressing them against the iron.
4. Pull out the last stitch so that the thread is on the outside where you can get to it. Continue sewing the opening closed following the front stitching lines. Fill the bag at some point during this final sewing with a funnel.

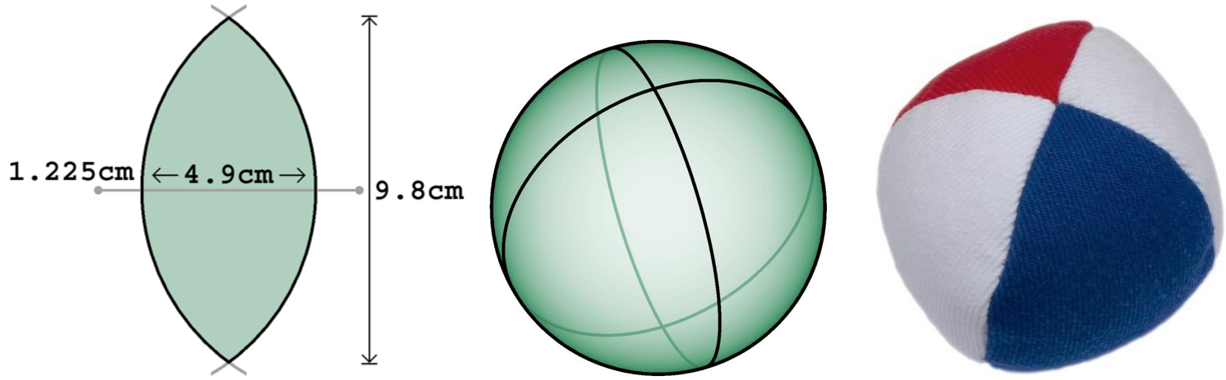
[14] http://modified.in/footbag/viewtopic.php?t=14125

4-PANEL BEACH BALL INSTRUCTIONS



Stitching pattern dimensions for a $2^{5}/_{8}$" ball

Design notes

4 seams, 44.5cm (17.5") of stitching.

Research: The shape of this type of beanbag (with any number of panels) is technically called a hosohedron[15] and the shape of the panels, when laid flat, is called a lens, or, more specifically, a symmetric lens[16]. The portion of a beach ball the lens panel covers is called a spherical lune[17].

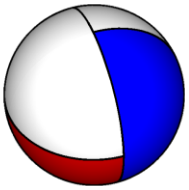
These are the simplest panels to draw and quickest beanbags to make, but, having only four panels, they are somewhat cubic around the equator as shown on the right (at least when made with the thick, stiff denim I use) and have a non-uniform feel due to the sparse and strictly longitudinal seam arrangement. The beanbag pictured is not at all broken in, though. After some heavy use the bag will probably round out somewhat. A flexible, stretchy fabric would probably be better for this design because it would allow the panel faces to bulge out and more closely match the seams, which are circular, and produce a better sphere. Filling the bag loosely also helps it feel rounder. For information about how I designed this panel shape, see the *How I Developed My Designs* chapter under "4-Panel beach ball".

You can make a beach ball style bag with more panels to improve its roundness (I describe how to do this in the *Other Designs and Variations* chapter under "How to design beach balls with any number of panels"), but if you're willing to sew more panels, there are designs with better seam uniformity than the beach ball. The only merit I see in a six or eight-panel beach ball is the attractive stripiness of it. The beach ball design's value, as I see it, is that you can make an easy, nearly spherical bag with just four simple panels, which is ideal for someone who isn't concerned about visual or tactile elegance but just wants something to toss around or for kids to play with. You can make a bunch of these in a day or two. (It is also possible to make a three-panel ball, but unless you use a very stretchy fabric, it will feel less spherical and symmetrical than the four-panel, and

[15] http://en.wikipedia.org/wiki/Hosohedron
[16] http://en.wikipedia.org/wiki/Lens_%28geometry%29
[17] http://en.wikipedia.org/wiki/Lune_%28mathematics%29#Spherical_geometry

it has three fewer color arrangement possibilities.)

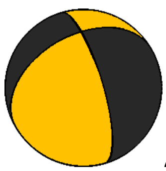
This design is larger along the seams than between them. Mathematically, using an arc length formula[18], it is 15.91% larger. By my measurements, the actual difference in my beanbag is 3.5%.



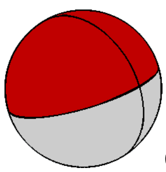
Here's a silly idea: Try misaligning the hemispheres by 90°. I haven't tried it, but I think it will work.

I have compiled four good color arrangements for this design. The images give examples of each arrangement in order.

2 colors

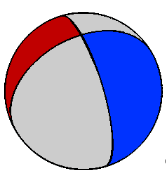


Alternating stripes of contrasting colors.



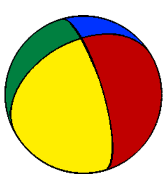
One hemisphere (two adjacent panels) of each color (Pokéball style).

3 colors



Color A on a pair of opposite panels and colors B and C on the other pair.

4 colors



Each panel a different color.

[18] $2r(\sin^{-1}(c/2r))$, where r = radius and c = chord length. Automatic online calculator available at: http://www.handymath.com/cgi-bin/arc18.cgi?submit=Entry.

Supplies

- **For the templates (if you draw them by hand)**
    - Cardboard or Template Plastic, fine-point Pencil, metric Ruler, Compass, Scissors, Paper & Adhesive Tape (optional).
- **For the beanbag**
    - Fabric, Needle and Thread, Scissors, Fabric Marker or Pencil, bean bag Filler, Funnel, Safety Pins (optional).
- **For your information**
    - Unless you're experienced with this sort of thing, I recommend that you browse through the *General Notes and Techniques* chapter before you get started. You may find some tips there that will improve your experience and your beanbags.

How to draw the panel shape for a tennis ball sized bag (diameter ≈ 67mm, $2^5/_8$")

The panel shape is a pointed ellipse or lemon shape whose height is twice its width and has circular sides.

- **Manual directions**
    1. Draw a 4.9cm line, mark each end of it (this is the panel's width), then continue the line on both ends of it and mark compass points 1.23cm (25%) out beyond each end of the 4.9cm section. (You don't really have to mark off the 4.9cm section, or even draw a line at all. As long as you have the compass extended to the correct radius, 6.13cm, all you need are two points that are 7.35cm apart (width * 1.5), each of which are compass points. I left the additional steps in to help you understand what you are accomplishing.)
    2. Place a compass on one of the two outer points and extend the arm until it reaches the opposite end of the 4.9cm section (it will extend 6.13cm, which is panel width × 1.25). Draw an arc that extends from directly above to directly below the center of the 4.9cm section.
    3. Without changing the compass radius, repeat the previous step on the other side. This should result in a pointed ellipse that is 4.9cm wide and 9.8cm tall. Cut this out and use it as the stitching template.
    4. To make a cutting template, draw everything the same but increase the compass radius by the desired seam allowance (I use 8mm) and then draw the two arcs from the same two points using that new radius.
- **SketchUp directions (using millimeter units)**
    1. Draw a 73.5mm line (panel width × 1.5).
    2. Draw circles of radius 61.25mm (panel width × 1.25) centered at each end of this line. The intersection of the circles forms the 49mm x 98mm lemon-shaped panel.
    3. To make a cutting template, start with the same 73.5mm line, but increase the circle radii by the desired seam allowance (I use 8mm).

Altering the size of the bag

The circumference of the bag is composed of two panel lengths around the "poles" or vertices and four panel widths around the equator, which is why the length must be exactly twice the width. I define the panel size by its width and express the other measurements in terms of the width. Its length is determined by the radius of the circular curves that form its shape. If you want to skip all the math, use the table of pre-calculated measurements below.

Measured between the seams or around the equator, this design will end up **4.6-9.4%** larger (depending on whether I fill it loosely or over-fill it) than a calculation based on the width/length of the template when made with the thick, stiff denim I use and with my stitching practices. I target halfway between the min and max inflations when sizing my patterns.

Template sizing formulas

To make a bag of diameter d with an assumed 7 % (0.07) inflation, multiply d by pi (3.1416) to get the circumference, divide by 4 to get the template width, and finally divide by 1+the decimal inflation (in red) to get the pre-inflated size. (If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may have to substitute a different inflation or deflation factor in the formula. For a method of figuring out your adjustment factor, see the *General Notes and Techniques* chapter under "Calculating your pattern size".) **(Don't forget to multiply the final result by 2.54 if you need to convert inches to centimeters.)** Thus, the formula is

Template Width $= \mathrm{d} \times \pi \div 4 \div 1.07$

Based on this formula, to adjust the bag's finished diameter by $^1/_8$" (3.175mm), change the width of the template by **2.298mm - 2.341mm** (accounting for the 8.5% and 6.5% inflations, respectively).

Now you need to know the relationship between the width of the panel, w, and the radius of the curves, r, (which is determined by the outer, compass points).

Curve Radius $= 1.25\mathrm{w}$

Distance between compass points $= 2\mathrm{r} - \mathrm{w} = 1.5\mathrm{w}$

This means that the compass points must be marked **r – w = 0.25w** beyond each end of the initial line that marks off the panel width.

Table of pre-calculated template measurements

Below is a table of template measurements for each $^1/_8$" diameter increment from 2" to 3" using the 7% adjustment factor.

Finished Diameter	**Width** (cm)	**Curve Radius** (cm)	**Distance Between Compass Points** (cm)
2" (5.08cm)	3.7288	4.6610	5.5932
$2^1/_8$" (5.40cm)	3.9619	4.9523	5.9428
$2^1/_4$" (5.72cm)	4.1949	5.2436	6.2924
$2^3/_8$" (6.03cm)	4.4280	5.5350	6.6420
$2^1/_2$" (6.35cm)	4.6610	5.8263	6.9915

Finished Diameter	**Width** (cm)	**Curve Radius** (cm)	**Distance Between Compass Points** (cm)
2$^5/_8$" (6.67cm)	4.8941	6.1176	7.3411
2$^3/_4$" (6.99cm)	5.1271	6.4089	7.6907
2$^7/_8$" (7.30cm)	5.3602	6.7002	8.0403
3" (7.62cm)	5.5932	6.9915	8.3898

Cutting pattern adjustments

To make the cutting template, draw everything the same, but extend the compass farther than you did before (increase the circle radii) by the desired seam allowance and then draw the two arcs from the same two points using that new radius. The cutting template will be larger than, but parallel to, the stitching template.

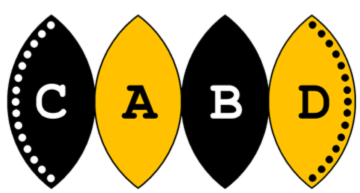
Making the panels

Depending on the type of template you're using and whether it is translucent or not, you must be careful which pattern, cutting or stitching, you trace first so that the second template doesn't hide the lines of the first and prevent you from aligning the two. Do not necessarily use them in the sequence below.

1. Use the larger, cutting template to trace the cutting pattern onto back of the fabric. You will need 4 panels.
2. Use the smaller, stitching template to trace the stitching pattern within each cutting pattern, being sure to center it well.
3. Cut out the panels.

Assembly

My method below can use as few as one thread if you cut it long enough and continue it from one seam to the next. The method consists of sewing A and B together, turning the pair right side out to expose the front faces, and then sewing C onto one side and D onto the other. The final seam between C and D will be partially closed from the outside along the dotted lines.

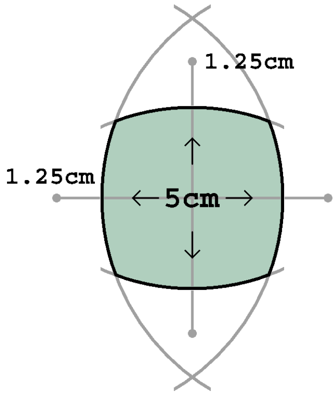


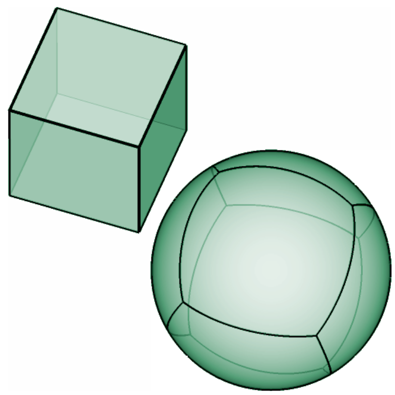
1. Lay the panels out as shown in the diagram above (I prefer to place them front face up) and arrange them according to your color pattern.
2. Use the stitching template to draw stitching lines on the *fronts* of the two panel edges shown with dotted lines in the diagram. Be sure to center the template well (it should align as well as possible with the stitching patterns on the backs; you can use a needle poked through the panel at a specific point on the stitching line to match the position of the pattern on the front to the

one on the back). My stitching pathway will leave these two edges partially unsewn so the bag can be turned out between them. They will then be sewn from the outside using the front stitching lines.

3. Sew panels A and B front faces together.
4. Turn the A+B pair inside out (actually right side out for the finished bag) so the front faces will be exposed for attaching C and D to them.
5. Add panel C and sew it to A, front faces together, along the edge of C that does not have the front stitching line. That is, orient C so that the edge with the front stitching line is along the seam between A and B.
6. Repeat the previous step on the other side with D and B.
7. Turn the A+B pair inside out so that the entire bag is inside out. You should now have a nearly complete bag with one open seam, and that seam should have front stitching lines on both sides (currently on the inward faces).
8. If you did not leave a hanging thread at one end of the opening, start a new one. Either way, sew the opening closed until you have just enough through which to turn the bag out.
9. Turn the bag right side out through the opening. A good method for this is to use a pen or other slender tool to push the fabric through the opening from the opposite side and then gently invert the bag around the tool. Consider pressing the seams; see the *General Notes and Techniques* chapter under "Better seams by ironing". For this design the seams can be pressed fairly easily *after* turning the bag out by sticking your finger inside the bag, separating the seam allowances, and pressing them against the iron.
10. Pull out the last stitch so that the thread is on the outside where you can get to it. Continue sewing the opening closed following the front stitching lines. Fill the bag at some point during this final sewing with a funnel.

6-PANEL SPHERICAL CUBE INSTRUCTIONS





Stitching pattern dimensions for a $2^{5}/_{8}$" ball

Design notes

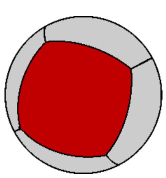
12 seams, 52.0cm (20.5") of stitching.

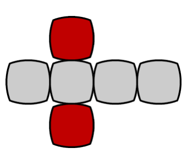
This design is based in part on the four-panel beach ball design. It can be thought of as that design but with each set of four panel tips at the top and bottom transformed into one new panel. Around the widths of the panels, both bags are the same. As you can see from the panel design diagram above, this panel is simply the middle portion of the lemon curve panel even though, technically, it is based on a square. For more discussion on this design, see my *How I Developed My Designs* chapter under "Spherical cube".

This design feels and looks almost perfectly spherical – much more so than my denim, 4-panel beach balls. At the beginning I could feel a hint of cubeness when I caught it, but it rounded out with use (I helped break it in by dampening it and putting it in a tumble dryer). If you use a stretchier fabric than denim, the bag will probably feel more spherical at the beginning. The arrangement of seams and the overall shape is more uniform than the beach ball design and I like the feel of it a lot better.

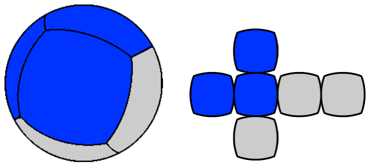
There are five good color arrangements for this design (possibly more that I haven't thought of), not including the obvious 1 and 6-color options. The arrangements are grouped according to the number of colors they contain.

2 colors

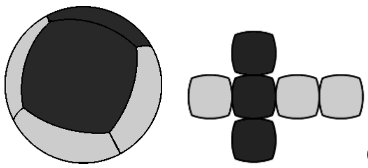




Color A on a pair of opposite panels and a belt of color B around the middle.

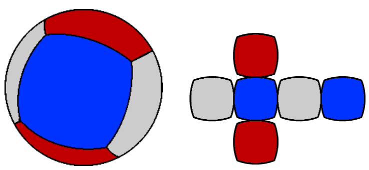


Color A on one hemisphere (three panels that share a corner) and color B on the other hemisphere.



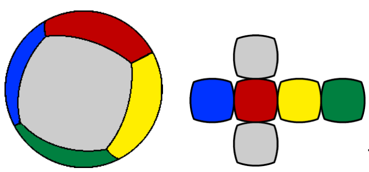
Color A on a row of three panels and color B on the other three, forming a tennis ball/baseball layout.

3 colors



Each color on pairs of opposite panels (my beanbag photo at the beginning of this chapter and my assembly diagram depict this arrangement).

5 colors



This is a variation of the first 2-color arrangement. Use a neutral color like white on two opposite panels and make a belt of four bold colors around the middle.

Supplies

- **For the templates (if you draw them by hand)**
  - Cardboard or Template Plastic, fine-point Pencil, metric Ruler, Compass, Scissors, Paper & Adhesive Tape (optional).
- **For the beanbag**
  - Fabric, Needle and Thread, Scissors, Fabric Marker or Pencil, bean bag Filler, Funnel, Safety Pins (optional).
- **For your information**
  - Unless you're experienced with this sort of thing, I recommend that you browse through the *General Notes and Techniques* chapter before you get started. You may find some tips there that will improve your experience and your beanbags.

How to draw the panel shape for a tennis ball sized bag (diameter ≈ 67mm, $2^5/_8$")

The panel shape is a circular square. The radius of the circle that produces the curves comes from the four-panel beach ball design. I draw this shape in a manner similar to the beach ball panel. Precision is important because if the compass points or the compass radius are not consistent across all four arcs, the sides of the panel will be unequal lengths and the stitching lines will not match up.

- **Manual directions**
    1. Draw a horizontal 5cm line, mark each end of it (this is the panel's width), and then continue the line to either side of it and mark compass points 1.25cm (25%) out beyond each end of the 5cm section.
    2. Place a compass on one of the two outer points and extend the arm until it reaches the opposite end of the 5cm section (it will extend 6.25cm, which is panel width × 1.25). Draw an arc that extends from directly above to directly below the center of the 5cm section. Then, keeping the compass radius unchanged, repeat this on the other side. This should result in a pointed ellipse or lemon shape.
    3. Use the intersections of the arcs above and below the center line to draw a vertical line through the center of the horizontal line and mark the same four points on this line as you did on the horizontal line (that is, mark a point 2.5cm above and below the center point to produce a 5cm segment centered on the horizontal line, and then mark compass points 1.25cm farther above and below those points). (Note that, once you have the compass set to the correct radius, you don't need to mark the 5cm portion of the lines. Just draw 7.5cm lines, or just the end points, and use the end points as compass points. I left that step in because I think it makes the process more understandable.)
    4. Keeping the compass at the same radius as before, place it on each vertical compass point (it should extend to the opposite side of the vertical, 5cm segment) and draw two horizontal arcs that intersect the first pair of arcs. You now have your circular square between the four arcs. That is the part you want; cut it out.
    5. To make a cutting template, draw everything the same but increase the compass radius by the desired seam allowance (I use 8mm) and then draw the four arcs from the same four points using that new radius.
- **SketchUp directions (using millimeter units)**
    1. Draw a horizontal 75mm line (panel width × 1.5).
    2. Draw a vertical line of the same length and center it on the horizontal line.
    3. Draw circles of radius 62.5mm (panel width × 1.25) centered on all four ends of the two lines. The intersection of the four circles forms the circular square which should be 50mm wide.
    4. To make a cutting template, draw everything the same but increase the radius of the four circles by the desired seam allowance (I use 8mm).

Altering the size of the bag

The circumference of this bag is composed of four panel widths. I define the other template measurements in terms of the panel width. If you want to skip all the math, use the table of pre-calculated measurements below.

Measured around the widths of the panels (around the diagonals is a larger circumference), this design will end up **2.5-6%** larger (depending on whether I fill it loosely or over-fill it) than a calculation based on the width of the template when made with the thick, stiff denim I use and with my stitching practices. I target halfway between the min and max inflations when sizing my patterns.

Template sizing formulas

To make a bag of diameter d with an assumed 4.25% (0.0425) inflation, multiply d by pi (3.1416) to get the circumference, divide by 4 to get the template width, and finally divide by 1+the decimal inflation (in red) to get the pre-inflated size. (If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may have to substitute a different inflation or deflation factor in the formula. For a method of figuring out your adjustment factor, see the *General Notes and Techniques* chapter under "Calculating your pattern size".) **(Don't forget to multiply the final result by 2.54 if you need to convert inches to centimeters.)** Thus, the formula is

Template Width $= d \times \pi \div 4 \div 1.0425$

Based on this formula, to adjust the bag's finished diameter by $^1/_8$" (3.175mm), change the width of the template by **2.352mm - 2.433mm** (accounting for the 6% and 2.5% inflations, respectively).

Now you need to know the relationship between the width of the panel, w, and the radius of the curves, r, that form its shape (which is determined by the outer, compass points). I use the same curve radius as for the four-panel beach ball panel. The formula is

Curve Radius $= 1.25w$

Distance between compass points $= 2r - w = 1.5w$

This means that the compass points must be marked **r – w = 0.25w** beyond each end of the initial line that marks off the panel width. Below is a table of template measurements for each $^1/_8$" increment from 2" to 3" using the 7.5% inflation.

(For my own reference, the diagonal of the 5cm panel is 6.02cm and the chord of the arcs is 4.25cm. This makes the diagonal circumference greater than the parallel circumference by 3.56%.)

Table of pre-calculated template measurements

Below is a table of template measurements for each $^1/_8$" diameter increment from 2" to 3" using the 4.25% adjustment factor.

Finished Diameter	**Width** (cm)	**Curve Radius** (cm)	**Distance Between Compass Points** (cm)
2" (5.08cm)	3.8272	4.7840	5.7408
$2^1/_8$" (5.40cm)	4.0664	5.0830	6.0996
$2^1/_4$" (5.72cm)	4.3056	5.3820	6.4584

Finished Diameter	**Width** (cm)	**Curve Radius** (cm)	**Distance Between Compass Points** (cm)
$2^{3}/_{8}$” (6.03cm)	4.5448	5.6810	6.8172
$2^{1}/_{2}$” (6.35cm)	4.7840	5.9800	7.1760
$2^{5}/_{8}$” (6.67cm)	5.0232	6.2790	7.5348
$2^{3}/_{4}$” (6.99cm)	5.2624	6.5780	7.8936
$2^{7}/_{8}$” (7.30cm)	5.5016	6.8770	8.2524
3” (7.62cm)	5.7408	7.1760	8.6111

Cutting pattern adjustments

To make the cutting template, draw everything the same, but extend the compass farther than you did before (increase the circle radii) by the desired seam allowance and then draw the four arcs from the same four points using that new radius. The cutting template will be larger than, but parallel to, the stitching template.

Making the panels

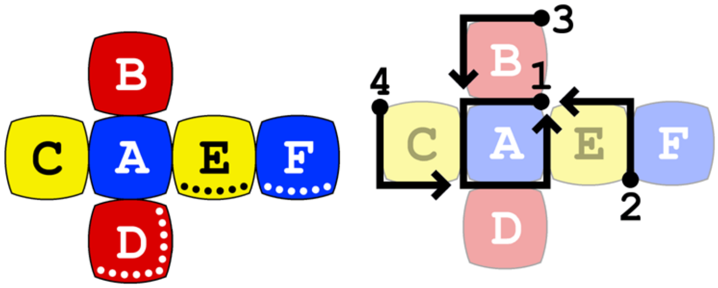
Depending on the type of template you're using and whether it is translucent or not, you must be careful which pattern, cutting or stitching, you trace first so that the second template doesn't hide the lines of the first and prevent you from aligning the two. Do not necessarily use them in the sequence below.

1. Use the larger, cutting template to trace the cutting pattern onto the back of the fabric. You will need 6 panels.
2. Use the smaller, stitching template to trace the stitching pattern within each cutting pattern, being sure to center it well.
3. Cut out the panels.

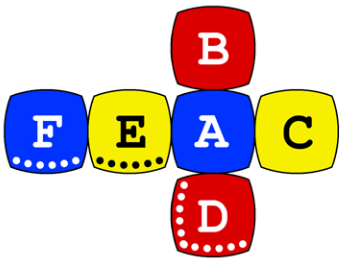
Assembly

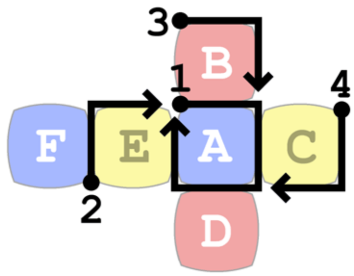
Below is my current favorite method of assembly which uses 4-5 threads. The diagrams depict my method. The letters indicate the sequence in which the panels are attached and each numbered path in the stitching map is a new thread. The final two seams will be sewn from the outside along the dotted lines in the layout diagram. I am right-handed and so the diagrams are oriented for stitching toward the left. In case you are left-handed, I included left-handed versions below the instructions.

If you don't mind using a very long thread, you can continue #1 into #2 (you'll have to reverse the direction of #2).



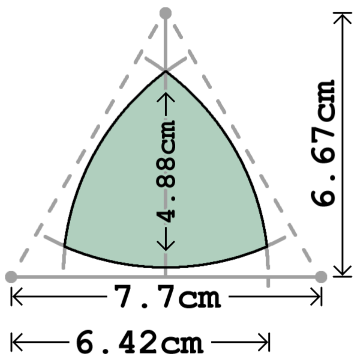
1. Lay the panels out as shown in the diagram above (I prefer to place them front face up) and arrange them according to your color pattern.
2. Use the stitching template to draw stitching lines on the *fronts* of the four panel edges shown with dotted lines in the diagram. Be sure to center the template well (it should align as well as possible with the stitching patterns on the backs; you can use a needle poked through the panel at a specific point on the stitching line to match the position of the pattern on the front to the one on the back). My stitching pathway will leave these four edges partially unsewn so the bag can be turned out between them. They will then be sewn from the outside using the front stitching lines. (If you use a thin fabric and don't need such a large opening, just skip marking two of the edges.)
3. Start with panel A and sew side panels B-E to each of its edges. Sew them with their front faces together so the bag will be inside out.
4. Add panel F and sew it to E, stitching toward B, and then continue down the side of E, attaching it to B. Tie and trim the thread.
5. Sew B to F, stitching toward C, and then continue down the side of B, attaching it to C. Tie and trim the thread.
6. Proceeding around the cube in the same manner as the previous two steps, sew C to F and then continue down the side of C, attaching it to D. Tie and trim the thread. You should now have just two open seams (four edges) and they should be the ones with the front stitching lines.
7. Start a new thread at either end of the final pair of adjacent seams and sew a few starter stitches to make it easier to continue from the outside. You probably won't need the entire two open seams to turn the bag out, so you may continue to sew as much as you don't need.
8. Turn the bag right side out through the opening. A good method for this is to use a pen or other slender tool to push the fabric through the opening from the opposite side and then gently invert the bag around the tool. (Consider pressing the seams first; see the *General Notes and Techniques* chapter under "Better seams by ironing".)
9. Pull out the last stitch so that the thread is on the outside where you can get to it. Continue sewing the opening closed following the front stitching lines. Fill the bag at some point during this final sewing with a funnel.

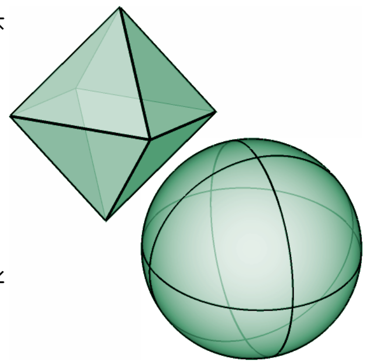




Left-handed diagrams

8-PANEL SPHERICAL OCTAHEDRON INSTRUCTIONS






Stitching pattern dimensions for a $2^{5}/_{8}$" ball

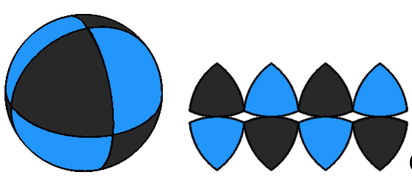
Design notes

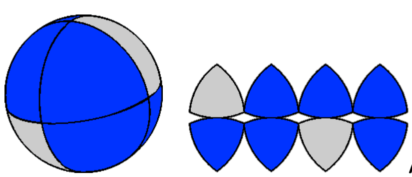
12 seams, 62.2cm (24.5") of stitching.

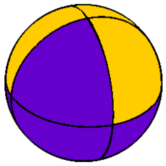
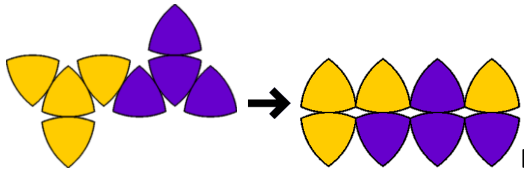
This design is based conceptually on dividing each panel of the four-panel beach ball in half widthwise, and the radius for the circularly curved edges comes from the beach ball design. The design is technically based on the regular octahedron, which is composed of eight equilateral triangular faces, but the use of circular triangles transforms the otherwise sharp vertices into continuously circular seams like those on the beach ball. For a fuller discussion of this design, see the *How I Developed My Designs* chapter under "Spherical octahedron". This design is a good balance between the uniform seam arrangement, roundness, and visual elegance of the higher panel count designs, and the ease of construction of the four-panel beach ball.

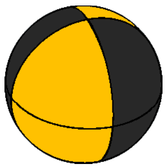
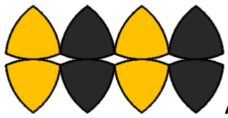
There are many color arrangement possibilities in this design. Following are all of the reasonable ones I could think of, grouped according to the number of colors they contain.

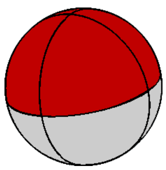
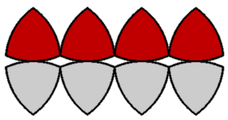
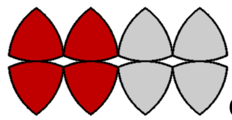
2 colors


Checkered pattern of contrasting colors (my assembly diagram depicts this arrangement).

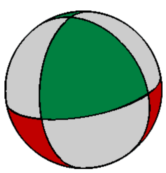
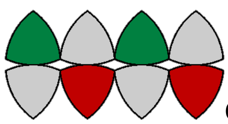

A ring of 6 panels of color A around two opposite panels of color B (billiard ball style).

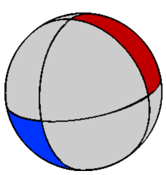
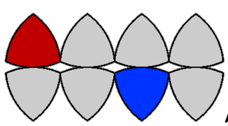
  Each color on a triangular patch of four panels.

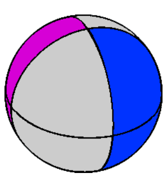
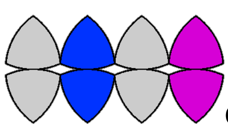
  Alternating two-panel stripes of contrasting colors (beach ball style).

  **OR**  One hemisphere (four panels that share a corner) of each color (Pokéball style).

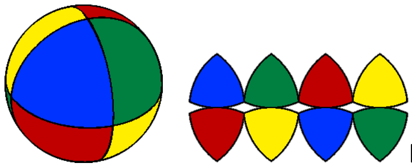
3 colors

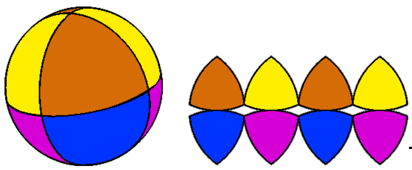
  Colors A and B alternate on one hemisphere and A and C alternate on the other with no panel having a neighbor of the same color. There will be four panels of color A, which can be thought of as a background color for B and C which are checkered throughout it. (See photo at the beginning of this chapter.)

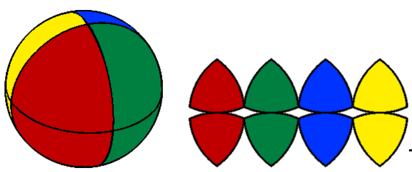
  A ring of 6 panels of color A around colors B and C which are on opposite panels (variation of the two-color billiard ball style).

  Color A on a pair of opposite, two-panel stripes and colors B and C on the other pair of two-panel stripes (beach ball style).

4 colors


Each color on pairs of opposite panels. No panel has a neighbor of the same color and all four colors are visible at any angle.


Two colors alternate on one hemisphere and the other two alternate on the other hemisphere. Each hemisphere (when viewed from the “pole”) has a distinct color personality. Around the equator between them all four colors are visible.


Two-panel stripes of each color (beach ball style).

Supplies

- **For the templates (if you draw them by hand)**
    - Cardboard or Template Plastic, fine-point Pencil, metric Ruler, Compass, Scissors, Paper & Adhesive Tape (optional).
- **For the beanbag**
    - Fabric, Needle and Thread, Scissors, Fabric Marker or Pencil, bean bag Filler, Funnel, Safety Pins (optional).
- **For your information**
    - Unless you’re experienced with this sort of thing, I recommend that you browse through the *General Notes and Techniques* chapter before you get started. You may find some tips there that will improve your experience and your beanbags.

How to draw the panel shape for a tennis ball sized bag (diameter ≈ 67mm, $2^{5}/_{8}$")

The panel shape is based on a regular (equilateral) triangle but has circular sides to produce a more spherical bag. I draw this shape by constructing an equilateral triangle (actually just enough of it to get the three corners), setting the compass radius to 83.33% of the length of the triangle’s sides, and then, using the three corners as compass points, inscribing the circular triangle inside the regular triangle.

Precision is important because if the compass points or the compass radius are not consistent across all three arcs, the sides of the panel will be unequal lengths and the stitching lines will not match up.

For more information on this design, see below under "Altering the size of the bag".

- **Manual directions**
  1. Draw a 7.7cm horizontal line and mark each end of it. This is the base of an imaginary equilateral guide triangle. Mark another point 6.42cm in from one end which will be used to extend the compass to the correct radius.
  2. Extend the compass to the 6.42cm radius, place it on each of the end points of the initial line, and draw arcs that extend from the line up to directly above the center. Draw partial arcs below the line as well to form an X that marks the horizontal center below the line.
  3. Using the two arc intersections to align the ruler, draw a 6.67cm vertical line up from the center of the base line. The top end of this line is the third corner of the imaginary equilateral triangle and the length can be found by multiplying the side length by $\sqrt{3}/2$ or 0.8660).
  4. Place the compass on this new point (keeping its radius unchanged) and draw a third arc between the first two, completing the circular triangle. The circular triangle should be 4.88cm from corner to opposite side.
  5. To make a cutting template, draw everything the same but increase the compass radius by the desired seam allowance (I use 8mm) and then draw the three arcs from the same three points using that new radius.
- **SketchUp directions (with greater precision and using millimeter units)**
  1. Draw a 77mm horizontal line which is the base of an imaginary equilateral guide triangle.
  2. Draw a 66.682mm vertical line with one end point on the center point of the first line. This forms a T shape whose outer endpoints are the three corners of an imaginary equilateral triangle. (The length of the vertical line is the height of the triangle which is found by multiplying the side length by $\sqrt{3}/2$ or 0.8660.)
  3. Draw circles of radius 64.164mm centered on the three endpoints of the skeleton. The intersection of the circles forms the circular triangle panel shape, which should be 48.811mm from corner to opposite side.
  4. To make a cutting template, draw everything the same but increase the circle radii by the desired seam allowance (I use 8mm).

Altering the size of the bag

This panel is formed by creating the three corners of an equilateral guide triangle to use as compass points and then using the same curve as the four-panel beach ball to draw a circular triangle. If you don't care about the rest of the mathematical explanation of the panel sizing formulas, just skip to where I provide sizing formulas and a table of pre-calculated measurements.

If the two compass points of the beach ball design are thought of as two corners of an equilateral triangle, this makes the ratio of the compass radius, r, to the length of the guide triangle's sides, s, 5:6, that is,

$$\mathbf{r = 0.8333s}$$

The height of the triangle (the location of the third corner or compass point) can be found using the following formula (s = side length):

Equilateral Triangle Height $= \sqrt{3}s/2 = \mathbf{0.8660s}$

The circumference of the bag (measured between the seams) is 4 × panel height, not the outer triangle height. So, to alter the size of the bag, you need a relationship between the length of the imaginary triangle's sides and the height of the circular triangle within it. I do not know a formula for this, but by using SketchUp to draw this template at a very large size for precision and measuring it I have the following, non-mathematical formula (s_t = side of the triangle and h_p = height of the panel):

$$\mathbf{s_t = 1.5775h_p}$$

(For my own interest: I originally calculated the ratio above using a hand-drawn template with 14.4cm triangle sides and measuring it with a ruler. The ratio I calculated using that method was 1.5824, which is only off by 0.0049!)

This design has a more uniform circumference than the previous two when it is tightly filled and can be measured between the seams or along them. By my measurements it ends up **6-8.5%** larger (depending on whether I fill it loosely or over-fill it) than a calculation based on the height (corner to side) of the template when made with the thick, stiff denim I use and with my stitching practices. I target halfway between the min and max inflations when sizing my patterns.

Template sizing formulas

To make a bag of diameter d with an assumed 7.25% (0.0725) inflation, multiply d by pi (3.1416) to get the circumference, divide by 4 to get the template height, multiply by 1.5775 to get the guide triangle's side length, and finally divide by 1+the decimal inflation (in red) to get the pre-inflated size. (If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may have to substitute a different inflation or deflation factor in the formula. For a method of figuring out your adjustment factor, see the *General Notes and Techniques* chapter under "Calculating your pattern size".) The remaining template measurements are calculated from the side length. **(Don't forget to multiply the final result by 2.54 if you need to convert inches to centimeters.)** Thus, the formulas are:

Guide Triangle Side Length $= \mathbf{d \times \pi \div 4 \times 1.5775 \div 1.0725}$

Guide Triangle Height $= \mathbf{side\ length \times 0.8660}$

Radius of curves $= \mathbf{side\ length \times 0.8333}$

Based on this formula, to adjust the bag's finished diameter by $^1/_8$" (3.175mm), change the length of the triangle's sides by **3.626mm - 3.711mm** (accounting for the 8.5% and 6% inflations, respectively).

For my own reference, the chord of the 4.88cm high panel is 5.04cm. This makes the seam circumference greater than the panel height circumference by 6.14%.

I used the octahedron design to make pocket-sized beanbags, which could also be used as footbags. I used a guide triangle with 6cm sides making the bags' filled diameter 2" (51mm). I filled them loosely using primarily plastic pellets and increased their weight to 75g using metal BBs. This size is large enough to juggle easily (when made heavy enough), but small enough to fit three of them in my jeans pocket (filling them loosely helps with this).

Table of pre-calculated template measurements

Below is a table of template measurements for each $^{1}/_{8}$" diameter increment from 2" to 3" using the 7.25% adjustment factor.

Finished Diameter	**Guide Triangle Side Length** (cm)	**Guide Triangle Height** (cm)	**Curve Radius** (cm)	**Height of the Panel (for double-checking)** (cm)
2" (5.08cm)	5.8685	5.0821	4.8902	3.7201
2$^{1}/_{8}$" (5.40cm)	6.2353	5.3997	5.1959	3.9526
2$^{1}/_{4}$" (5.72cm)	6.6021	5.5015	5.5015	4.1851
2$^{3}/_{8}$" (6.03cm)	6.9688	5.7174	5.8071	4.4176
2$^{1}/_{2}$" (6.35cm)	7.3356	6.3526	6.1128	4.6502
2$^{5}/_{8}$" (6.67cm)	7.7024	6.6703	6.4184	4.8827
2$^{3}/_{4}$" (6.99cm)	8.0692	6.9879	6.7240	5.1152
2$^{7}/_{8}$" (7.30cm)	8.4360	7.3055	7.0297	5.3477
3" (7.62cm)	8.8027	7.6232	7.3353	5.5802

Cutting pattern adjustments

To make the cutting template, draw everything the same, but extend the compass farther than you did before (increase the circle radii) by the desired seam allowance and then draw the three arcs from the same three points using that new radius. The cutting template will be larger than, but parallel to, the stitching template.

Making the panels

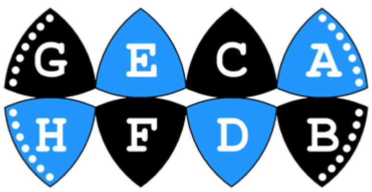
Depending on the type of template you're using and whether it is translucent or not, you must be careful which pattern, cutting or stitching, you trace first so that the second template doesn't hide the lines of the first and prevent you from aligning the two. Do not necessarily use them in the sequence below.

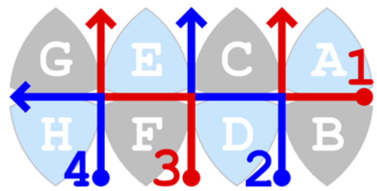
1. Use the larger, cutting template to trace the cutting pattern onto back of the fabric. You will need 8 panels.
2. Use the smaller, stitching template to trace the stitching pattern within each cutting pattern, being sure to center it well.
3. Cut out the panels.

Assembly

Below is my current favorite method of assembly which uses 2-5 threads. The diagrams depict my method. The letters indicate the sequence in which the panels are attached and each numbered path in the stitching map is a new thread. The final two seams will be sewn from the outside along the dotted lines in the layout diagram. I am right-handed and so the diagrams are oriented for stitching toward the left. In case you are left-handed or find this orientation confusing, I included left-handed versions below the instructions.

If you don't mind using a very long thread you can assemble this bag with just two threads by using a single thread for paths 2-4 (this requires reversing the direction of #3). Path #1 can be continued for the final two seams sewn from the outside.

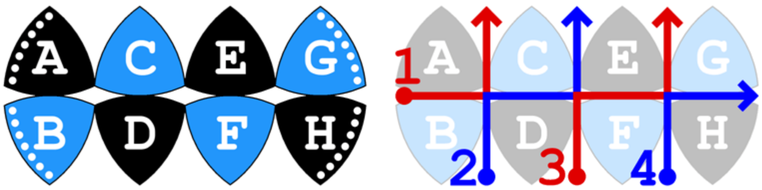




1. Lay the panels out as shown above (I prefer to place them front face up) and arrange them according to your color pattern.
2. Use the stitching template to draw stitching lines on the *fronts* of the four outer panel edges shown with dotted lines in the diagram. Be sure to center the template well (it should align as well as possible with the stitching patterns on the backs; you can use a needle poked through the panel at a specific point on the stitching line to match the position of the pattern on the front to the one on the back). My stitching pathway leaves these four edges partially unsewn so the bag can be turned out between them. They will then be sewn from the outside using the front stitching lines.
3. Place panels A and B front faces together and sew them together toward C and D, then add C and sew it to A. Tie off the thread at the end and trim it - unless... Tip: If you make this thread long enough for four seams instead of two, you can leave it hanging and continue it on the outside for the final two seams rather than starting a new thread. If you do this, I recommend that you still tie the thread so that if you break it while sewing outside you won't have to turn the bag inside out again to fix the stitches.
4. Continue adding panels in alphabetical order as in the previous step and sewing them according to the pathways depicted above. Sew them front faces together so the bag will be inside out. Tie and trim the threads when you reach the end of each path. At the end you should have an inside-out bag with two adjacent, parallel seams open (four panel edges), and these should have stitching lines on the fronts.
5. Either continue the first thread from where it left off or start a new thread at either end of the final pair of adjacent seams. Either way, sew a few starter stitches to make it easier to continue from the outside. You probably won't need the entire two open seams to turn the bag out, so you may continue to sew as much as you don't need.
6. Turn the bag right side out through the opening. A good method for this is to use a pen or other slender tool to push the fabric through the opening from the opposite side and then gently

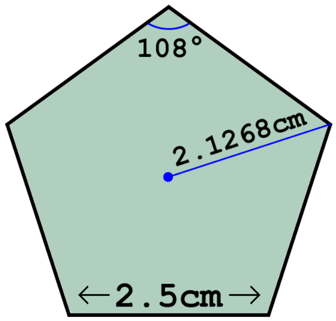
invert the bag around the tool. (Consider pressing the seams first; see the *General Notes and Techniques* chapter under "Better seams by ironing".)

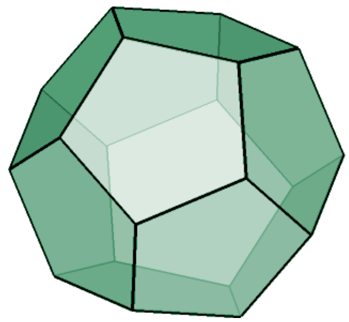
7. Pull out the last stitch so that the thread is on the outside where you can get to it. Continue sewing the opening closed following the front stitching lines. Fill the bag at some point during this final sewing with a funnel.



Left-handed diagrams

12-PANEL DODECAHEDRON INSTRUCTIONS





Stitching pattern dimensions for a $2^5/_8$" ball

Design notes

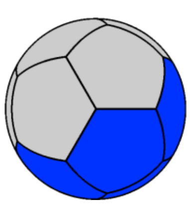
30 seams, 75cm (29.5") of stitching plus 5cm (2") of overlap if you use my assembly method.

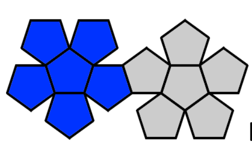
Unlike the previous two designs (the spherical octahedron and spherical cube), this design is not at all related to the beach ball design. There are no curves here and the panel layout is very different. I discuss the possibility of using a circular or otherwise bulged pentagon for the panel shape in the *Other Designs and Variations* chapter under "Dodecahedron with bulged faces". In short, I think it would add unnecessary complication since this design is already very round and smooth.

This design has the advantage over the previous designs of feeling wonderfully spherical and uniform in the hand and looking the most elegant, but the disadvantage of being tedious and difficult to make because of having so many panels. It also has an advantage in the number of color arrangements that are possible. 12 is divisible by 1, 2, 3, 4, 6, and 12 which means you can use 6 different numbers of colors in many different patterns and still have a balanced look.

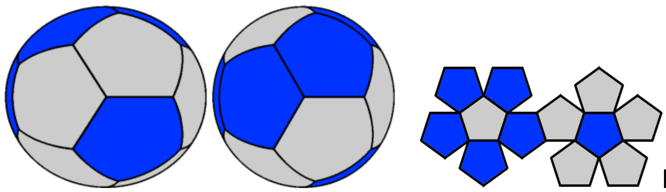
Following are thirteen color arrangement ideas of which seven are mine and six are from http://www.pjb.com.au/jug/leatherballs.html (the latter are the ones with the British spelling of "colour"). The arrangements are grouped according to the number of colors they contain. To help me design the more complicated of my arrangements and create the diagrams, I stuck colored thumbtacks into each panel of an old, solid beige, dodecahedron beanbag. This worked very well and I highly recommend it if you want to have some fun playing around with color arrangements. I did this before I knew about SketchUp. SketchUp would be a better way to experiment with color arrangements if you know how to use it.

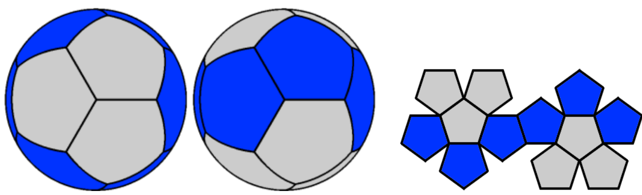
2 colors

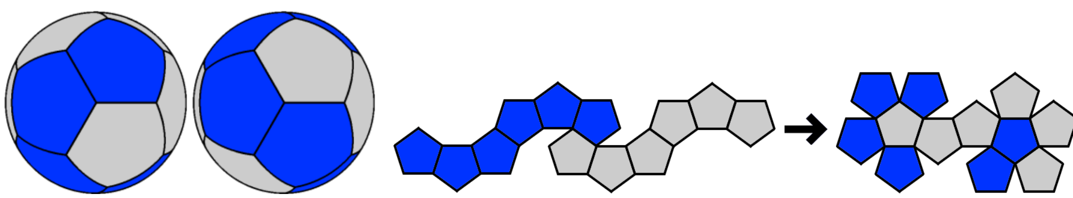




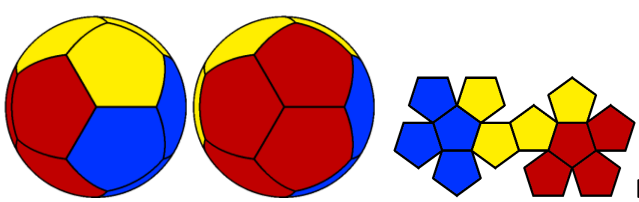
Each color covering an entire hemisphere.

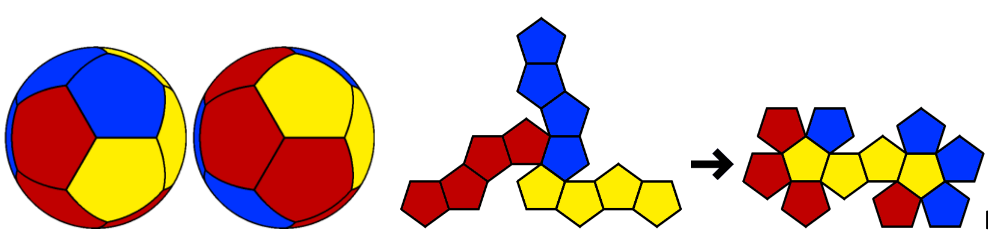

Five-fold rotational symmetry arises with one face of colour A at the top surrounded by five faces of colour B, and one face of colour B at the bottom surrounded by five faces of colour A.


Three-fold rotational symmetry arises with a patch of three faces of colour A at the top, a patch of three faces of colour A at the bottom, and a belt of six faces of colour B around the middle.

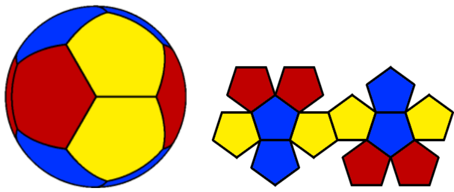

Two intertwining spirals [I would describe them as serpentine curves] of six faces can give a pattern with chirality, either left-handed or right-handed. This can be derived from the four-colour pattern by pairing the colours, and so the four-colour patterns also have chirality, though this is not obvious at first glance. [If you lay the panels out as shown above with the fronts facing up, you will get S-curves on the finished bag. If you lay them out with the backs facing up, you will get “2”-curves. My beanbag shown at the beginning of this chapter has the S-curve version of this arrangement.]

3 colors


Each color on a diamond-shaped patch of four panels.

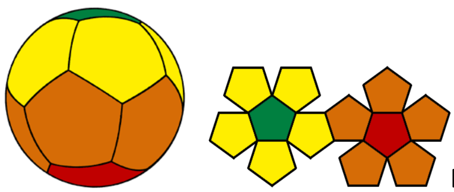

Each color on a serpentine row of four panels. If you lay the panels out as shown above with the fronts facing

up, you will get S-curves on the finished bag. If you lay them out with the backs facing up, you will get “2”-curves.

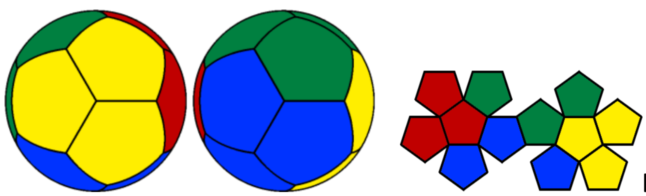


Three separate two-fold reflectional symmetries arise if each colour is arranged in two opposite patches of two faces each.

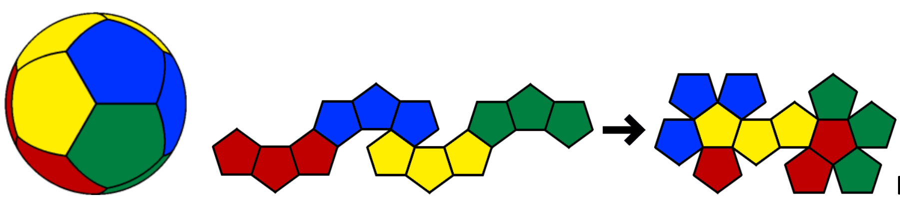
4 colors



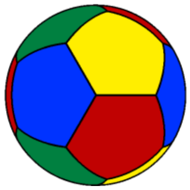
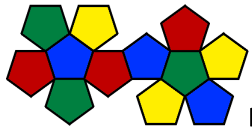
Based on the 2-color alternating ring pattern. One face of color A at the top surrounded by five faces of color B, and one face of color C at the bottom surrounded by five faces of color D.



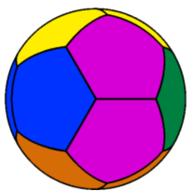
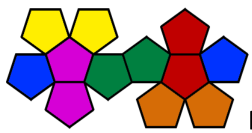
Each color on a patch of three panels that share a corner.

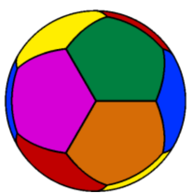
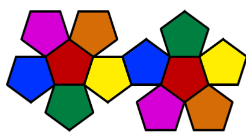


Each color on a row of three panels. By using the two-color intertwining spiral arrangement to form this arrangement, you will get a pattern as follows (using abbreviations for the diagram colors): colors in the pairs B&Y and R&G curve into each other and each pair is opposite and perpendicular to the other while colors in the pairs B&G and Y&R curve away from each other and these pairs are also opposite and perpendicular to each other. There are other, random ways to position 4 three-panel curves, but this method produces a balanced pattern. If you lay the panels out as shown above with the fronts facing up, you will get (bi-color) S-curves on the finished bag. If you lay them out with the backs facing up, you will get “2”-curves.



Four different colours arranged so that no side has a neighbour of the same colour. This would illustrate the Four-Colour theorem except that the Four-Colour theorem applies to maps on a plane rather than maps on a sphere; still, it refers to it. The four-colour patterns have chirality, though it is not obvious at first glance; it can be seen by pairing the four colours into two groups of two.

6 colors



Patches of two panels of each color such that each pair is perpendicular to the pairs that surround it. That is, the end of one pair fits into the side of another like the panel layout of a traditional volleyball. This is similar to the third three-color arrangement.



The two faces of each colour are placed opposite each other, and whichever angle the ball is viewed from, all six colours are visible.

Supplies

- **For the templates (if you draw them by hand)**
  - Cardboard or Template Plastic, fine-point Pencil, metric Ruler, Protractor, X-acto Knife or Scissors, Paper & Adhesive Tape (optional).
- **For the beanbag**
  - Fabric, Needle and Thread, Scissors, Fabric Marker or Pencil, bean bag Filler, Funnel, Safety Pins (optional).
- **For your information**
  - Unless you're experienced with this sort of thing, I recommend that you browse through the *General Notes and Techniques* chapter before you get started. You may find some tips there that will improve your experience and your beanbags.

How to draw the panel shape for a tennis ball sized bag (diameter ≈ 67mm, $2^5/_8$")

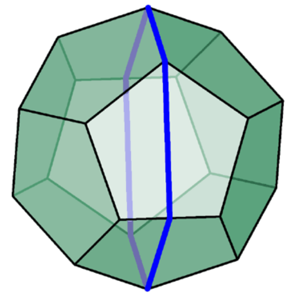
The panel shape is a regular (equilateral) pentagon with 2.5cm sides and 108° corners. There is more than one way to draw a regular pentagon. Following is how I do it. Precision is important because if the sides or angles of the panel are not equal the stitching lines will not match up and you may get a visibly skewed bag.

- **Manual directions**
    1. Draw a 2.5cm line (the length of the pentagon's sides), mark each end of it, and then continue the line to either side of it for several centimeters to aid in accurately aligning the protractor along it.
    2. Place a protractor on the line, center it on each of the endpoint marks in turn, and mark points at 108° angles to the base line.
    3. Draw a line through each pair of points, forming half of the pentagon. Mark a point 2.5cm out from each origin point and continue the line on both ends as in the first step.
    4. Place the protractor on each new line, centered on the new endpoints, and mark points at 108° as in step 2.
    5. Draw lines through the two new pairs of points, completing the pentagon. Make sure the lines meet exactly 2.5cm from their origin points. If they do, you drew a perfect pentagon. Any error you make will be compounded several times in the juggling bag, so be as precise as you can.
    6. Take advantage of the straight sides and cut the template out with an X-acto knife and straight edge.
    7. To make a cutting template that includes an 8mm seam allowance, draw a pentagon with 3.7cm sides. For a different allowance, multiply the desired allowance by 1.4531 and add that to the side length. (Why? See the cutting template alteration instructions below.)
- **SketchUp directions (using millimeter units)**
    1. Use the polygon tool (**Draw** menu -> **Polygon**) set to 5 sides and draw a pentagon with radius = 21.266mm, which will result in a pentagon with 25mm sides. The radius is explained in the alteration section below.
    2. To make a cutting template that includes an 8mm seam allowance, draw a pentagon with 31.155mm radius. For a different allowance, multiply the desired allowance by 1.2361 and add that to the radius. (Why? See the cutting template alteration instructions below.) You could also just draw the cutting pattern around the stitching pattern, using its edges as guides.

Altering the size of the bag

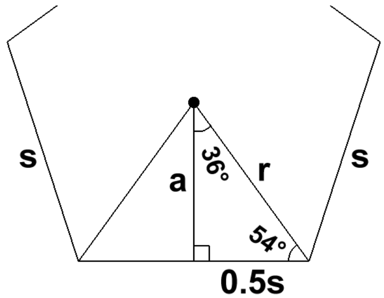
If you don't care about the mathematical explanation of the panel sizing formulas, just skip to where I provide sizing formulas and a table of pre-calculated measurements.

Mathematical background for the sizing formulas



The circumference of the bag is composed of 4 panel heights and 2 side lengths. To alter the size of the bag, you need the relationship between the pentagon's side length and its height so that the circumference can be expressed solely in terms of the side length, enabling you to draw the right template size for a desired bag size. Following is a series of formulas that accomplish this. In the formulas, s = side length. (To draw a pentagon in SketchUp, which uses a defined radius instead of a defined side length, you will need the first formula.)

Note: I only display four decimal places (three for millimeter measurements), but I used full precision (as supplied by my calculator) in all my calculations.



- **Pentagon radius, r** (center to corner) $= 0.5\text{s}/\sin 36° = 0.5\text{s}/\cos 54° = \mathbf{0.8507}\text{s}$
- **Pentagon apothem, a** (center to midpoint of side) $= 0.5\text{s}/\tan 36° = 0.5\text{s}(\tan 54°) = \mathbf{0.6882}\text{s}$
- **Pentagon height** (sum of above) $= \mathbf{1.5388}\text{s}$
- **Dodecahedron circumference** $= 4(1.5388\text{s}) + 2\text{s} = \mathbf{8.1554}\text{s}$
- **Dodecahedron diameter** $\approx 8.1552\text{s}/\pi = \mathbf{2.5959}\text{s}$

This bag has a completely uniform circumference and can be measured in any orientation. By my measurements it will end up **1-4%** larger (depending on whether I fill it loosely or over-fill it) than a calculation based on the above formulas when made with the thick, stiff denim I use and with my stitching practices. I target halfway between the min and max inflations when sizing my patterns.

Template sizing formulas

To make a bag of diameter d with an assumed 2.5% (0.025) inflation, multiply d by pi (3.1416) to get the circumference, divide by 8.1552 to get the pentagon's side length or by 9.5872 to get the pentagon's radius, and finally divide by 1+the decimal inflation (in red) to get the pre-inflated size. (If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may have to substitute a different inflation or deflation factor in the formula. For a method of figuring out your adjustment factor, see the *General Notes and Techniques* chapter under "Calculating your pattern size".) **(Don't forget to multiply the final result by 2.54 if you need to convert inches to centimeters.)** Thus, the formulas are:

Pentagon Side Length $= \text{d} \times \pi \div 8.1554 \div 1.025$

Pentagon Radius $= \text{d} \times \pi \div 9.5872 \div 1.025$

Based on this formula, to adjust the bag's diameter by $^{1}/_{8}$" (3.175mm), change the length of the template's sides by **1.176mm - 1.211mm** (accounting for the 4% and 1% inflations, respectively).

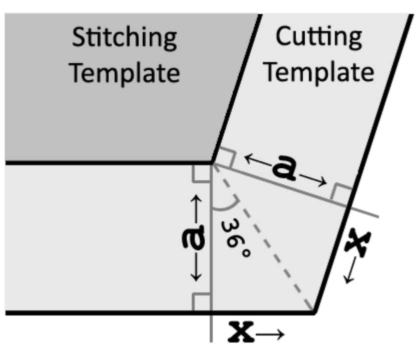
Table of pre-calculated template measurements

Below is a table of template measurements for each $^{1}/_{8}$" diameter increment from 2" to 3" using the 2.5% adjustment factor.

Finished Diameter	**Pentagon Side Length** (cm)	**Pentagon Radius** (cm)
2" (5.08cm)	1.9092	1.6240
2$^{1}/_{8}$" (5.40cm)	2.0285	1.7256
2$^{1}/_{4}$" (5.72cm)	2.1478	1.8271
2$^{3}/_{8}$" (6.03cm)	2.2671	1.9286
2$^{1}/_{2}$" (6.35cm)	2.3865	2.0301
2$^{5}/_{8}$" (6.67cm)	2.5058	2.1316
2$^{3}/_{4}$" (6.99cm)	2.6251	2.2331
2$^{7}/_{8}$" (7.30cm)	2.7444	2.3346
3" (7.62cm)	2.8638	2.4361

Cutting pattern adjustments

If you want to draw the cutting pattern directly (not using the stitching pattern as a guide), you need to use more trigonometry. In the diagram below, x is the amount to extend one end of each of the two sides shown to get a seam allowance "a" on those two sides. You would have to double that to get the full amount by which to extend each side for the cutting template. The full formula for the cutting template extension is below the diagram.



Side Length increase, $2x = 2(\tan 36°)a = \mathbf{1.4531}a$

Radius increase $= 1.4531 \times 0.8507 = \mathbf{1.2361}a$

In the case of the 2.5cm pentagon and 0.8cm seam allowance, you would want the sides of the cutting template to be

$2.5 + 2(\tan 36°)0.8 = 3.6625\text{cm}$

$\text{Radius} = 3.6625 \times 0.8507 = \mathbf{3.1155}\text{cm}$

Making the panels

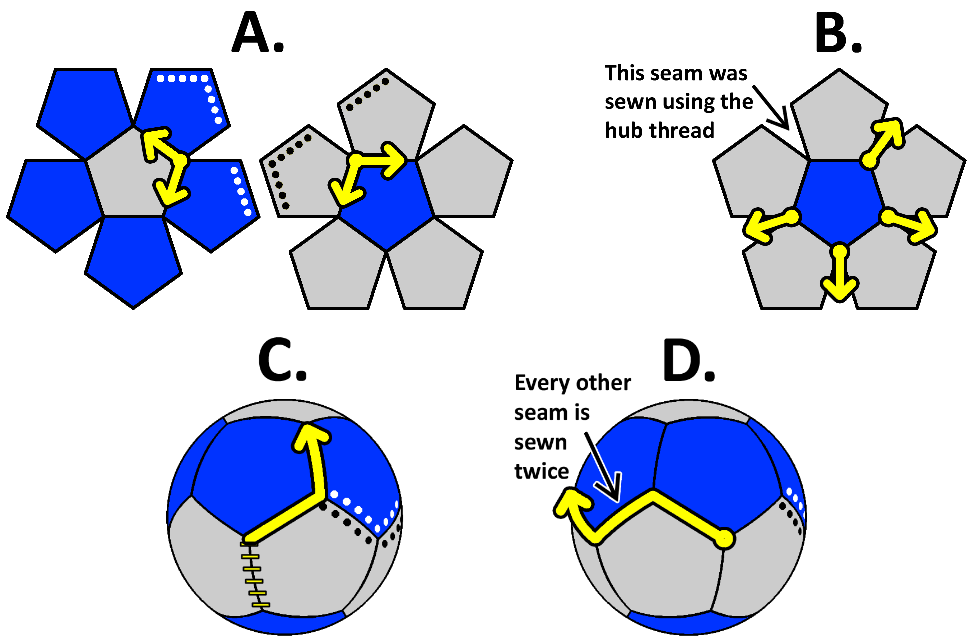
Depending on the type of template you're using and whether it is translucent or not, you must be careful which pattern, cutting or stitching, you trace first so that the second template doesn't hide the lines of the first and prevent you from aligning the two. Do not necessarily use them in the sequence below.

1. Use the larger, cutting template to trace the cutting pattern onto back of the fabric. You will need 12 panels.
2. Use the smaller, stitching template to trace the stitching pattern within each cutting pattern, being sure to center it well.
3. Cut out the panels.

Assembly

Following is my current favorite method of assembling the panels which uses 7-8 threads. The diagrams depict my method. You will be forming two separate hemispheres and then joining one to the other using the "spoke" threads from one to continue up the spoke seams of the other (Illustration C). You will then use a new thread to sew around the "equator" (Illustration D). Your stitching for each hemisphere will begin at the corner indicated by the spot in the center of the arrows in Illustration A and proceed around the "hub" seams and then out the spoke seam. Four additional threads will be used for the four remaining spoke seams on one hemisphere (Illustration B). You will be sewing 3 equatorial seams from the outside (indicated by the dotted lines).

If you use multiple colors, be careful to assemble the panels correctly to form your chosen pattern, especially when you join the two hemispheres. With so many panels and seams, it's easy to make a mistake and misalign the panels. I found it helpful to make a cardboard dodecahedron with colored panels or labels to help me keep track of what I was doing. I recommend building a model anyway to help you visualize the stitching sequence unless you can see it from my description. Index cards, file folders, or something of similar thickness work well for this. Just use your stitching template and cut several layers at a time to produce the panels faster. You could also build a model in SketchUp and color the faces according to your chosen color arrangement.

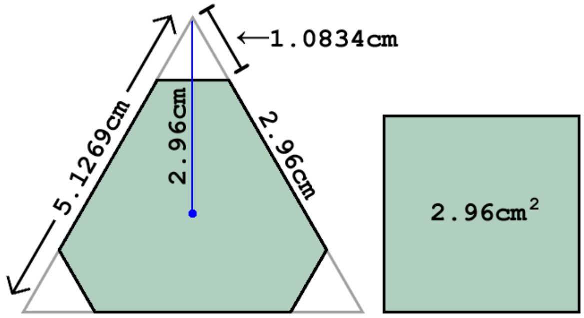


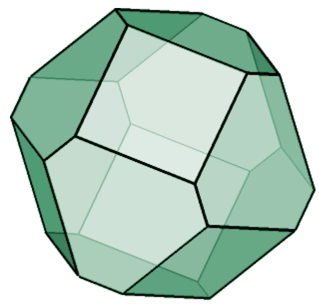
Note that in illustrations C and D the ball is still inside out and so the front stitching lines (the dotted lines) won't actually be visible.

1. Lay the panels out as shown in Illustration A and arrange them according to your color pattern. Note that my color arrangement diagrams assume that the fronts face upward (the orientation makes a difference in some of the arrangements, and this is noted in the descriptions of those arrangements).
2. Use the stitching template to draw stitching lines on the *fronts* of the six panel edges shown with dotted lines to form 3 seams in a row around the equator that will be stitched from the outside. Be sure to center the template well (it should align as well as possible with the stitching patterns on the backs; you can use a needle poked through the panel at a specific point on the stitching line to match the position of the pattern on the front to the one on the back).
3. Cut a thread that is long enough to sew 6 seams.
4. See Illustration A. Start with a center panel and sew a panel to each of its sides beginning at the corner at the center of the pair of arrows in the diagram and proceeding in either direction. Sew the panels with their front faces together so the bag will be inside out. Note that the starting point, which will also be the ending point, is at the corner where the front stitching path splits across two panels. This is important.
5. When you have attached all five panels and the thread has reached the starting point again, sew the adjacent sides of the two outer panels together, connecting the two segments of the front stitching lines, and then tie off the thread and trim it. You are done with this hemisphere for now. The spoke seams (the adjacent edges of the outer panels) will be sewn with threads that continue from the other hemisphere.
6. Construct the other hemisphere in the same way.
7. See Illustration B. Using one new thread per seam, sew the four remaining "spoke" seams of either hemisphere starting at the center panel and sewing outward. Leave the threads hanging at the ends of their seams (there is no need to tie them off, but you may). The threads will have to be long enough to sew two more seams.

8. Before starting the next step, make sure you know how you are going to align the two hemispheres when you join them. They must be joined in such a way as to form your intended color pattern and to make the three front stitching lines on each half meet each other.
9. See Illustration C. Sew the two hemispheres together (still inside out) using the hanging threads by sewing across the equator and up the spoke seams of the hemisphere that didn't have its spoke seams sewn. Note the stitching path in Illustration C. On that side of the front stitching lines you should angle toward them; don't take the other branch and go away from them. Tie each thread and trim it when it reaches the hub of the opposing hemisphere.
10. See Illustration D. Start a thread at one end of the set of seams with the front stitching lines (skipping the seam you already sewed) and sew away from them around the equator until you reach the other end of the front-stitched seams. You will be re-stitching two seams which is annoying, but to avoid double stitching you would have to use more threads or a more complicated assembly method (unless there is a better way I haven't thought of). If you are using the backstitch you can make these duplicate stitches pretty long without causing the fabric to ripple. You can leave the thread hanging and continue it from the outside or tie and trim it and start a new one for the outside. (If you continue it on the outside, you will have to double-sew a third seam to reach the final seams.)
11. Either start a new thread or continue the previous one and sew a few stitches into the front-sewn seams to make it easier to continue from the outside. If you don't need the entire three open seams to turn the bag out you may continue to sew as much as you don't need.
12. Turn the bag right side out through the opening. A good method for this is to use a pen or other slender tool to push the fabric through the opening from the opposite side and then gently invert the bag around the tool. (Consider pressing the seams first; see the *General Notes and Techniques* chapter under "Better seams by ironing".)
13. Pull out the last stitch so that the thread is on the outside where you can get to it. Continue sewing that seam and proceed to the second and third, following the front stitching lines. Fill the bag at some point during this final sewing with a funnel.

14-PANEL EQUIDISTANT CUBOCTAHEDRON (AND POLKA DOT) INSTRUCTIONS






Stitching pattern dimensions for a $2^5/_8$" ball

Design notes

24 long seams, 12 short seams, 84cm (33.1") of stitching plus 3.3cm (1.3") of overlap if you use my assembly method. This design is composed of 6 squares and 8 hexagons.

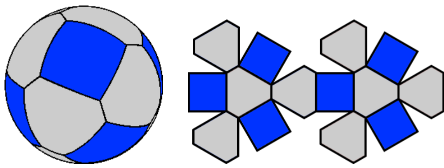
See the section in this chapter titled “Polka dot version” for a variation of the 14-panel structure.

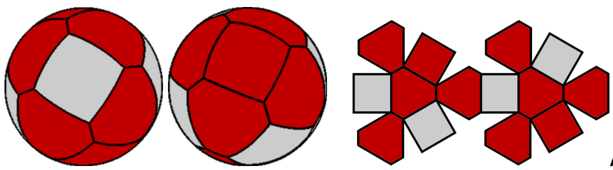
This is my first design that uses two different shapes for the panels. I like the visual variety that introduces. I created this design in April, 2013 to emulate the 14-panel beanbags I often see in online juggling stores. A true cuboctahedron has triangles and squares, but mine is modified so that both face shapes are the same distance from the center, and this truncates the triangles into semiregular hexagons. This modification also reduces the length of the edges which should produce a more spherical shape, and makes the two faces more nearly the same size, producing a more uniform feel and appearance. It also creates new edges which increases the bag’s visual elegance and complexity, but also increases the difficulty in assembling the bag.

This is a more complex and difficult design than the previous ones. The semiregular polygon and the need for two different shapes for the templates make the template-making phase of the manufacturing more complicated, and the large number of seams and the very short seams alternating with the long ones make the assembly phase more difficult than the dodecahedron. But, if you enjoy the process, I think the extra work is worth the beauty of this design. Because this design is based on the cube and the octahedron, you can create a symmetrical checker pattern on this bag similar to those designs. This is something you can’t do on the dodecahedron because it is based on fives, not fours. The dodecahedron is beautiful, but I miss this checkering aesthetic. This was part of my motivation for creating this design. Also, the dodecahedron requires at least four different colors to have no neighboring panels of the same color while the cuboctahedron requires only three.

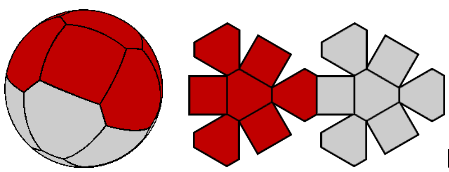
Following are some good color arrangement ideas. Several of the dodecahedron color arrangements are possible with this design, though I feel that the only reason to make the 14-panel design instead of the 12-panel is to show off the panel structure, in which case you'd want to use one of the unique arrangements.

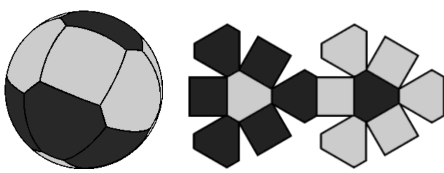
2 colors

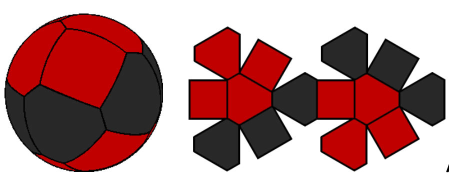

The squares one color and the hexagons another.

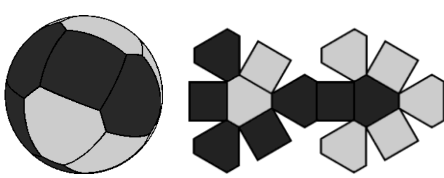

A ring of four diamonds/squares of color A around the middle surrounded by a contrasting color B on the two "caps" above and below this ring. Each cap is composed of a square surrounded by four hexes.

All of the four 2-color arrangements of the dodecahedron design.

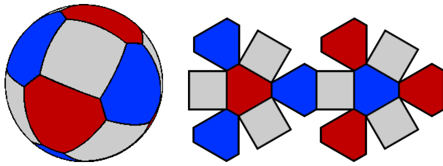

Each hemisphere a different color.

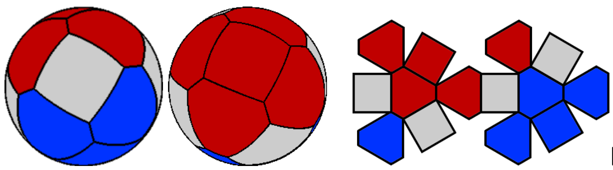

Alternating rings.


A belt of color A around two caps of color B.

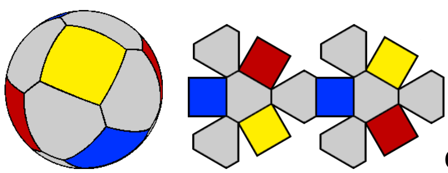

Two intertwining spirals or serpentine curves of contrasting colors.

3 colors

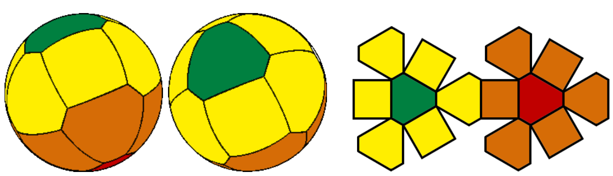
 The squares are color A and the hexagons alternate between colors B and C so that no panel has a neighbor of the same color.

 Based on the second two-color arrangement, this has the same ring of four diamonds/squares of color A around the middle, but with color B on the five panels above it and color C on the five below it. Each “cap” above and below the ring of diamonds is composed of a square surrounded by four hexes.

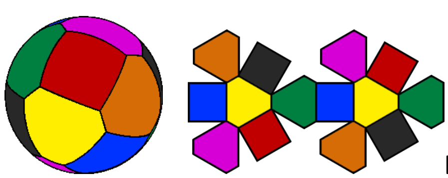
4 colors

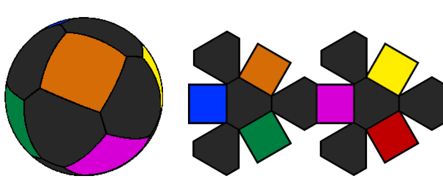
 Colors A, B, and C each on a pair of opposite square faces, and color D on the hexagonal faces. This results in color D framing each of the other colors.

The concentric rings arrangement from the dodecahedron design.



7 colors

 Each color on a pair of opposite panels.

 The hexagons all one color and each square a unique color. This is based on the same concept as the first 4-color arrangement.

Supplies

- **For the templates (if you draw them by hand)**
    - Cardboard or Template Plastic, fine-point Pencil, metric Ruler, Protractor, X-acto Knife or Scissors, Paper & Adhesive Tape (optional).
- **For the beanbag**
    - Fabric, Needle and Thread, Scissors, Fabric Marker or Pencil, bean bag Filler, Funnel, Safety Pins (optional).
- **For your information**
    - Unless you're experienced with this sort of thing, I recommend that you browse through the *General Notes and Techniques* chapter before you get started. You may find some tips there that will improve your experience and your beanbags.

How to draw the panel shapes for a tennis ball sized bag (diameter ≈ 67mm, $2^5/_8$")

There are two panel shapes: a semiregular hexagon and a square. The hexagon is formed by first drawing an equilateral triangle and then truncating the corners. For this design I only provide SketchUp directions, but they can be used for drawing the patterns manually. Note that they use millimeter units while the diagrams at the beginning of the chapter use centimeter units. To convert the triangle radius, r, to a side length, s (which I do for you), the formula is s = $3r/\sqrt{3}$ *=* ***1.7321****r.*

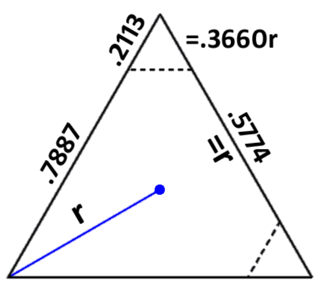
To draw a square manually, you can either use a protractor to measure the 90° angles or you can use a compass placed at equal distances from each end of the initial side to produce small arcs forming two Xs that mark points that are perfectly perpendicular to the endpoints of the initial side. You can use those to align the ruler. The Introduction chapter has illustrated instructions for this method under How to draw perpendicular lines without a protractor.

You can use a similar method to draw the triangle. Instead of measuring out the 60° angles from the initial side, you can draw a vertical line at the center of the base of the triangle whose length is the triangle's height, and then draw the two remaining sides of the triangle so that they meet the end of that line. The length of that line (the height of the triangle) can be found by multiplying the side length by $\sqrt{3}/2$ or 0.8660 *.*

- **SketchUp-centric directions (can be followed manually)**
    1. Use the polygon tool (**Draw** menu -> **Polygon**) set to 3 sides and draw an equilateral triangle with radius = 29.6mm (making the sides 51.269mm long).
    2. Mark points on both ends of all three sides that are 10.834mm distant from each corner; these are the truncation points that form the hexagon. I do this by drawing lines from each corner down each side and using the endpoints of those lines as the truncation points.
    3. Draw a line across each pair of truncation points to produce the semiregular hexagon. Erase the superfluous triangle corners.

4. Draw a square with sides the same length as the triangle's radius, which is also the same as the hexagon's long sides (29.6mm).
5. To make a cutting template that includes an 8mm seam allowance, draw the triangle with a 45.6mm radius and mark the truncation points at 20.072mm from the corners. For a different allowance, multiply the allowance by 2 and add that to the radius (or by 3.4641 and add that to the side length), then multiply the allowance by 1.1547 and add that to the truncation. (Why? See the cutting template alteration instructions below). Draw the square with 45.6mm sides (which is the old width + 2 × allowance). You could also just draw the cutting patterns around the stitching patterns, using their edges as guides.

Altering the size of the bag



The high precision values are 0.2113248654, 0.3660254038, 0.5773502692, and 0.7886751346.

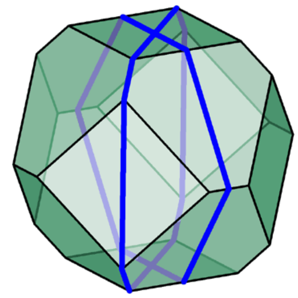
Mathematical background for the sizing formulas

The key element of the design is the hexagon, which is derived from an equilateral triangle. The square is based simply on the hexagon's long side, which happens to be the same as the triangle's radius. For a full discussion of this design, see the *How I Developed My Designs* chapter under "Equidistant cuboctahedron". Here I will just repeat the panel diagram with the side length proportions. If you don't care about the mathematical explanation of the panel sizing formulas, just skip to where I provide sizing formulas and a table of pre-calculated measurements.

Once you have used the circumference formulas that follow to decide on a panel size for your beanbag, multiply the triangle radius by 0.3660 to get the amount of truncation for each end of each side (or multiply the side length by 0.2113) and make the square the same width as the triangle's radius (which is the triangle's side length × 0.5774).

For the design formulas, I'll define the following variables:
$\boldsymbol{h_t}$ = height of the triangle
$\boldsymbol{h_h}$ = height of the hexagon
$\boldsymbol{s_t}$ = side length of the triangle
$\boldsymbol{s_{hl}}$ = long side length of the hexagon (same as width of the square)
$\boldsymbol{s_{hs}}$ = short side length of the hexagon
$\boldsymbol{d}$ = diagonal of the square
r = radius of the triangle = $\boldsymbol{s_{hl}}$

Note: I only display four decimal places (three for millimeter measurements), but I used full precision in all my calculations. Also, if you want to understand where the unexplained expressions come from (those that don't come from the diagram above), read about equilateral triangles and, for the cutting template, 30-60-90 right triangles. I learned/relearned about them using Wikipedia (http://en.wikipedia.org/wiki/Equilateral_triangle, http://en.wikipedia.org/wiki/30_60_90_triangle#30.E2.80.9360.E2.80.9390_triangle).



There are two simple ways to define the circumference of the polyhedron. One is 4 x height of the hexagon + 2 x width of the square (which is the same as the long side of the hexagon) and the other is 4 x diagonal of the square + 4 x short side of the hexagon.

$$\text{Circumference A} = 4h_h + 2s_{hl}$$

$$\text{Circumference B} = 4d + 4s_{hs}$$

I want everything to be in terms of s_{hl} so I will use the following conversions for circumference A: $h_h = 0.7887h_t$, $h_t = (\sqrt{3}/2)s_t$, and $s_t = s_{hl}/0.5774$. For circumference B I will use the following conversions: $= \sqrt{2}s_{hl}$, $s_{hs} = 0.3660s_{hl}$. So the circumference formulas are now

$$\text{Circumference A} = 4\left[(0.7887)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{0.5774}\right)\right]s_{hl} + 2s_{hl} \;=\; 4\left(\frac{1.3660\sqrt{3}}{2}\right)s_{hl} + 2s_{hl}$$
$$= \; 6.7321s_{hl}$$

$$\text{Circumference B} = 4\sqrt{2}s_{hl} + 4(0.3660)s_{hl} \;=\; 7.1210s_{hl}$$

The two measurements differ by 5.47%, which is significant enough that I will use the average of the two.

$$\textbf{Average Circumference} = \; \mathbf{6.9162s_{hl}} = \mathbf{6.9162}\boldsymbol{r}$$

I repeat the above value to emphasize that the long side of the hexagon, s_{hl}, is the same as the radius of the triangle, *r*. This is useful for drawing it in SketchUp. I don't know why this is and I can't prove it mathematically, but it's a convenient property of the shape.

To draw the template manually, you will have to express the circumference in terms of the length of the triangle's side. To do that, multiply the radius by $\sqrt{3}/3$ = 0.5774.

$$\textbf{Average Circumference} = \; \mathbf{3.9931s_t}$$

This bag has a completely uniform circumference (when tightly filled) and can be measured in any orientation. By my measurements it will end up **0.8-3.8%** larger (depending on whether I fill it loosely or over-fill it) than a calculation based on the above formulas when made with the thick, stiff denim I use and with my stitching practices. I target halfway between the min and max inflations when sizing my patterns.

Template sizing formulas

To make a bag of diameter d with an assumed 2.3% (0.023) inflation, multiply d by pi (3.1416) to get the circumference, divide by 6.9162 to get the triangle's radius and the square's width, or by 3.9931 to get the triangle's side length, and finally divide by 1+the decimal inflation (in red) to get the pre-inflated size. (If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may have to substitute a different inflation or deflation factor in the formula. For a method of figuring out your adjustment factor, see the *General Notes and Techniques* chapter under "Calculating your pattern size".) **(Don't forget to multiply the final result by 2.54 if you need to convert inches to centimeters.)** Thus, the formulas are:

Guide triangle radius and square width $= d \times \pi \div 6.9162 \div 1.023$

Guide triangle side length $= d \times \pi \div 3.9931 \div 1.023$

Guide triangle truncation to form the hexagon $= \text{radius} \times 0.3660$ or $\text{side length} \times 0.2113$

Based on this formula, to adjust the bag's diameter by 1/8" (3.175mm), change the triangle's radius (same as the length of the templates' long sides) by **1.389mm - 1.431mm** (accounting for the 3.8% and 0.8% inflations, respectively).

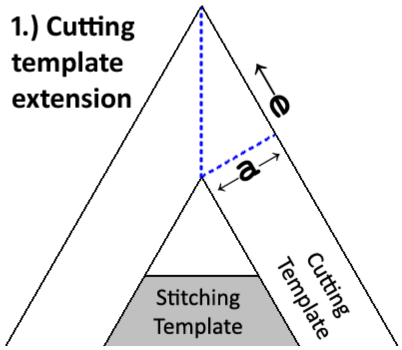
Table of pre-calculated template measurements

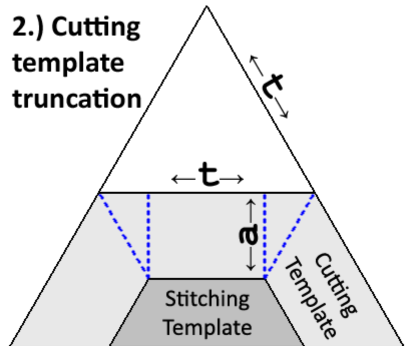
Below is a table of template measurements for each 1/8" diameter increment from 2" to 3" using the 2.3% adjustment factor.

Finished Diameter	**Guide Triangle Radius and Square Width** (cm)	**Guide Triangle Side Length** (cm)	**Guide Triangle Truncation to Form the Hexagon** (cm)
2" (5.08cm)	2.2556	3.9069	0.8255
2 1/8" (5.40cm)	2.3966	4.1510	0.8771
2 1/4" (5.72cm)	2.5376	4.3952	0.9287
2 3/8" (6.03cm)	2.6786	4.6394	0.9803
2 1/2" (6.35cm)	2.8196	4.8836	1.0319
2 5/8" (6.67cm)	2.9605	5.1278	1.0835
2 3/4" (6.99cm)	3.1015	5.3719	1.1351
2 7/8" (7.30cm)	3.2425	5.6161	1.1867
3" (7.62cm)	3.3835	5.8603	1.2383

Cutting pattern adjustments

If you want to draw the hexagon cutting pattern directly (not using the stitching pattern as a guide), you need to use more trigonometry. In diagram 1 below, *e* is the amount to extend one end of each side to get a seam allowance "*a*". You would have to double that to get the full amount by which to extend each side for the cutting template and then, if you're drawing it in SketchUp, convert it to a change in radius. The full formula for the cutting template extension is below the diagram. In diagram 2, t is the amount of truncation to apply to each end of each side. In both diagrams, the triangles outlined in blue dotted lines are 30-60-90 right triangles, which helps in solving them. I won't go through all the trigonometry; I'll just give the formulas.





1. **Side Length Extension,** $\mathbf{2e} = 2\sqrt{3}a = \mathbf{3.4641a}$

   Radius increase $= 2\sqrt{3}\left(\frac{\sqrt{3}}{3}\right)a = \mathbf{2a}$

   In the case of my design with a seam allowance, *"a"*, of 8mm this is 16mm

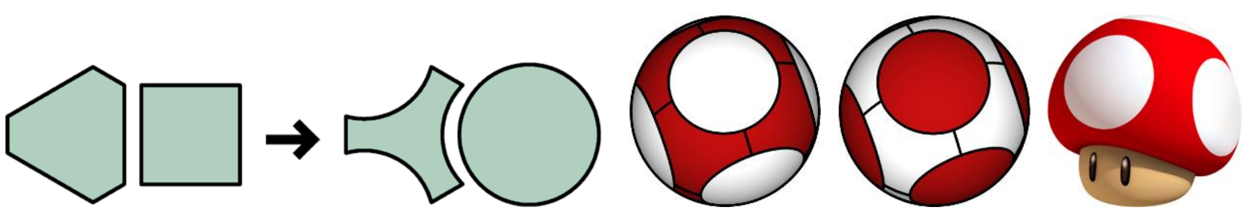
2. t equals the short side of the stitching template plus the tops of the two triangles in blue. The short side is 0.3660×radius (of the stitching template triangle) which is just the stitching template truncation. The triangle tops are each $a/\sqrt{3}$. So the formula for t is

   Truncation, t $= (\text{stitching truncation}) + {}^{2a}/_{\sqrt{3}} = (\text{stitching truncation}) + \mathbf{1.1547}\text{a}$

   In the case of my design this is

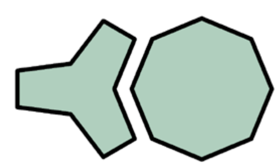
   $0.3660(29.6) + 1.1547(8) = 20.072\text{mm}$

Polka dot version



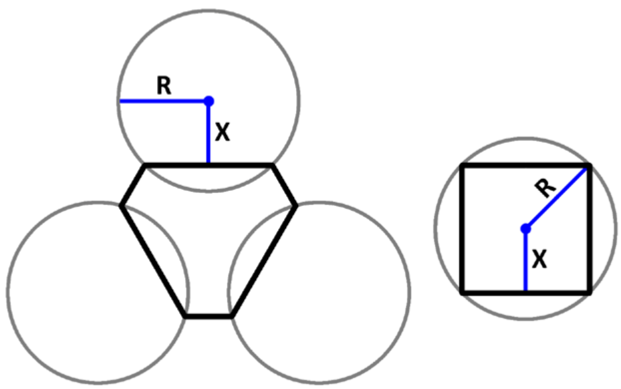
Super Mario Bros. mushroom from http://mario.wikia.com/wiki/File:Mushroom-SM3DL.png

By converting the square panels into circles and making corresponding circular cuts in the long sides of the hexagon, you can get a Super Mario Bros. mushroom look. Be warned that sewing a convex curve to a concave curve, especially such tight curves, is more difficult than sewing two matching patterns together. You cannot simply lay the panels flat together and sew along the line; you will have to continually adjust the panels as you sew to keep the point of each pattern you are sewing matched up. This is especially difficult when sewing from the outside. An alternative version of this concept that would be easier to assemble would have an octagon instead of a circle and corresponding 135° angular cuts in the sides of the hexagon as shown on the right.

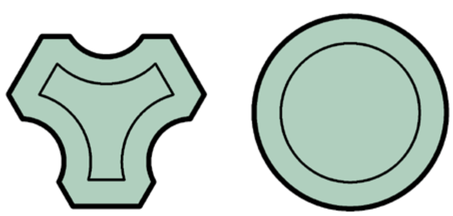


Below are photos of my proof-of-concept bag followed by the bag that inspired this design (which has smaller polka dots). I'm not as happy with the look of my bag as I thought I would be, but it's kind of cool. It was quite difficult and frustrating to make and it looked rather lumpy when it was done due to all the puckering of the seams caused by their curvature. I had to spend a lot of time ironing the seams and adjusting the seam allowances to make it look smooth and pretty like my others. It might not have been so difficult if I had used a thinner, more flexible fabric, but take warning that this is not an easy bag to make. It does have a great tactile quality to it, though. The six large, soft, round pads separated by the firmer, narrow, slightly recessed areas feel good in the hand and give it a good grip.

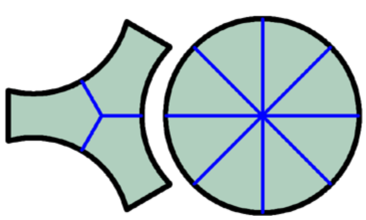
Third photo from http://www.footbagshop.com/reaper-14-net-red-white.html



The radius of the curves is equal to half the diagonal of the square. The length of X (from the hexagon's side to the center of the circle) is equal to half the square's side length.



To make cutting templates for this pattern, increase the radius of the circular panel by the desired seam allowance and decrease the hex curve radius by the same amount.



You will need four markers on the circle corresponding to the corners of the original square so you can align those points with the corners of the tri-curve panels as you sew. Because of the difficulty of sewing concave and convex curves to each other, I recommend adding a second set of four markers in between the other four for a total of eight, and making corresponding marks in the center of each curve of the tri-curve panels. These markers will aid in progressing your stitching equally along each curve of a seam and preventing the seam from becoming distorted.

Making the panels

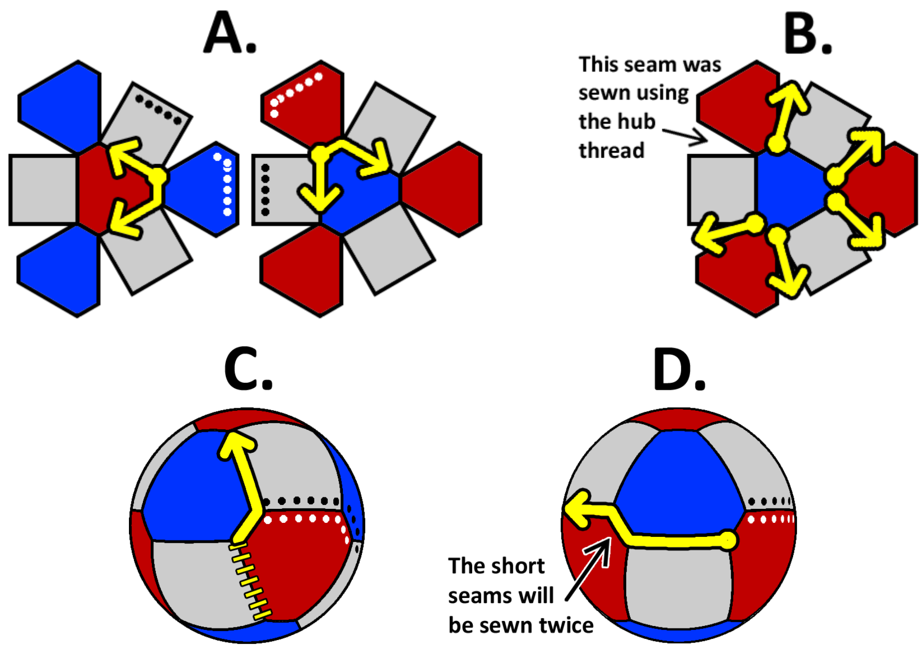
Depending on the type of template you're using and whether it is translucent or not, you must be careful which pattern, cutting or stitching, you trace first so that the second template doesn't hide the lines of the first and prevent you from aligning the two. Do not necessarily use them in the sequence below.

1. Use the larger, cutting templates to trace the cutting patterns onto back of the fabric. You will need 6 squares and 8 hexagons.
2. Use the smaller, stitching templates to trace the stitching patterns within each cutting pattern, being sure to center them well.
3. Cut out the panels.

Assembly

Following is my current favorite method of assembling the panels which is similar to the method I use for the dodecahedron. It uses 7-8 threads. The diagrams depict my method. You will be forming two separate hemispheres and then joining one to the other using the "spoke" threads from one to continue up the spoke seams of the other (Illustration C). You will then use a new thread to sew around the "equator" (Illustration D). Your stitching for each hemisphere will begin at the corner indicated by the spot in the center of the arrows in Illustration A and proceed around the "hub" seams and then out the spoke seam. Five additional threads will be used for the five remaining spoke seams on one hemisphere (Illustration B). You will be sewing 3 equatorial seams (two longs and a short) from the outside (indicated by the dotted lines).

If you use multiple colors, be careful to assemble the panels correctly to form your chosen pattern, especially when you join the two hemispheres. With so many panels and seams, it's easy to make a mistake and misalign the panels. I found it helpful to make a cardboard model with colored panels or labels to help me keep track of what I was doing. I recommend building a model anyway to help you visualize the stitching sequence unless you can see it from my description. Index cards, file folders, or something of similar thickness work well for this. Just use your stitching template and cut several layers at a time to produce the panels faster. You could also build a model in SketchUp and color the faces according to your chosen color arrangement.

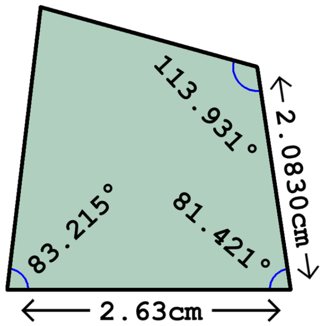


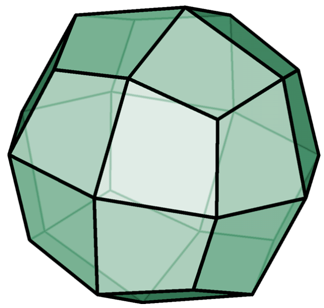
Note that in illustrations C and D the ball is still inside out and so the front stitching lines (the dotted lines) won't actually be visible.

1. Lay the panels out as shown in Illustration A above and arrange them according to your color pattern. I find it easier to lay them front faces upward so I can more easily draw the front stitching lines in the correct positions.
2. Use the stitching template to draw stitching lines on the *fronts* of the six panel edges shown with dotted lines to form 3 seams in a row (a long, a short, and a long) around the equator that will be stitched from the outside. Be sure to center the template well (it should align as well as possible with the stitching patterns on the backs; you can use a needle poked through the panel at a specific point on the stitching line to match the position of the pattern on the front to the one on the back).
3. Cut a thread that is long enough to sew 4 long seams (plus 3 shorts).
4. See Illustration A. Start with a center panel and sew a panel to each of its sides beginning at the corner in the center of the pair of arrows in the diagram and proceeding in either direction. Sew the panels with their front faces together so the bag will be inside out. Note that the starting point, which will also be the ending point, is at the corner where the front stitching path splits across two panels. This is important.
5. When you have attached all six panels and the thread has reached the starting point again, sew the adjacent sides of the two outer panels together, connecting the two segments of the front stitching lines, and then tie off the thread and trim it. You are done with this hemisphere for now. The spoke seams (the adjacent edges of the outer panels) will be sewn with threads that continue from the other hemisphere.
6. Construct the other hemisphere in the same way.
7. See Illustration B. Using one new thread per seam, sew the five remaining "spoke" seams of either hemisphere starting at the center panel and sewing outward. Leave the threads hanging at the ends of their seams (there is no need to tie them off, but you may). The threads will have to be long enough to sew another long and short seam.

8. Before starting the next step, make sure you know how you are going to align the two hemispheres when you join them. They must be joined in such a way as to form your intended color pattern and to make the three front stitching lines on each half meet each other.
9. See Illustration C. Sew the two hemispheres together (still inside out) using the hanging threads by sewing across the equator and up the spoke seams of the hemisphere that didn't have its spoke seams sewn. Remember that each hanging thread will sew first a short seam and then a long. This will help you not to make a mistake and sew the wrong seam. Tie each thread and trim it when it reaches the hub of the opposing hemisphere.
10. See Illustration D. Start a thread at one end of the set of seams with the front stitching lines and sew away from them around the equator until you reach the other end of the front-stitched seams. You will be re-stitching three short seams which is annoying, but to avoid double stitching you would have to use more threads or a more complicated assembly method (unless there is a better way I haven't thought of). If you are using the backstitch you can make these duplicate stitches pretty long without causing the fabric to ripple. You can leave the thread hanging and continue it from the outside or tie and trim it and start a new one for the outside. (If you continue it on the outside, you will have to double-sew a forth seam to reach the final seams.)
11. Either start a new thread or continue the previous one and sew a few stitches into the front-sewn seams to make it easier to continue from the outside. If you don't need the entire three open seams to turn the bag out you may continue to sew as much as you don't need.
12. Turn the bag right side out through the opening. A good method for this is to use a pen or other slender tool to push the fabric through the opening from the opposite side and then gently invert the bag around the tool. (Consider pressing the seams first; see the *General Notes and Techniques* chapter under "Better seams by ironing". For my proof-of-concept bag of this design I decided that pressing the short seams would be too difficult and probably not very important and so I only pressed the long seams. The bag turned out very well as you can see from the photo at the beginning of the chapter. A few short seams puckered where you can't see, but they look good enough.)
13. Pull out the last stitch so that the thread is on the outside where you can get to it. Continue sewing that seam and proceed to the second and third, following the front stitching lines. Fill the bag at some point during this final sewing with a funnel.

24-PANEL CUSTOM DELTOIDAL ICOSITETRAHEDRON INSTRUCTIONS



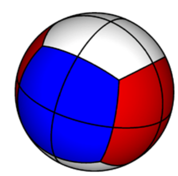
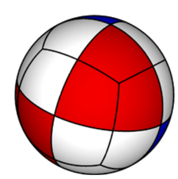


Stitching pattern dimensions for a $2\,^5/_8$" ball

Design notes

24 long seams, 24 short seams, 26 vertices, 113.1cm (44.5") of stitching if you don't overlap any seams. This design is composed of 24 kites (definition: http://en.wikipedia.org/wiki/Kite_%28geometry%29).

I haven't yet figured out an optimal stitching pathway for this design. If I ever do, I'll add a diagram and instructions for it.

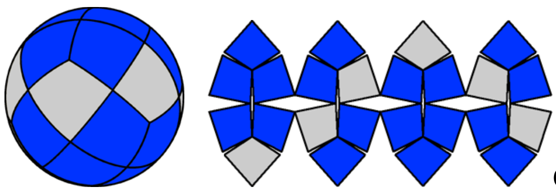



I was inspired to design this bag by a footbag design I found (see the *Other Designs and Variations* chapter under "A lineup of footbag panel structures" and look for the 24-panel design). I have found many footbag designs, but this one stood out to me. It has a comparatively reasonable number of panels, it simply looks attractive to me, and I discovered that it is effectively a combination of the cube and octahedron designs and so can be made to look like either one of those but with more uniformity and roundness because of the greater number of panels. I designed my variation of the polyhedron to have vertices that are as uniform and blunt as possible. For an explanation of how I designed it, look up the section on that design in the *How I Developed My Designs* chapter.

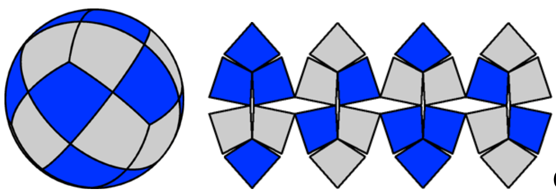
I made my second proof-of-concept bag (the larger photo above) with denim and an 8mm seam allowance, but that is not really an ideal fabric to use for this design. A thin, non-fraying fabric like Ultrasuede would probably be much easier to work with for a design with so many small panels. I ironed the seams flat, but there are so many of them that it was very tedious. A thin fabric would make this task less important. I made my first bag (the smaller photo) with a 4mm seam allowance and I only half-heartedly ironed the seams. It turned out okay, but it wasn't as smooth.

Following are some color arrangement ideas. These arrangements are unique to this panel structure, but keep in mind that the structure also supports all of the arrangements of the 4-panel beach ball, cube, and octahedron. On the right is an octahedron arrangement.

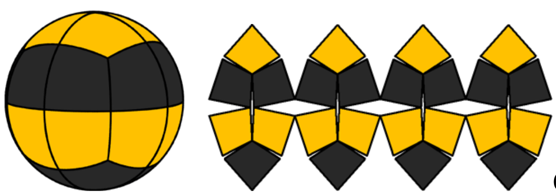
2 colors



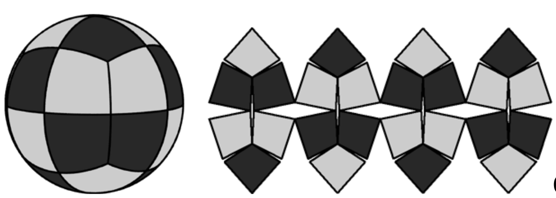
Color A on a ring of 6 panels that join at their lateral corners and color B on the remaining 18 panels.



Color A on two opposite patches of three panels and on each panel touching the corners of each patch. Color B on the remaining 12 panels.

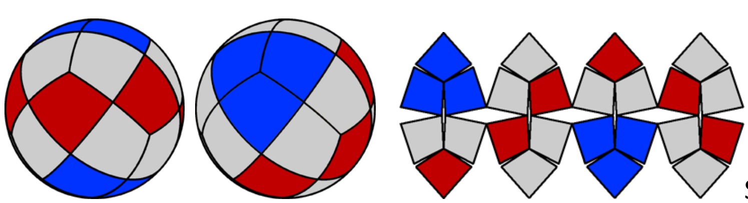


Color A on a patch of 4 panels, color B on a ring of 8 panels around that patch, color A on the adjacent ring of 8 panels, and color B on the final patch of 4 panels.

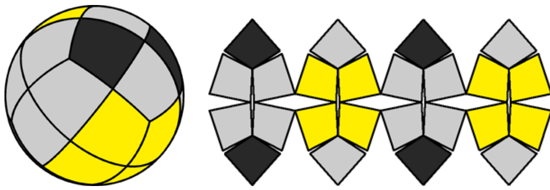


One way to design this arrangement is to divide the panel structure into four 6-panel patches in a beach ball layout and arrange the two colors on alternating horizontal stripes that also alternate from patch to patch.

3 colors

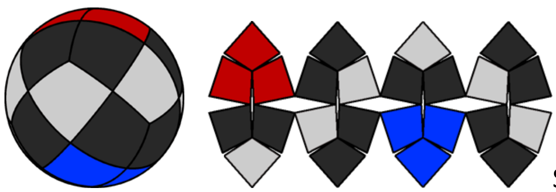


Same concept as the second two-color arrangement. Color A on opposite triangular patches of three panels, color B surrounding these patches with two panels on each side of the three-panel patches (12 panels total), color C on the remaining 6 panels, forming a ring in which each panel is joined to the adjacent panels by the lateral corners.



This is an arrangement I came across on a footbag store's website[19]. I'm not going to try to describe it verbally, but the diagram and the ball illustration should give you the idea.

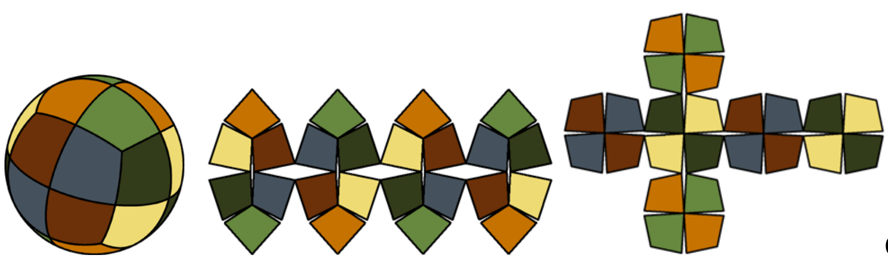
4 colors



Same concept as the first 3-color arrangement except that the two opposite patches of color A now have two unique colors.

Same concept as the third 2-color arrangement. Color A on a patch of 4 panels, color B on a ring of 8 panels around the patch, color C on the adjacent ring of 8 panels, and color D on the final patch of 4 panels.

6 colors



One way to thnk about this arrangement is as an octahedron with an alternating two-color arrangement, but with each 3-panel patch's color replaced with a distinct, coordinated triplet of colors which are sequenced in the same way for each matching patch in terms of clockwise/counterclockwise directions. You can also think of this as a 3-color cube with each color replaced with a checkered pattern of two colors. Each distinct set of two colors lies on opposite patches of four panels.

[19] http://footbagcanada.com/detail.aspx?ID=93

Supplies

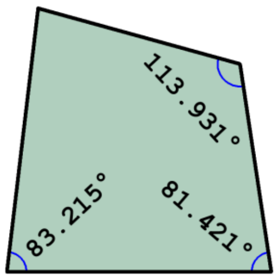
- **For the templates (if you draw them by hand)**
  - Cardboard or Template Plastic, fine-point Pencil, metric Ruler, Protractor, X-acto Knife or Scissors, Paper & Adhesive Tape (optional).
- **For the beanbag**
  - Fabric, Needle and Thread, Scissors, Fabric Marker or Pencil, bean bag Filler, Funnel, Safety Pins (optional).
- **For your information**
  - Unless you're experienced with this sort of thing, I recommend that you browse through the *General Notes and Techniques* chapter before you get started. You may find some tips there that will improve your experience and your beanbags.

How to draw the panel shape for a tennis ball sized bag (diameter ≈ 67mm, 2 $^{5}/_{8}$")

The panel shape is a kite with specific angles. It is very simple to draw either by hand or with SketchUp. SketchUp displays up to three-digit precision for angles and I include that precision because I like precision, not because the beanbag needs it. You can round the angles to whole or half degrees if you're drawing the panel by hand. This will alter the length of the sides a bit, but probably not enough to matter.

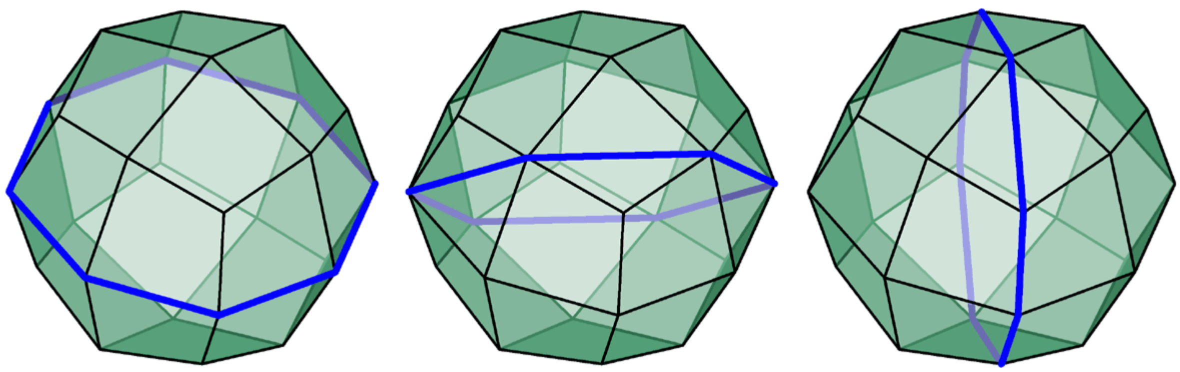
- **Directions**
  1. Draw a 2.63cm line (if you're drawing it by hand, include some excess at each end to aid in aligning the protractor). This is one of the two longer sides of the kite.
  2. Measure an 83.215° angle at one end of the line and draw a matching line at that angle.
  3. Measure 81.421° angles at the outer ends of each line and draw lines at those angles. The two new lines should meet at an angle of 113.931° and should be 2.083cm long (79.2% the length of the long sides).
  4. I drew my pattern with SketchUp and made the cutting template simply by measuring 8mm at perpendicular angles to each side of the stitching pattern and drawing the cutting pattern around the stitching pattern. I don't feel like doing the math for this design, but if you want to, look at the dodecahedron chapter cutting template instructions to help you get started figuring it out.

Altering the size of the bag



Above is the generic definition of the panel shape. The short sides are 79.20% the length of the long and the panel length (from obtuse corner to opposite) is 88.81% of the width. If you don't care about the mathematical explanation of the panel sizing formulas, just skip to where I provide sizing formulas and a table of pre-calculated measurements.

There are three simple ways to define the circumference of the polyhedron. I will express them below as ratios of each other, with my preferred method, the first, being the basis of comparison.



Long Side × 8 = 1

Kite Width × 6 = 0.9960

Kite Length × 4 + Short Side × 4 = 0.9851

The circumferences are pretty close and the first is easiest to use for drawing the panel. Once you have calculated the length of the kite's long side needed to produce the bag size you want, draw that side and the matching side at an 83.215° angle to each other, measure an 81.421° angle at the ends of each side and the short sides and final angle should result.

This bag, when made with thick, nearly non-stretch denim I use and with my stitching practices, has a negative inflation when loosely filled and very little inflation when tightly filled and stretched (compared to a calculation based on the above formulas). The inflation as I measured it is **-2.05 – +0.67%**. I target halfway between the min and max inflation when sizing my patterns, which results in -0.7%.

Template sizing formulas

To make a bag of diameter d with an assumed -0.7% (-0.007) inflation, multiply d by pi (3.1416) to get the circumference, divide by 8 to get the kite's long side length, and finally divide by 1+the decimal inflation (in red) to get the pre-inflated size. (If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may have to substitute a different inflation or deflation factor in the formula. For a method of figuring out your adjustment factor, see the *General Notes and Techniques* chapter under "Calculating your pattern size".) The kite's short side is calculated from the long side. **(Don't forget to multiply the final result by 2.54 if you need to convert inches to centimeters.)** Thus, the formulas are:

Kite's Long Side $= d \times \pi \div 8 \div 0.993$

Kite's Short Side $= \text{Long Side} \times 0.7920$

Based on this formula, to adjust the bag's diameter by $^1/_8$" (3.175mm), change the kite's long side by **1.239mm - 1.273mm** (accounting for the 0.67% and -2.05% inflations, respectively).

Table of pre-calculated template measurements

Below is a table of template measurements for each $^1/_8$" diameter increment from 2" to 3" using the -0.7% adjustment factor.

Finished Diameter	Kite's Long Side (cm)	Kite's Short Side (cm)
2" (5.08cm)	2.0090	1.5911
2$^1/_8$" (5.40cm)	2.1345	1.6906
2$^1/_4$" (5.72cm)	2.2601	1.7900
2$^3/_8$" (6.03cm)	2.3857	1.8894
2$^1/_2$" (6.35cm)	2.5112	1.9889
2$^5/_8$" (6.67cm)	2.6368	2.0883
2$^3/_4$" (6.99cm)	2.7623	2.1878
2$^7/_8$" (7.30cm)	2.8879	2.2872
3" (7.62cm)	3.0135	2.3867

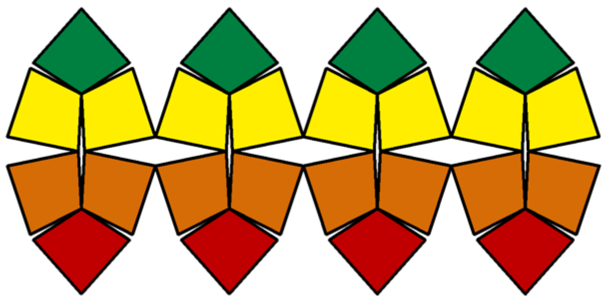
Making the panels

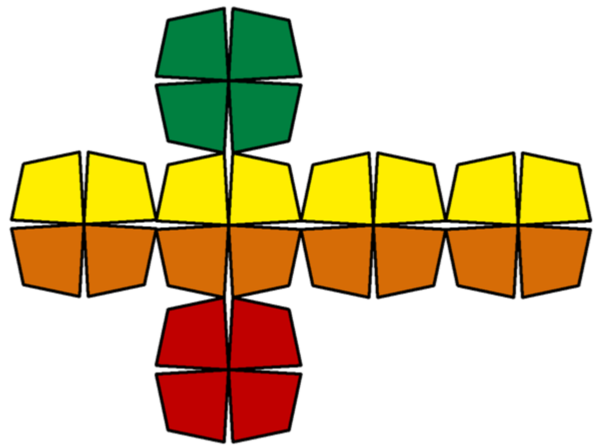
Depending on the type of template you're using and whether it is translucent or not, you must be careful which pattern, cutting or stitching, you trace first so that the second template doesn't hide the lines of the first and prevent you from aligning the two. Do not necessarily use them in the sequence below.

1. Use the larger, cutting template to trace the cutting pattern onto back of the fabric. You will need 24 panels.
2. Use the smaller, stitching templates to trace the stitching patterns within each cutting pattern, being sure to center them well.
3. Cut out the panels.

Assembly

I have not yet figured out an optimal stitching pathway for this panel structure. One way to plan your stitching and color arrangement is to consider that the panels can be thought of either as eight sets of three panels arranged like the triangular faces of an octahedron or as six sets of four panels arranged like the faces of a cube. The former set has a three-way vertex in the center of each patch and the latter a four-way. The illustrations below depict this. There may also be other good ways to lay them out.



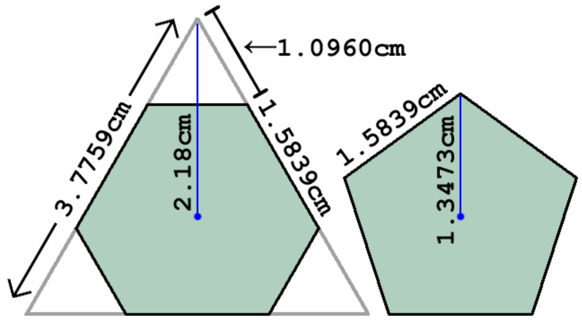


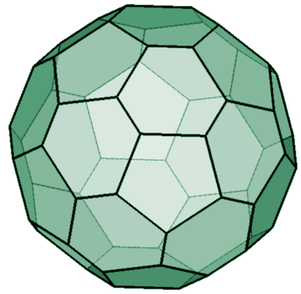
To help you correctly assemble the panels, remember that each of the three types of vertices of the structure is composed purely of one type of panel corner.   The three-way vertices are formed by the obtuse corners of the panels, the four-way vertices corresponding to the faces of a cube or corners of an octahedron are composed of the corners opposite the obtuse corners, and the vertices corresponding to the edges of the cube or octahedron are composed of the lateral corners.

If you use multiple colors, be careful to assemble the panels correctly to form your chosen pattern. With so many panels and seams, it's easy to make a mistake and misalign the panels. I recommend making a SketchUp model or a cardboard model to help you keep track of what you're doing. I also used small, numbered strips of masking tape on each edge of my cardboard model to help me follow a stitching sequence I wanted to try. Index cards, file folders, or something of similar thickness work well for a model. Just use your stitching template and cut several layers at a time to produce the panels faster. To learn how to draw this polyhedron in SketchUp, see the section on this design in the *How I Developed My Designs* chapter.

I was able to turn my denim bag out through an opening of two long seams in a row, but it was a tight fit.

32-PANEL EQUIDISTANT ICOSIDODECAHEDRON (AND VARIATIONS) INSTRUCTIONS






Stitching pattern dimensions for a $2^{5}/_{8}$" ball

Design notes

60 long seams, 30 short seams, 60 vertices, 127.9cm (50.4") of stitching if you don't overlap any seams. This design is composed of 12 pentagons and 20 hexagons.

This is an advanced design with multiple variations and I'm not including step-by-step instructions for drawing the panel shapes. If you need help with that, look at the relevant sections of the 12-panel and 14-panel instructional chapters as they have similar face shapes. Also, I haven't yet figured out an optimal stitching pathway. If I ever do, I'll add a diagram and instructions for it.

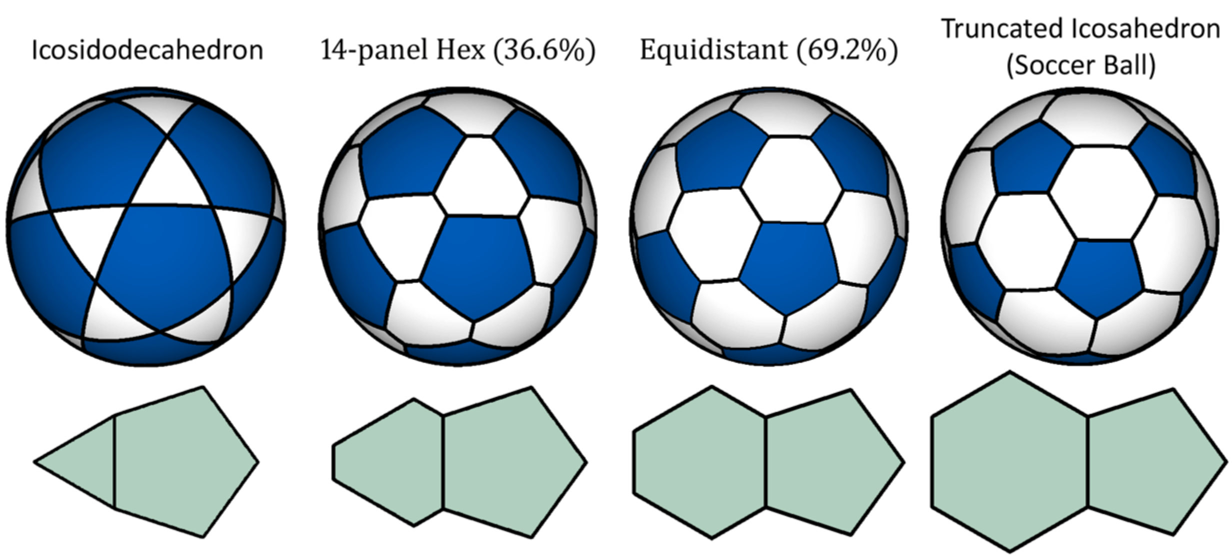
This design is rather difficult and tedious to make, but it is beautiful and fun to show off. I made mine with denim and a 4mm seam allowance, but that is not really an ideal fabric to use for this design. The panels are so small and denim frays so much and is so thick that the bag becomes a frizzy mess and is difficult to sew precisely. It was definitely doable, but I wouldn't want to do it again. A thin, non-fraying fabric like Ultrasuede would probably be much easier to work with. Also, I did *not* iron the seam allowances flat. It would be way too difficult and time-consuming, but also it turns out that I really like the texture of the dense network of prominent seams.

I was inspired to design this bag by the Tossaball Phat Tyre 32-panel juggling bag made by Flying Clipper (shown at the beginning of the color arrangement section below). I love this design. This type of panel structure is commonly used for footbags (hacky sacks), but Flying Clipper is the only manufacturer I have seen using it for a juggling bag. It is nearly identical to the traditional soccer ball design, but soccer balls are truncated icosahedrons which have equilateral hexagons rather than the slightly triangular hexes my design uses. My hex design accomplishes two things: it makes the solid equidistant, and it makes the two face shapes nearly the same size. By equidistant I mean that both face shapes are the same distance from the center of the solid (which is the same as to say that all opposing faces are the same distance from each other), giving the solid a more constant diameter. For an explanation of how I designed the equidistant icosidodecahedron, look up the section on that design in the *How I Developed My Designs* chapter.

I could have called my design an "equidistant truncated icosahedron", but I happened to use the icosidodecahedron as the starting point for creating the shape, so I named it after that. The series of images below show the progression from an icosidodecahedron to a truncated icosahedron, both of which are Archimedean solids, meaning (among other things) that they are composed of regular polygonal faces. My designs form two transition points at roughly $^{1}/_{3}$ intervals between the two Archimedean solids and have hexes that are only semiregular (the percentages indicate the ratio of the short side to the long).

It turns out that the equidistant icosidodecahedron has hexes that are more nearly equilateral (less triangular) than Flying Clipper's appear to be. The short side is 69.2% the length of the long. I tried building a CG version that uses the hexes from the 14-panel Equidistant Cuboctahedron (in which the radius of the guide triangle equals the long side of the hex) to see how that would look. That hex has a short side that is 36.6% the length of the long. I think Flying Clipper's hexes are somewhere in between, but closer to the 36.6% hex.

Below is a comparison of the four versions of the 32-panel design with the panel shapes to show the relative size of the two panels. The equidistant version has nearly equal face sizes (the hexes are larger in area by only 6.9%). The truncated icosahedron has hexes that are 51% larger than the pents (the distance between the pents is 2.7% greater than between the hexes). The version with the 14-panel hex has pents that are larger than the hexes by 52.9% (the distance between opposing hexes is 3.7% greater than between the pents).



I did some research on the proportions of the face shapes in footbags and there seems to be no consensus. The photos I've found appear to have many different hexes, and some use triangles. Each stitcher seems to have his own preference. I have so far found three people on forums who have given a specific ratio of the short side to the long side. Two of them prefer 40% and one prefers 33%. My equidistant cuboctahedron hex, as I said before, is 36.6% (the second column above). I think when the stitchers discuss their patterns they are talking about the cutting pattern, not the stitching pattern, and this means that the shape their stitching produces will have a smaller ratio.

Since a 32-panel bag is going to be pretty much perfectly spherical regardless of the shape and proportion of the panels, the decision of the shape of the hex panel should be more based on the look you want. The design is composed of pents surrounded by five hexes, which means that if all the pents are one color and all the hexes another as in the illustrations above, a larger pent means larger "polka dots" and a larger proportion of that color. If you use a color arrangement that does not distinguish between the two face shapes such as rings or stripes, a design with both shapes the same size will give the pattern cleaner lines and will draw attention to the pattern rather than to the differing panel shapes. Some designs such as the big-eyed ladybug shown among the photos below work better with specific panel proportions. If the ladybug was made with my equidistant hexes, the eyes would be smaller and farther apart and not have quite the stern expression that the triangular panels give them. I provide design calculations for all four design variations later in this chapter.

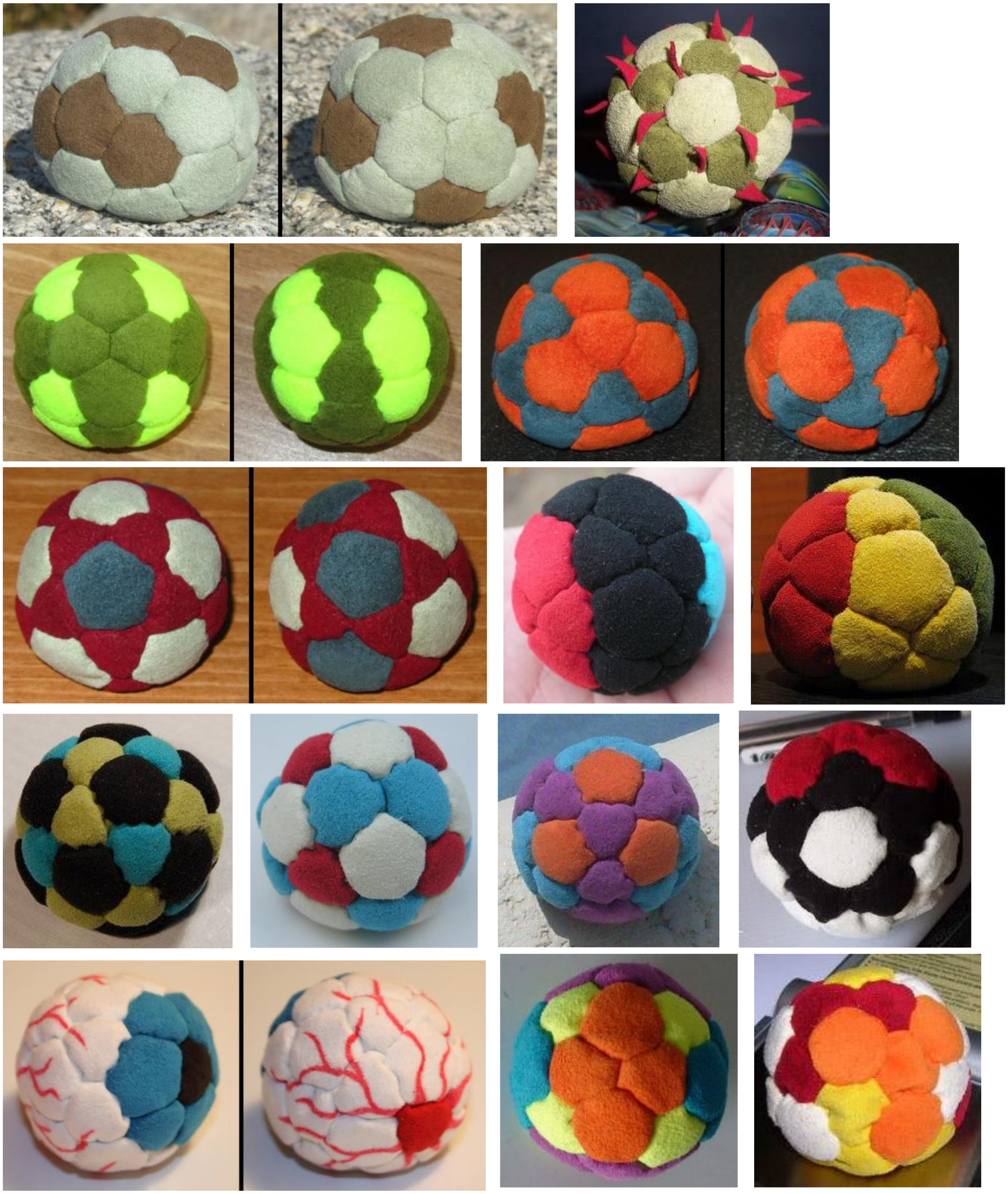
Color arrangements

There are very many color arrangement possibilities for this design and rather than create diagrams of them as I have for the other chapters I'll just share some photos I've found online (ordered, with a few exceptions, by the number of colors they contain). Some of these photos show duplicate arrangement patterns but with different colors, and the pairs with black lines between them are two views of the same bag. Except for the first row of images below, which are the Flying Clipper juggling bags that first inspired me, I found these photos on http://modified.in/footbag/viewtopic.php?t=21008, http://modified.in/footbag/viewtopic.php?p=436817, and http://umbrellabags.wordpress.com/. Google Images will find many of them for you as well. The big-eyed ladybug design is from http://modified.in/footbag/viewtopic.php?f=11&t=22702.

Images from http://www.flyingclipper.com/home/fly/page_1024_134/tossaball_phat_tyre_32_panel_juggle_ball.html





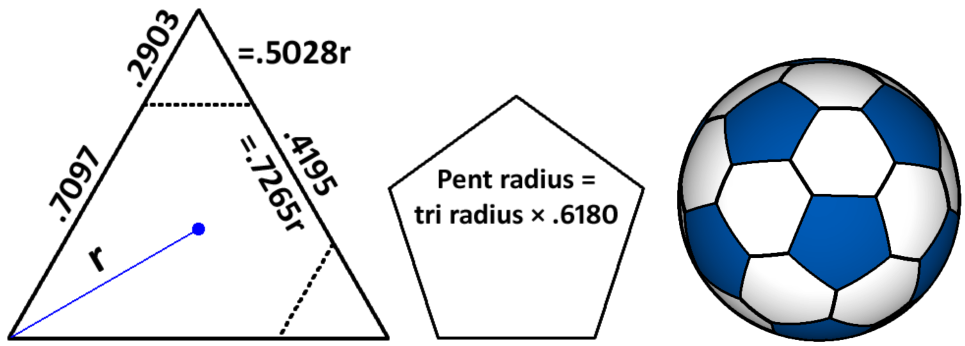



Supplies

- **For the templates (if you draw them by hand)**
    - Cardboard or Template Plastic, fine-point Pencil, metric Ruler, Protractor, X-acto Knife or Scissors, Paper & Adhesive Tape (optional).
- **For the beanbag**
    - Fabric, Needle and Thread, Scissors, Fabric Marker or Pencil, bean bag Filler, Funnel, Safety Pins (optional).

- **For your information**
    - Unless you're experienced with this sort of thing, I recommend that you browse through the *General Notes and Techniques* chapter before you get started. You may find some tips there that will improve your experience and your beanbags.

Panel design – equidistant version



The high precision values are 0.7097347622, 0.2902652378, 0.5027541395, 0.4194695244, 0.7265425284, and 0.6180339892.

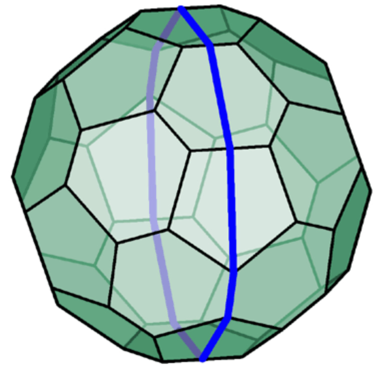
The most important element of the design is the hexagon, which is derived from an equilateral triangle. The pentagon is based simply on the hexagon's long side. For a full discussion of how I created this design, see the *How I Developed My Designs* chapter under "Equidistant icosidodecahedron design". If you don't care about the mathematical explanation of the panel sizing formulas, just skip to where I provide sizing formulas and a table of pre-calculated measurements.

Once you have used the circumference formulas that follow to decide on a panel size for your beanbag, use the diagram above to derive the hexagon from an equilateral triangle. Multiply the triangle's radius by 0.5028 (or its side length by 0.2903) to get the amount of truncation for each end of each side, and multiply the radius by 0.7265 (or the side length by 0.4195) to get the length of the hexagon's long sides and the pentagon's sides. To draw the pentagon, use a radius equal to the triangle's radius multiplied by 0.618 or a side length equal to the hexagon's long side length.

Mathematical background for the sizing formulas

For the design formulas, I'll define the following variables:
$\boldsymbol{h_p}$ = height of the pentagon
$\boldsymbol{h_h}$ = height of the hexagon
$\boldsymbol{h_t}$ = height of the triangle
$\boldsymbol{s_{hl}}$ = long side length of the hexagon which is the same as the pentagon's side length
$\boldsymbol{s_{hs}}$ = short side length of the hexagon
$\boldsymbol{s_t}$ = side length of the triangle
$\boldsymbol{r_p}$ = radius of the pentagon
$\boldsymbol{r_t}$ = radius of the triangle

Note: I only display four decimal places (three for millimeter measurements), but I used full precision in all my calculations.



I will define the circumference of the polyhedron as 4 x height of the pentagon + 4 x height of the hexagon + 2 x short side of the hexagon.

$\text{Circumference} = 4h_p + 4h_h + 2s_{hs}$

I want everything to be in terms of s_{hl} (I can easily derive all the other values from that), so I will use the following conversions: $h_p = (0.5/\sin 36° + 0.5/\tan 36°)s_{hl}$, $h_h = 0.7097h_t$, $h_t = (\sqrt{3}/2)s_t$, $s_t = s_{hl}/0.4195$, and $s_{hs} = 0.6920s_{hl}$. So the circumference formula is now

$$\textbf{Circumference} = 4\left(\frac{0.5}{\sin(36)} + \frac{0.5}{\tan(36)}\right)s_{hl} + 4\left[(0.7097)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{0.4195}\right)\right]s_{hl} + 2(0.6920)s_{hl}$$
$$= \mathbf{13.4005}\boldsymbol{s_{hl}}$$

To express the circumference in terms of the side length of the guide triangle, multiply the above value by 0.4195.

$$\textbf{Circumference} = \mathbf{5.6211}\boldsymbol{s_t}$$

To express it in terms of the triangle's radius, divide the above value by $\sqrt{3}/3$ = 0.5774.

$$\textbf{Circumference} = \mathbf{9.7361}\boldsymbol{r_t}$$

To express it in terms of the pentagon's radius, divide the hexagon side length formula by 0.5/sin(36) = 0.8507 (the long side of the hexagon is the same length as the pentagon's sides).

$$\textbf{Circumference} = \mathbf{15.7533}\boldsymbol{r_p}$$

This bag, when made with thick, nearly non-stretch denim I use and with my stitching practices, has a negative inflation when loosely filled and very little inflation when tightly filled and stretched. The inflation as I measured it is **-4 – +1%** (depending on whether I fill it loosely or over-fill it) than a calculation based on the above formulas. I target halfway between the min and max inflation when sizing my patterns, which results in -1.5%.

Template sizing formulas

To make a bag of diameter d with an assumed -1.5% (-0.015) inflation, multiply d by pi (3.1416) to get the circumference, divide by correct value above to get the desired panel measure, and finally divide by 1+the decimal inflation (in red) to get the pre-inflated size. (If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may have to substitute a different inflation or deflation factor in the formula. For a method of figuring out your adjustment factor, see the *General Notes and Techniques* chapter under "Calculating your pattern size".) **(Don't forget to multiply the final result by 2.54 if you need to convert inches to centimeters.)** Thus, the formulas are:

Triangle radius = d × π ÷ 9.7361 ÷ 0.985

Pentagon radius = d × π ÷ 15.7533 ÷ 0.985

Triangle side length = d × π ÷ 5.6211 ÷ 0.985

Guide triangle truncation to form the hexagon $= \text{radius} \times 0.5028$ or $\text{side length} \times 0.2903$

Table of pre-calculated template measurements

Below is a table of template measurements for each $^1/_8$" diameter increment from 2" to 3" using the -1.5% adjustment factor.

Finished Diameter	**Guide Triangle Radius** (cm)	**Pentagon Radius** (cm)	**Guide Triangle and Pentagon Side Length** (cm)	**Guide Triangle Truncation to Form the Hexagon** (cm)
2" (5.08cm)	1.6642	1.0285	2.8824	0.8368
2$^1/_8$" (5.40cm)	1.7682	1.0928	3.0626	0.8891
2$^1/_4$" (5.72cm)	1.8722	1.1571	3.2427	0.9414
2$^3/_8$" (6.03cm)	1.9762	1.2214	3.4229	0.9937
2$^1/_2$" (6.35cm)	2.0802	1.2856	3.6030	1.0460
2$^5/_8$" (6.67cm)	2.1842	1.3499	3.7832	1.0983
2$^3/_4$" (6.99cm)	2.2882	1.4142	3.9633	1.1506
2$^7/_8$" (7.30cm)	2.3922	1.4785	4.1435	1.2029
3" (7.62cm)	2.4962	1.5428	4.3236	1.2551

Cutting pattern adjustments

To make the cutting patterns, you can either just draw them around the stitching patterns or you can use the same calculations from the previous chapters. The triangle calculations can be found in the 14-panel chapter and the pent calculations can be found in the 12-panel chapter. Here are the values (a = seam allowance). Remember that these are the increases over the stitching pattern dimensions and must be added to those dimensions:

Hex Side Length increase $= 2\sqrt{3}\text{a} = \mathbf{3.4641a}$

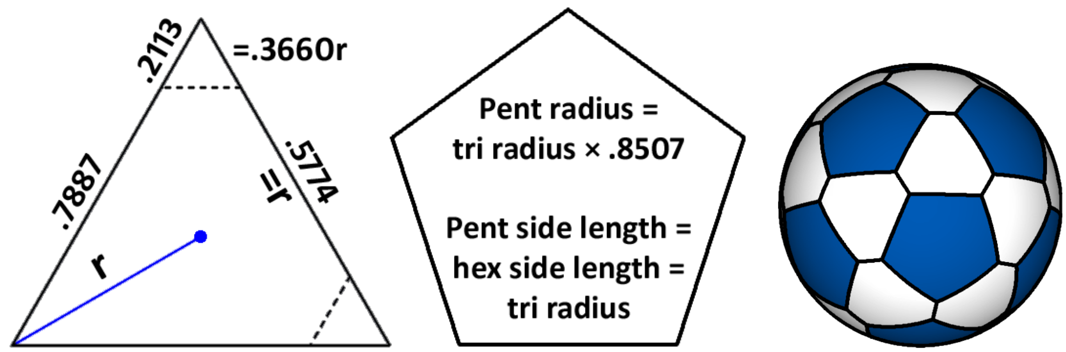
Hex Radius increase $= 2\sqrt{3}\left(\frac{\sqrt{3}}{3}\right)\text{a} = \mathbf{2a}$

Hex Truncation increase = $^{2a}/_{\sqrt{3}} = \mathbf{1.1547}\text{a}$

Pent Side Length increase, $2\text{x} = 2(\tan 36°)\text{a} = \mathbf{1.4531}\text{a}$

Pent Radius increase $= 1.4531 \times 0.8507 = \mathbf{1.2361}\text{a}$

Panel design – 14-panel hex version



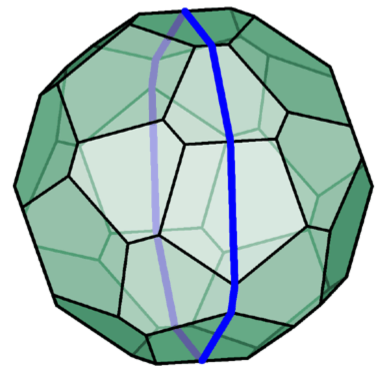
The high precision values are 0.7886751346, 0.2113248654, 0.3660254038, 0.5773502692, and 0.8506508084.

If you like the look of a smaller and more triangular hexagon, the hexagon I designed for the equidistant cuboctahedron will work well. The benefit of this particular hexagon is that the length of the long side is the same as the radius of the guide triangle. This makes it simpler to draw in SketchUp because once you have calculated the side length you want you can just use that as the radius to draw the triangle. The truncation to derive the hexagon from the equilateral triangle, as you can see in the diagram, is the triangle's radius multiplied by 0.366 or its side length multiplied by 0.2113. You can draw the pentagon by either using a radius equal to the triangle's radius multiplied by 0.8507 or a side length equal to the hexagon's long side length or the triangle's radius. If you don't care about the rest of the mathematical explanation of the panel sizing formulas, just skip to where I provide sizing formulas and a table of pre-calculated measurements.

Mathematical background for the sizing formulas

For the design formulas, I'll define the following variables:
h_p = height of the pentagon
h_h = height of the hexagon
h_t = height of the triangle
s_{hl} = long side length of the hexagon = pentagon's side length = r_t
s_{hs} = short side length of the hexagon
s_t = side length of the triangle
r_p = radius of the pentagon
r_t = radius of the triangle = s_h

Note: I only display four decimal places (three for millimeter measurements), but I used full precision in all my calculations.



To reiterate, I define the circumference of the polyhedron as 4 x height of the pentagon + 4 x height of the hexagon + 2 x short side of the hexagon.

$\text{Circumference} = 4h_p + 4h_h + 2s_{hs}$

I want everything to be in terms of s_{hl} (I can easily derive all the other values from that), so I will use the following conversions: $h_p = (0.5/\sin 36° + 0.5/\tan 36°)s_{hl}$, $h_h = 0.7887h_t$, $h_t = (\sqrt{3}/2)s_t$, $s_t = s_{hl}/0.5774$, and $s_{hs} = 0.3660s_h$. So the circumference formula is now

$$\textbf{Circumference} = 4\left(\frac{0.5}{\sin(36)} + \frac{0.5}{\tan(36)}\right)s_{hl} + 4\left[(0.7887)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{0.5774}\right)\right]s_{hl} + 2(0.3660)s_{hl}$$
$$= \mathbf{11.6195}\boldsymbol{s_{hl}} = \mathbf{11.6195}\boldsymbol{r_t}$$

I repeat the above value to emphasize that the long side of the hexagon, s_{hl}, is the same as the radius of the triangle, r_t; this is useful for drawing it in SketchUp. I don't know why this is and I can't prove it mathematically, but it's a convenient property of the shape.

To express the circumference in terms of the side length of the guide triangle, multiply the above value by 0.5774.

$\textbf{Circumference} = \mathbf{6.7085}\boldsymbol{s_t}$

To express it in terms of the pentagon's radius, divide the hexagon side length formula by 0.5/sin(36) = 0.8507 (the long side of the hexagon is the same length as the pentagon's sides).

$\textbf{Circumference} = \mathbf{13.6595}\boldsymbol{r_p}$

The equidistant version of this bag, when made with thick, nearly non-stretch denim I use and with my stitching practices, has a negative inflation when loosely filled and very little inflation when tightly filled and stretched. The inflation as I measured it is **-4 – +1%** (depending on whether I fill it loosely or over-fill it) than a calculation based on the above formulas. I'll assume this design has the same inflation. I target halfway between the min and max inflation when sizing my patterns, which results in -1.5%.

Template sizing formulas

To make a bag of diameter d with an assumed -1.5% (-0.015) inflation, multiply d by pi (3.1416) to get the circumference, divide by correct value above to get the desired panel measure, and finally divide by 1+the decimal inflation (in red) to get the pre-inflated size. (If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may have to substitute a different inflation or deflation factor in the formula. For a method of figuring out your adjustment factor, see the *General Notes and Techniques* chapter under "Calculating your pattern size".) **(Don't forget to multiply the final result by 2.54 if you need to convert inches to centimeters.)** Thus, the formulas are:

Triangle radius = d × π ÷ 11.6195 ÷ 0.985

Pentagon radius = d × π ÷ 13.6595 ÷ 0.985

Triangle side length $= \mathrm{d} \times \pi \div 5.6211 \div 0.985$

Guide triangle truncation to form the hexagon $= \text{radius} \times 0.3660$ or $\text{side length} \times 0.2113$

Table of pre-calculated template measurements

Below is a table of template measurements for each $^{1}/_{8}$" diameter increment from 2" to 3" using the -1.5% adjustment factor.

Finished Diameter	Guide Triangle Radius (cm)	Pentagon Radius (cm)	Guide Triangle and Pentagon Side Length (cm)	Guide Triangle Truncation to Form the Hexagon (cm)
2" (5.08cm)	1.3944	1.1862	2.8824	0.6091
$2^{1}/_{8}$" (5.40cm)	1.4816	1.2603	3.0626	0.6471
$2^{1}/_{4}$" (5.72cm)	1.5687	1.3344	3.2427	0.6852
$2^{3}/_{8}$" (6.03cm)	1.6559	1.4086	3.4229	0.7233
$2^{1}/_{2}$" (6.35cm)	1.7430	1.4827	3.6030	0.7613
$2^{5}/_{8}$" (6.67cm)	1.8302	1.5568	3.7832	0.7994
$2^{3}/_{4}$" (6.99cm)	1.9173	1.6310	3.9633	0.8375
$2^{7}/_{8}$" (7.30cm)	2.0045	1.7051	4.1435	0.8755
3" (7.62cm)	2.0916	1.7792	4.3236	0.9136

Cutting pattern adjustments

To make the cutting patterns, you can either just draw them around the stitching patterns or you can use the same calculations from the previous chapters. The triangle calculations can be found in the 14-panel chapter and the pent calculations can be found in the 12-panel chapter. Here are the values (a = seam allowance). Remember that these are the increases over the stitching pattern dimensions and must be added to those dimensions:

Hex Side Length increase $= 2\sqrt{3}\mathrm{a} = \mathbf{3.4641a}$

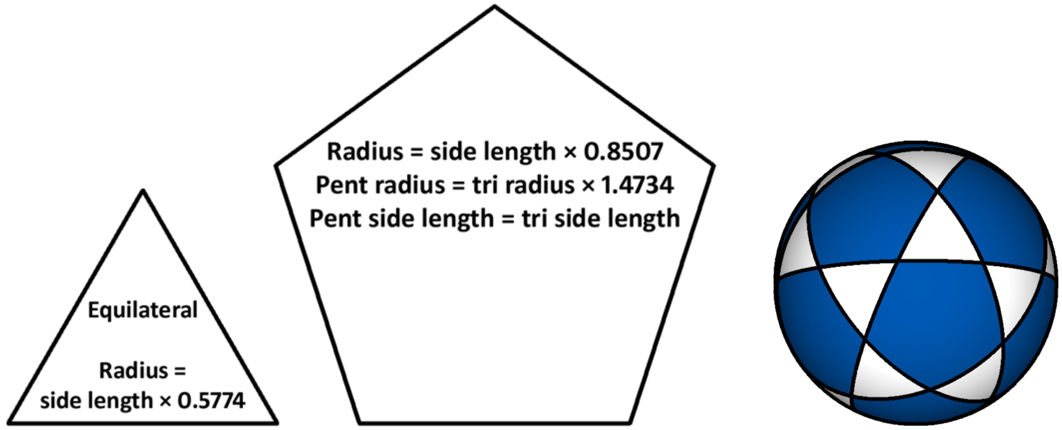
Hex Radius increase $= 2\sqrt{3}\left(\frac{\sqrt{3}}{3}\right)\mathrm{a} = \mathbf{2a}$

Hex Truncation increase $= {}^{2a}/_{\sqrt{3}} = \mathbf{1.1547}\mathrm{a}$

Pent Side Length increase, $2\mathrm{x} = 2(\tan 36°)\mathrm{a} = \mathbf{1.4531}\mathrm{a}$

Pent Radius increase $= 1.4531 \times 0.8507 = \mathbf{1.2361}\mathrm{a}$

Panel design – true icosidodecahedron (equilateral pentagons and triangles)



The high precision values are 1.732050808 and 1.47337042

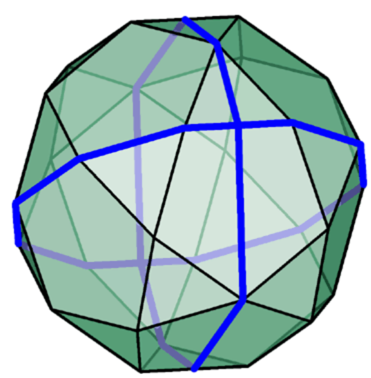
If you like the star patterns of the true icosidodecahedron, simply use equilateral pentagons and triangles. It's a simpler design, but not as good for creative color arrangements. If you don't care about the mathematical explanation of the panel sizing formulas, just skip to where I provide sizing formulas and a table of pre-calculated measurements.

Mathematical background for the sizing formulas

For the design formulas, I'll define the following variables:
$\boldsymbol{h_p}$ = height of the pentagon
$\boldsymbol{h_t}$ = height of the triangle
$\boldsymbol{s}$ = side length of the triangle and pentagon
$\boldsymbol{r_p}$ = radius of the pentagon
$\boldsymbol{r_t}$ = radius of the triangle

Note: I only display four decimal places (three for millimeter measurements), but I used full precision in all my calculations.

There are two basic ways to calculate the circumference of the polyhedron. The simpler of the two is 10 x edge length. The more complicated is 4 x height of the pentagon + 4 x height of the triangle.



$$\text{Circumference} = 10s$$

$$\text{Circumference} = 4h_p + 4h_t$$

I want everything to be in terms of s, so I will use the following conversions for the second formula: $h_p = (0.5/\sin 36° + 0.5/\tan 36°)s$, $h_t = (\sqrt{3}/2)s$. So the circumference formula is now

$$\text{Circumference} = 4\left(\frac{0.5}{\sin(36)} + \frac{0.5}{\tan(36)}\right)s + 4\left(\frac{\sqrt{3}}{2}\right)s \quad = \quad 9.6195s$$

The two methods differ by 3.96%, so I'll use the average.

$$\textbf{Average Circumference} = \frac{10s + 9.6195s}{2} = \mathbf{9.8097s}$$

To express the circumference in terms of the triangle's radius, divide the above value by $\sqrt{3}/3$ = 0.5774.

$$\textbf{Circumference} = \mathbf{16.9910}r_t$$

To express it in terms of the pentagon's radius, divide the side length expression by 0.5/sin(36) = 0.8507.

$$\textbf{Circumference} = \mathbf{11.5320}r_p$$

The equidistant version of this bag, when made with thick, nearly non-stretch denim I use and with my stitching practices, has a negative inflation when loosely filled and very little inflation when tightly filled and stretched. The inflation as I measured it is **-4 – +1%** (depending on whether I fill it loosely or over-fill it) than a calculation based on the above formulas. I'll assume this design has the same inflation. I target halfway between the min and max inflation when sizing my patterns, which results in -1.5%.

Template sizing formulas

To make a bag of diameter d with an assumed -1.5% (-0.015) inflation, multiply d by pi (3.1416) to get the circumference, divide by correct value above to get the desired panel measure, and finally divide by 1+the decimal inflation (in red) to get the pre-inflated size. (If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may have to substitute a different inflation or deflation factor in the formula. For a method of figuring out your adjustment factor, see the *General Notes and Techniques* chapter under "Calculating your pattern size".) **(Don't forget to multiply the final result by 2.54 if you need to convert inches to centimeters.)** Thus, the formulas are:

Triangle radius $= \mathrm{d} \times \pi \div 16.9910 \div 0.985$

Pentagon radius $= \mathrm{d} \times \pi \div 11.5320 \div 0.985$

Side length $= \mathrm{d} \times \pi \div 9.8097 \div 0.985$

Table of pre-calculated template measurements

Below is a table of template measurements for each $^1/_8$" diameter increment from 2" to 3" using the -1.5% adjustment factor.

Finished Diameter	**Guide Triangle Radius** (cm)	**Pentagon Radius** (cm)	**Guide Triangle and Pentagon Side Length** (cm)
2" (5.08cm)	0.9536	1.4050	1.6517
2$^1/_8$" (5.40cm)	1.0132	1.4928	1.7549
2$^1/_4$" (5.72cm)	1.0728	1.5806	1.8581
2$^3/_8$" (6.03cm)	1.1324	1.6684	1.9614
2$^1/_2$" (6.35cm)	1.1920	1.7562	2.0646
2$^5/_8$" (6.67cm)	1.2516	1.8441	2.1678
2$^3/_4$" (6.99cm)	1.3112	1.9319	2.2710

Finished Diameter	**Guide Triangle Radius** (cm)	**Pentagon Radius** (cm)	**Guide Triangle and Pentagon Side Length** (cm)
$2^{7}/_{8}$” (7.30cm)	1.3708	2.0197	2.3743
3” (7.62cm)	1.4304	2.1075	2.4775

Cutting pattern adjustments

To make the cutting patterns, you can either just draw them around the stitching patterns or you can use the same calculations from the previous chapters. The triangle calculations can be found in the 14-panel chapter and the pent calculations can be found in the 12-panel chapter. Here are the values (a = seam allowance). Remember that these are the increases over the stitching pattern dimensions and must be added to those dimensions:

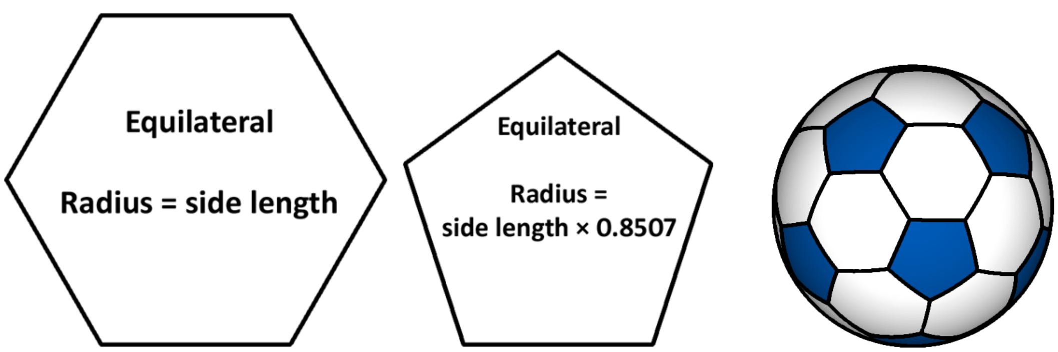
Triangle Side Length increase $= 2\sqrt{3}\mathrm{a} = \mathbf{3.4641a}$

Triangle Radius increase $= 2\sqrt{3}\left(\frac{\sqrt{3}}{3}\right)\mathrm{a} = \mathbf{2a}$

Pent Side Length increase, $2\mathrm{x} = 2(\tan 36°)\mathrm{a} = \mathbf{1.4531}\mathrm{a}$

Pent Radius increase $= 1.4531 \times 0.8507 = \mathbf{1.2361}\mathrm{a}$

Panel design – soccer ball (equilateral pentagons and hexagons)



The high precision value is 0.8506508084

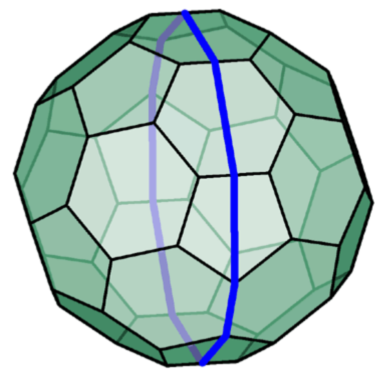
If you want to make a true soccer ball design (a truncated icosahedron), use equilateral pentagons and hexagons. If you draw the panels with SketchUp, which uses a defined radius to draw polygons, it is useful to know that the radius of a hexagon is equal to its side length. If you don’t care about the mathematical explanation of the panel sizing formulas, just skip to where I provide sizing formulas and a table of pre-calculated measurements.

Mathematical background for the sizing formulas

For the design formulas, I'll define the following variables:
$\boldsymbol{h_p}$ = height of the pentagon
$\boldsymbol{h_h}$ = height of the hexagon (from edge to edge)
$\boldsymbol{s}$ = side length of the hexagon and pentagon
$\boldsymbol{r_p}$ = radius of the pentagon
$\boldsymbol{r_h}$ = radius of the hexagon, which is the same as its side length

Note: I only display four decimal places (three for millimeter measurements), but I used full precision in all my calculations.

I will define the circumference of the polyhedron as 4 x height of the pentagon + 4 x height of the hexagon + 2 x edge length.



$$\text{Circumference} = 4h_p + 4h_h + 2s$$

I want everything to be in terms of s (I can derive the radii from that), so I will use the following conversions: $h_p = (0.5/\sin 36° + 0.5/\tan 36°)s$, $h_h = \sqrt{3}s$. So the circumference formula is now

$$\textbf{Circumference} = 4\left(\frac{0.5}{\sin(36)} + \frac{0.5}{\tan(36)}\right)s_h + 4(\sqrt{3}s) + 2s \quad = \quad \mathbf{15.0836}\boldsymbol{s}$$

The hexagon's radius is the same as its side length, so

$$\textbf{Circumference} = \mathbf{15.0836}\boldsymbol{r_h}$$

To express it in terms of the pentagon's radius, divide the hexagon side length formula by 0.5/sin(36) = 0.8507.

$$\textbf{Circumference} = \mathbf{17.7318}\boldsymbol{r_p}$$

The equidistant version of this bag, when made with thick, nearly non-stretch denim I use and with my stitching practices, has a negative inflation when loosely filled and very little inflation when tightly filled and stretched. The inflation as I measured it is **-4 – +1%** (depending on whether I fill it loosely or over-fill it) than a calculation based on the above formulas. I'll assume this design has the same inflation. I target halfway between the min and max inflation when sizing my patterns, which results in -1.5%.

Template sizing formulas

To make a bag of diameter d with an assumed -1.5% (-0.015) inflation, multiply d by pi (3.1416) to get the circumference, divide by correct value above to get the desired panel measure, and finally divide by 1+the decimal inflation (in red) to get the pre-inflated size. (If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may have to substitute a different inflation or deflation factor in the formula. For a method of figuring out your adjustment factor, see the *General Notes and Techniques* chapter under "Calculating your pattern size".) **(Don't forget to multiply the final result by 2.54 if you need to convert inches to centimeters.)** Thus, the formulas are:

Hexagon radius $= d \times \pi \div 15.0836 \div 0.985$

Pentagon radius $= d \times \pi \div 15.7318 \div 0.985$

Side length $= d \times \pi \div 15.0836 \div 0.985$

Cutting pattern adjustments

To make the cutting patterns, you can either just draw them around the stitching patterns or you can use the formulas below (a = seam allowance). Remember that these are the increases over the stitching pattern dimensions and must be added to those dimensions:

Hex Side Length increase $= {}^{4a}/_{\sqrt{3}} = 2.3094a$

Hex Radius increase $=$ same as above

Pent Side Length increase, $2x = 2(\tan 36°)a = \mathbf{1.4531}a$

Pent Radius increase $= 1.4531 \times 0.8507 = \mathbf{1.2361}a$

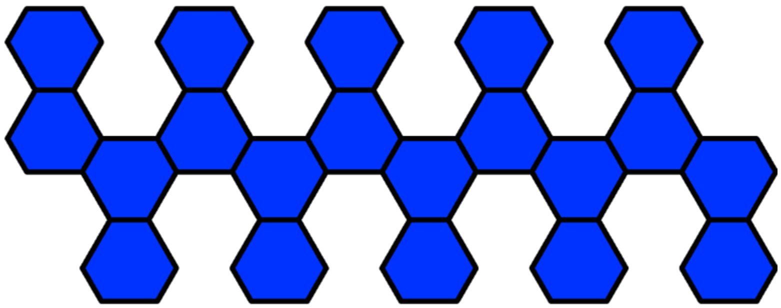
Making the panels

Depending on the type of template you're using and whether it is translucent or not, you must be careful which pattern, cutting or stitching, you trace first so that the second template doesn't hide the lines of the first and prevent you from aligning the two. Do not necessarily use them in the sequence below.

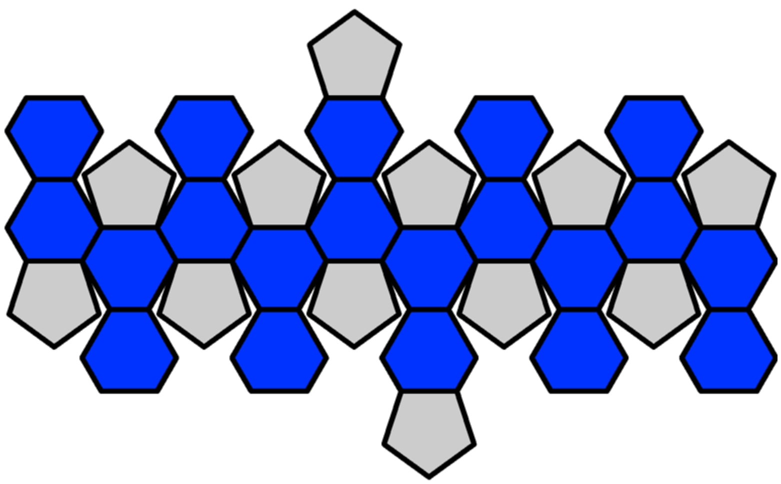
1. Use the larger, cutting template to trace the cutting pattern onto back of the fabric. You will need 12 pentagons and 20 hexagons (or triangles).
2. Use the smaller, stitching templates to trace the stitching patterns within each cutting pattern, being sure to center them well.
3. Cut out the panels.

Assembly

I have not yet figured out an optimal stitching pathway for this panel structure. I found[20] what looks like a good way to lay out the panels to help in planning the stitching and color arrangement. I haven't made use of it yet, though. For this layout, the hexagons are arranged in staggered pairs as shown below. If you're using hexagons with two different side lengths, make sure only the short sides touch each other. The long sides will all connect to pentagons.

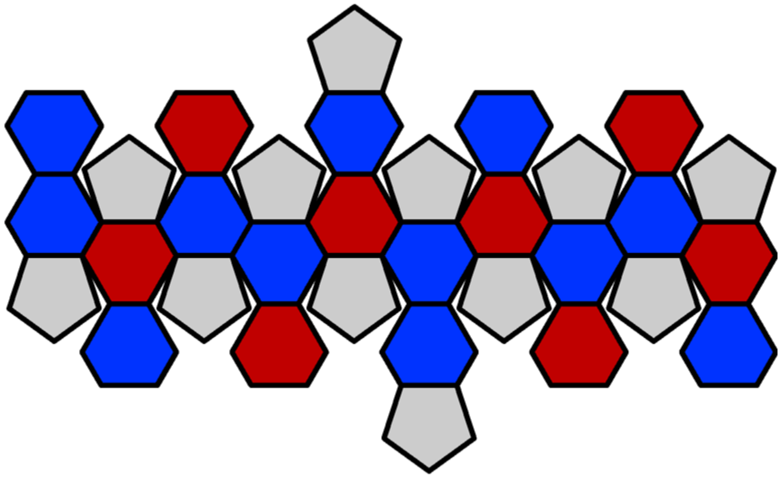


Ten pentagons fit into the gaps between the hex pairs, and the last two form caps on the top and bottom. Each cap pentagon will connect to the five hexagons on its side of the layout.



[20] http://crafting.squidoo.com/make-soccer-ball-pillow

To form the three-color pattern I used, which is a common pattern, the layout would look like the one below. The nature of this pattern is that the white panels correspond to (are in the same positions as) the faces of a dodecahedron, the blue pairs correspond to the six faces of a cube, and the red faces correspond to the eight corners of a cube. This is easier to see with an actual bag or a model.



If you use multiple colors, be careful to assemble the panels correctly to form your chosen pattern. With so many panels and seams, it's easy to make a mistake and misalign the panels. I recommend making a SketchUp model or a cardboard model to help you keep track of what you're doing. Index cards, file folders, or something of similar thickness work well for a model. Just use your stitching template and cut several layers at a time to produce the panels faster. To learn how to draw these polyhedrons in SketchUp, see the end of the section on this design in the *How I Developed My Designs* chapter. I draw all four versions of this bag shape by truncating an icosahedron to various degrees. My instructions tell you what truncation to use, and you can find instructions for drawing the icosahedron on the web.

OTHER DESIGNS AND VARIATIONS

How to design beach balls with any number of panels

Images sources, respectively: http://www.dube.com/beanbag/dube-pro-beanbags.php, http://www.higginsbrothers.com/html/juggling_balls.html, my own beanbag on the right.

You can use the beach ball concept to make juggling bags with any number of panels greater than two (it technically works for two, but a two-panel beach ball would be more of a pillow than a ball and the panel shape is simply a circle). The above examples use six and eight-panel configurations, and I read a blog in which a guy said he made one with five panels. (If you want to use alternating colors or a repeating sequence of colors, you will have to use a non-prime number of panels.) There are two rules regarding the design of the beach ball panel (the *stitching* pattern, not the *cutting* pattern):

1. Length × 2 = width × number of panels = circumference of the ball. That is, the length of the panel must be half of the ball's target circumference and the width must be equal to the circumference divided by the number of panels.
2. The curve is circular.

I do not know enough math to prove that a circular curve (as opposed to an elliptical or function-based one) is optimal and will work for any number of panels, but I have made four and eight-panel bags and they are quite spherical and the seams form what look like perfect circles when I look at them in profile. Also, the theory makes sense to me intuitively. Additionally, my theory has been supported (though still not proven) by a blogger I found (see *Appendix I*) who made a six-panel beach ball using the same panel design theory. Happily, I learned from the same source of a formula to calculate the radius of the circle that will form the curve for a beach ball panel. (Before I learned the formula I used a tedious, trial-and-error method of finding the radius.) I discuss two alternate types of curves at the end of the "4-panel beach ball" section of the *How I Developed My Designs* chapter. I believe the circular curve to be superior to both of them, both in the resulting bag shape and in ease of drawing the shape.

Once you have decided on the number of panels you want to use and their width (w), and calculated the circle radius (r), the easiest way to draw the panel (by hand or with SketchUp) is to draw a line of length **2r – w**, set the compass to the correct radius (if you're drawing the pattern manually), and then draw the arcs/circles centered on each end of the line. Their overlap zone forms the panel shape. These steps are depicted below the formulas. To confirm you drew the panel correctly, measure it and make sure it follows the first of the two design rules above. Remember that this method creates the stitching pattern. To make the cutting pattern, simply draw everything the same but increase the arc/circle radius by the desired seam allowance.

Panel design formulas

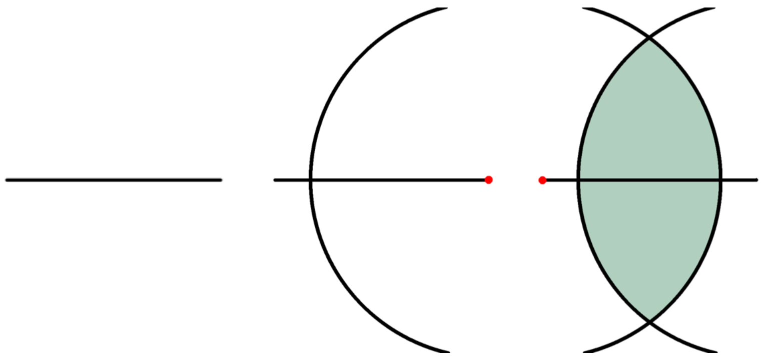
Panel Length $= \text{ball diameter} \times \pi \div 2$

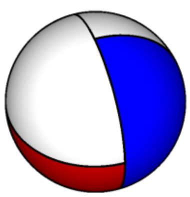
Panel Width $= \text{ball diameter} \times \pi \div n$

Curve Radius $= (w^2 + l^2) \div 4w$

Distance between compass points $= 2r - w$

(n = number of panels, w = panel width, l = panel length, and r = curve radius)





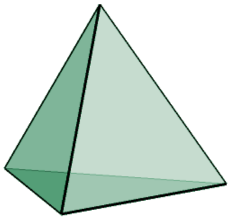
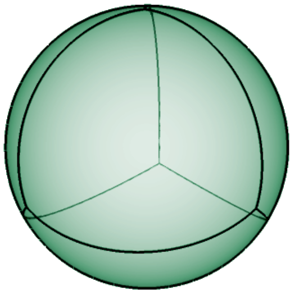
I thought of a silly idea for beach balls with even numbers of panels: Try misaligning the hemispheres by 90°. I haven't tried it, but I think it will work.

I made an eight-panel bag to confirm that the panel design theory worked for that many panels. The bag turned out well and did confirm the theory. A photo of it is at the beginning of this section. It had an inflation factor of 5.9% - 9.5% depending on whether I filled it loosely or over-filled it.

However, I made it with denim and the thickness of the fabric and the large number of panel tips crowded into each pole of the bag made the poles dimple inward slightly (shown below) so that the bag felt slightly like an apple. (I assume that's the reason for the dimples.) I recommend a thin fabric if you use that many panels, which I think will reduce this effect. I was able to mostly correct it by pinching the poles of the bag several times to compress the seam allowance tips and by breaking the bag in.

Another issue was getting all the panel tips to pull together and make a tight formation at each pole with each tip aligned with its opposing tip (I'm very picky). The way I sewed them allowed them to separate slightly and the poles looked messy as shown on the left below. I used a doubled thread for strength (my single thread broke) and from outside the bag I put stitches into each panel tip, stitched across to the opposite panel, and pulled the two tightly together. I continued this around the pole a couple of times to produce a tight, uniform formation. The result is shown below on the right. I could have done this from the inside as I was joining the panels together if I had thought of it. Instead of stitching each panel tip only to the adjacent one, I should have also stitched each one to the opposing tip.


4-panel spherical tetrahedron

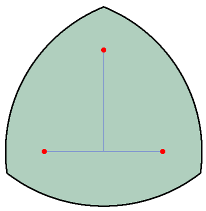
The beanbag photos are two angles of my broken-in denim model from 4/28/2013

For a discussion of this design, see the *How I Developed My Designs* chapter. Here I will just give a summary.

This design uses a curve derived from a 3-panel beach ball applied to an equilateral triangle. To make a bag of this design, use elements of my beach ball design theory (described in the previous section) combined with my octahedron method to draw the panel shape. Begin by designing a 3-panel beach ball. Use the $(w^2 + l^2) \div 4w$ formula to calculate a curve radius and the $2r - w$ formula to calculate the distance between the compass points. This results in

Curve radius $= 0.8125\text{w}$

Distance between compass points $= 0.625\text{w}$



Use the distance between compass points as the side length of the equilateral guide triangle. Complete the triangle and draw a third arc from the third corner. The inverted T within the panel diagram on the left is the guide I used (the three corners of the triangle - the red dots are the corners/compass points). By my calculations

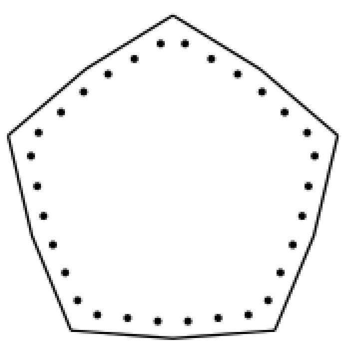
Panel height = 1.02122718w = 1.63396348 × (guide triangle side length)

Remember that this method creates the stitching pattern. To make the cutting pattern, simply draw everything the same but increase the arc/circle radius by the desired seam allowance.

From the limited experimentation I've done, it looks like the mathematical circumference of the tetrahedron will be just a few percent larger than the mathematical circumference of the 3-panel beach ball of the same panel size. My guide had a 3.976cm base and a 5.173cm curve radius producing a panel that was 6.506cm from corner to side. Using the arc length formula 2r(arcsin(c/2r)), where r = radius and c = chord length, the bag's mathematical circumference (measured across the heights of two faces and along one seam/arc) is 19.765cm. The beach ball of the same panel size (measured between the seams) would be 19.11cm. My bag's inflated size when tightly filled is 20.65cm, which is an inflation of 4.5% (I later removed some filler to soften the bag slightly and make it feel less angular and the inflation at that point was 4%). This is just a little smaller than a tennis ball which has a circumference of 20.95cm. For my own reference, my tetrahedron weighs 90.2g filled with Poly-Pellets.

This design is best made using a flexible, stretchy fabric unless you like a rounded angular shape. It is not as round as the 4-panel beach ball (at least not when using denim), and the corners and the acuteness of the edges can be felt. See the *How I Developed My Designs* chapter for a detailed description of this design's shape.

Dodecahedron with bulged faces



The inspiration for some of my designs came from a website by someone named Peter Billam[21]. Billam provides patterns for several polyhedral beanbag designs using bulged polygons for the panel shapes. One of these is a dodecahedron (shown on left). The bulge is very subtle and is angular rather than curved (which is the case for most of his panel shapes). I have not tried rounding/bulging the faces of my dodecahedron; I don't know what bulge/curve to use and it would be rather a pain to draw. (Billam does not provide mathematical definitions for the panel shapes and so I do not know what bulges he uses.) I don't feel that the dodecahedron needs bulged panels as it is wonderfully round and smooth already.

After reading an article on spherical geometry[22], I made a guess that Billam's pentagon bulges are designed to form 120° angles at the primary corners. The theory is that for polygons to form a sphere, the sum of the angles meeting at each vertex must be 360°. A dodecahedron has three-corner vertices, so that's 120° per corner. I measured an enlarged view of Billam's angle in PhotoShop to confirm this and it appeared to be 116.5°. Well, it's close. Perhaps Billam was attempting to make the primary

[21] http://www.pjb.com.au/jug/leatherballs.html
[22] http://euler.slu.edu/escher/index.php/Spherical_Geometry

vertices match the secondary vertices (the latter being the ones caused by the angled bulge on each edge and of which two meet at each vertex of the dodecahedron). By my crude measurements, the two are less than 7° different.

A lineup of footbag panel structures

There is a huge variety of paneled footbag designs, and most of them have higher panel counts than juggling bags which usually only go up to 14 panels. Those who use footbags evidently see a benefit to having more panels. Probably the most popular structure is the 32-panel on which I have written a chapter. Following is a lineup of panel structures that I have found, ordered by their panel counts. There are designs not represented here – the traditional juggling bag designs and some others. I included the designs that were unique to footbags and that I liked the look of.

1 Panel

From http://www.footbagshop.com/footbags/whirlpool-single-panel.html. This design is composed of a single long, slender panel that is sewn to itself in a spiral manner.

2 Panels

Original Hacky Sack from http://www.ebay.com/itm/221263945612?_trksid=p2048036. This is the same as the baseball structure. I have an instructional chapter about this design.

Another 2-panel design, this one from http://www.footbagshop.com/footbags/nova-2.html. Each panel is star-shaped.

6 Panels

From http://www.ebay.com/itm/6-Panels-sand-footbag-hacky-sack-item-6004-/181192436901?pt=Outdoor_Toys_Structures_US&hash=item2a2fe92ca5. Bomb Footbags also makes bags of this design, but their photos aren't as good. I actually designed a template that looks just like this while trying to invent a simpler version of the Volley Bag (see the section called "Volleyball style" below). This design is basically a dodecahedron with pairs of pentagons joined and their angles transformed into curves. That is how I designed mine. I got the curves wrong, though, and their lengths didn't match. That's why I never made a bag of that design.

14 Panels

From http://www.ebay.com/itm/Nut-Sack-14-panel-Sand-filled-hackysack-kickbag-footbag-/400084852917#vi-content and http://www.footbagshop.com/reaper-14-net-red-white.html. This is a variation of the traditional 14-panel structure in which the squares have become circles and the hexagons have concave circular edges in place of the three long edges. I like its resemblance to a ladybug. I designed my own version of this. It is described in the 14-panel instructional chapter.

"The Rocket" 14-panel footbag by Bomb Footbags http://bombfootbags.com/TheRocket.html.

20 Panels


"Elemental" footbag from http://www.footbagcentral.com/proddetail.php?prod=Elemental-Flow-Footbag-Hackysack. This is the fifth Platonic Solid, the icosahedron, composed of equilateral triangles.

24 Panels


From (respectively) http://www.expo-star.com/lview.asp?mainid=12&Subid=0&pid=52, http://www.jugglingstore.com/pyramid-footbag-765.html, and http://www.oddballs.co.uk/oddballs-sand-filled-footbags-24-panel-p-2933.html. I have written an instructional chapter on this design. My research turned up the interesting fact that this solid is called a deltoidal icositetrahedron and is the dual polyhedron of the 26-panel structure below (http://en.wikipedia.org/wiki/Deltoidal_icositetrahedron).

26 Panels

"Alpha" and "Dirtbag" footbags by Flying Clipper from http://www.flyingclipper.com/home/fly/page_272_99/alpha_footbag.html and http://www.flyingclipper.com/home/fly/page_461_171/dirtbag_26_footbag.html. Through research on the 24-panel structure I discovered that the 26-panel structure is similar to (and is probably based on) the rhombicuboctahedron which is an Archimedean Solid (http://en.wikipedia.org/wiki/Rhombicuboctahedron) composed of squares and triangles. It turns out that the two solids are duals of each other. This bag uses truncated triangles, though, and so it is actually composed of squares, semiregular hexagons, and semiregular octagons.

32 Panels

These are homemade footbags. The first two are from http://modified.in/footbag/viewtopic.php?t=21008. The spiky one is from http://modified.in/footbag/viewtopic.php?p=436817. The big-eyed ladybug is from http://modified.in/footbag/viewtopic.php?f=11&t=22702. See more color arrangements in the *32-Panel Equidistant Icosidodecahedron* chapter and on Google Images.

42 Panels

"Legend" footbag by Flying Clipper from http://www.flyingclipper.com/home/fly/page_274_99/legend_footbag.html.

50 Panels

"Dirtbag" footbag by Flying Clipper from
http://www.flyingclipper.com/home/fly/page_464_171/dirtbag_50_footbag.html.

52 Panels

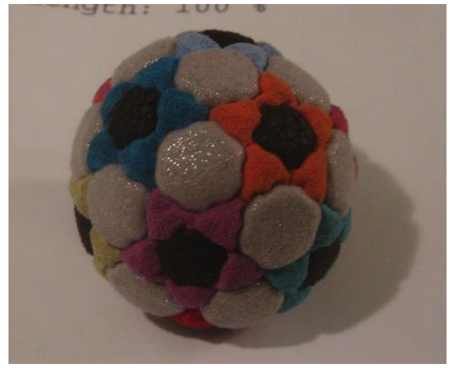
"The Bee" footbag from http://www.higginsbrothers.com/html/footbags.html.

62 Panels

"Hammer Net", "Sand Hammer", "Axe Hammer", and "Jack Hammer" footbags from http://worldfootbag.com/product/hammer-net-footbag/, http://worldfootbag.com/product/sand-hammer-hacky-sack-footbag/, http://worldfootbag.com/product/axe-hammer-footbag/, and http://worldfootbag.com/product/jack-hammer-hacky-sack-footbag/. These all have the same panel structure but very different color arrangements.

92 Panels





From (respectively) http://www.miusports.com.pk/detail.php?nod=320, http://www.expo-star.com/lview.asp?mainid=13&Subid=0&pid=39, homemade by Brian Bear http://modified.in/footbag/viewtopic.php?f=11&t=22702.

120 Panels



From http://worldfootbag.com/product/steely-120-footbag/.

152 Panels


“Super Hero” footbag from (respectively) http://www.ebay.com/itm/HACKY-SACK-FOOTBAG-SUPER-HERO-152-PANEL-RED-WHT-BLUE-/170521186119, http://www.amazon.com/Super-Hero-Yellow-Blues-152-Panel/dp/B005M35IUM/ref=sr_1_1?ie=UTF8&qid=1375798596&sr=8-1&keywords=super+hero+footbag, and http://www.sears.com/adventure-trading-super-hero-black-grey-white-152/p-SPM7003085208?prdNo=17. I like the Triforce designs in these bags (http://en.wikipedia.org/wiki/Triforce).

182 Panels

"BB King" and homemade footbags from http://worldfootbag.com/product/bb-king-footbag/ and http://www.leberknight.com/footbags.html.

Volleyball style

Images from http://www.jugglingstore.com/volley-bag-737.html

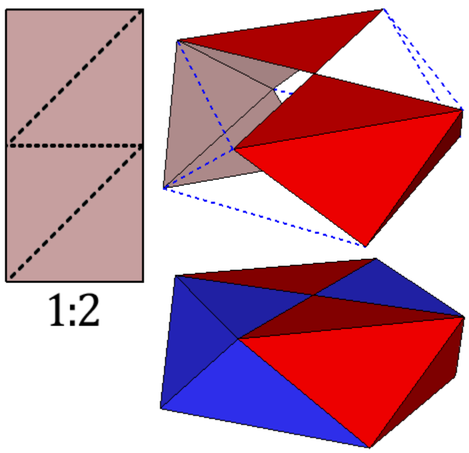
This 12-panel design is sold at various online juggling stores. It is called the "Volley Bag". I tried and failed to figure out how to design the panel. I do not know enough geometry to tessellate the surface of a sphere in this manner. I very much want to make a bag like this.

Japanese Otedama beanbags (a.k.a., ojami)

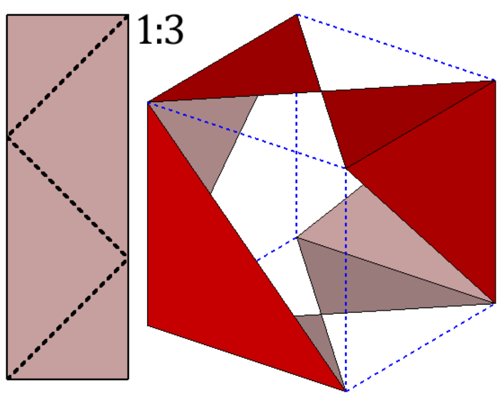
http://www.livingwithpunks.com/2011/06/sys-japanese-otedama-bags.html

I found a very interesting beanbag design in which four rectangles are sewn together in a pinwheel fashion to form a polyhedral patty, and which can be modified to form a cube. Further research identified this as the traditional design for "ojami", the small beanbags used in the Japanese children's game of “Otedama” (some sources say that both terms refer to the beanbags but are in different dialects). The game is reportedly similar to Jacks or Knucklebones and the bags are made small enough for a child to be able to hold five of them in one hand. The smallest pattern I found was 2.5cm by 5.5cm (not including seam allowance), and a commenter claiming to have experience with Otedama said that this was the correct panel size[23]. This would make the middle photo above roughly life-size when this document is zoomed to 100% or printed.

This design is angular rather than spherical and so doesn’t strictly belong in a document about spherical beanbag designs, but I like the look of it and I find the panel layout intriguing, so I included it anyway.



I think the panel proportions (the stitching pattern, not the cutting pattern) ought to be 1:2 as this ratio produces the square antiprism shown on the left in which the top and bottom square faces are rotated 45° from each other and have a ring of eight 45-45-90 triangular faces between them. This is what most of the photos of these beanbags look like, but each of the two Otedama patterns I’ve seen uses a different rectangle and they are both longer than the 1:2 rectangle. (The tutorial from which I obtained the photos above calls for a $1^{3}/_{4}$" x $3^{1}/_{2}$" cutting size which is 1:2, but subtracting the standard $^{1}/_{4}$" seam allowance from each side results in a 1:2.4 ratio. This may have been an oversight by the author. The 2.5cm x 5.5cm panel size I mentioned is a 1:2.2 ratio.) The panel diagram on the left includes the fold lines to help you understand how it wraps around the bag.



The first tutorial I found for this type of design used a pattern modified to produce cubes for juggling[24]. For a cubic bag the panel is a 1:3 rectangle. The diagrams above show the panel with the fold lines and how two opposing panels fit into the cube.

In both variations of the design, the short side of the panel runs one half the diagonal of the square face. This fact can be used to calculate the stitching pattern size for a desired beanbag size. A tennis ball-sized cube, for instance, would have faces about 5cm wide making the diagonals 7.07cm. So the short side of the panel would be roughly 3.5cm. The long side is calculated using the appropriate ratio which in this case is 1:3, making the long side 10.5cm.

[23] http://melyndahuskey.wordpress.com/2007/07/01/basic-otedama-a-tutorial/

[24] http://juggleballs.amielmartin.com/

The ojami/otedama beanbag design is used by some Japanese companies for sitting cushions or throw pillows. I really love this idea and the look of the cushions and so I'm presenting it here as a craft idea. Below are some examples.


http://www.jcrafts.com/eg/shop/special.asp?id=ojami


http://www.takaoka-kyoto.jp/english/products_page/iroiro_ojyami2.e.html

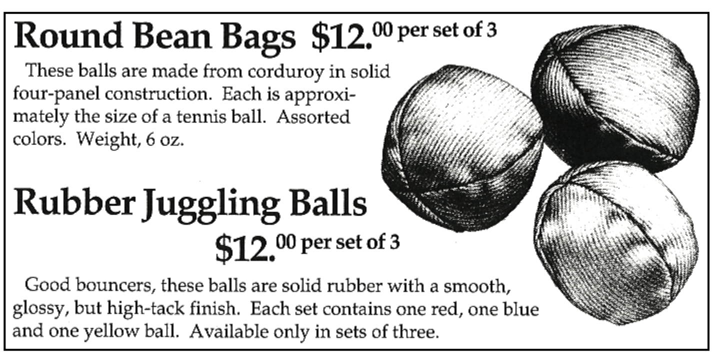
HOW I DEVELOPED MY DESIGNS

My denim, proof-of-concept bags

The sections of this chapter are in chronological order of when I developed each design.

4-panel beach ball

Note: For a concise statement of my theory behind the design of these panels, and for external support of this theory, see Appendix I.



Original advertisement from "The Flying Apparatus Catalogue" by Klutz, scanned by me

This is my first design which I developed in the mid-1990s when I was in my mid-teens. I got the idea for it from some corduroy, four-panel juggling bags sold in the Klutz Flying Apparatus catalog (shown above). I badly wanted them but I didn't want to cough up the dough ($12 for three plus shipping). All I currently had were angular cube bags by Jugglebug (right). The corduroy ones looked much rounder and more stylish. I decided to do what I often do

in this kind of situation: make them myself.

Unfortunately, I had no idea how to draw the panel shape. I had to use the small, black and white images of the bean bags from the catalog to figure out the shape and proportions of the panels. It took me weeks of experimentation and failed attempts to realize how simple it is to draw the panel shape.

My initial idea after some pondering was that the ratio of the width to the length should be 1:2, which is correct since the circumference of the finished bag would be composed of four panel widths around the equator but only two panel lengths across the poles. So I drew a stick skeleton of the panel with a vertical line forming the length and a horizontal line, half the length of the vertical, centered on it to form the width. Then all I had to do was draw the panel shape around this frame. I drew the curved sides of the panel free-hand because I had no better idea how to do it. I had rarely ever used a compass in my life and that is probably why it did not immediately occur to me to use one. I had also not studied whatever advanced mathematics would be required to understand that the edges of a circularly curved lemon shape, when wrapped around a sphere, would produce the correct, circumscribing curve I needed. I still do not really understand this; I just know it works. Despite all of this, I should have known that drawing an arbitrary curve had to be the wrong approach, but though I had subtle misgivings about it, I tried it anyway.[25]

In my first attempt, I drew the curves too steep at the ends which resulted in the panels and, consequently, the bean bag to be too narrow at the ends. When I saw the finished, slightly lemon/football-shaped bag, I unfortunately decided that my perfectly logical width:length ratio must be flawed. This began a series of experiments with different width:length ratios and styles of curves. I even tried drawing the ends of each side as a straight line to form a right angle at each end and only curving them in the middle. I hoped this would flatten those deformed ends.

After I had assembled or partially assembled over a dozen different bags and even bought a styrofoam ball around which I fitted paper models, I finally realized that the arbitrary curve was the problem. The only way to make a non-arbitrary curve was by using a compass. (After I began drawing the sides with a compass, I marked the templates “All Round” to differentiate them from the ones with partially straight sides.) It took a little longer and the assembly of a collection of five identical bean bags (which are now at my grandparents’ house for kids to play with) to come full circle with the realization that, with the now truly circular curves, my original width:length ratio should work (by this time I had settled on a different ratio with a slightly shorter length).

After that, there was just the matter of deciding on the best size of the bean bag. I wanted the inflated bag to be as close as possible to the size of a tennis ball because I liked that size for juggling (I also do not like to choose things arbitrarily and the tennis ball gave me a nice, non-arbitrary choice). So, I would make a bag, fill it to a comfortable firmness, and spend a few days tossing it around and comparing it to a tennis ball, trying to decide if I would like it better if it was a different size.

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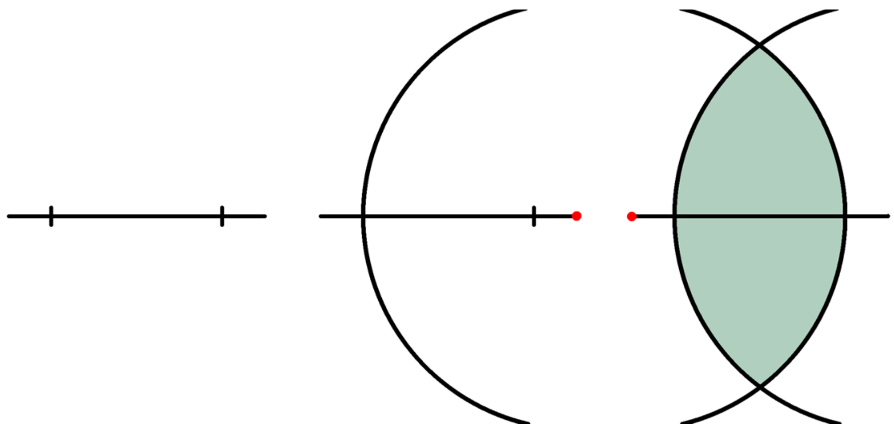
[25] When writing this document I felt foolish for having tried to draw a free-hand curve rather than thinking of using a compass, and I didn’t like admitting to it. But I have found through my web research that this is actually by far the most common way to draw this panel shape (by amateurs). I have seen PDF patterns that are obviously drawn free-hand (example: PDF pattern linked from http://ideas.stitchcraftcreate.co.uk/juggling-balls/), and I have seen written instructions that explicitly say to draw the curve by hand and give no formula or radius for the curve (example: http://jordanclan.net/cassandra/sewing/juggling.php). So far I have never seen anyone provide a mathematical definition of the beach ball panel or any explicit instruction to use a circular curve. (Edit: I finally found one six months after writing this; see *Appendix I*.)

I decided on a panel size of $1^7/_8$ by $3^3/_4$ inches and made nine bean bags of this size. When I began making twelve-panel bags, I decided on a panel size that made the new bags a little larger than the four-panel bags. When I compared these new bags to my old, smaller bags, I found that I liked the larger size better. Conveniently, the panel size that would make the four-panel bags the desired size happened to be a nice, simple, 2 by 4 inches.

Eventually, I figured out a shortcut to drawing the panels that did not involve drawing two perpendicular lines (one marking off the width and the other the length of the panel) as guides for the curves. I now only need one line (the width) because I know how to place the compass so that the arc formed therewith will be exactly the right vertical distance from the center of the panel. The illustrations below depict the basic steps. The red dots are the compass points or circle centers.



The arc radius is given by $\mathbf{r = (w^2 + l^2) \div 4w}$ and the length of the horizontal line is $\mathbf{2r - w}$ where w = panel width and l = panel length.

My stitching technique went through its own evolution. I started out with a basic running stitch (that and the whip stitch were the only stitches I knew as I had only a rudimentary knowledge of sewing), but I found that this resulted in the finished seams looking unappealingly rippled. I figured this was because each stitch pulled the fabric toward itself and had no opposing stitch to pull the fabric back. So I tried using a double running stitch: I stitched from one end to the other and then back the other direction so that each new stitch was on the opposite side from the first stitch. This helped, but not enough to satisfy my perfectionist nature.

Finally, after much thought, I came up with a new stitch that was based on the double running stitch concept and which worked perfectly. The method for this new stitch is described in the *General Notes and Techniques* chapter under "Stitching techniques". This new stitch made the seams perfectly smooth and, because half the stitches are backwards, they sort of locked themselves against the fabric and stayed tighter. When I decided to write the first, informal draft of these instructions (which I wrote during that time), I wanted to give a name as well as a description to my stitch. My mom had several sewing books from her younger years and I figured that since a stitch as good as mine probably didn't originate with me, I might find it (or something similar to it) in one of those books. Well, it was in one of the books and it was called a "Backstitch". I think that's pretty cool: *I invented the backstitch!* (I just wasn't the first to do so.)

Alternate methods of designing a beach ball panel

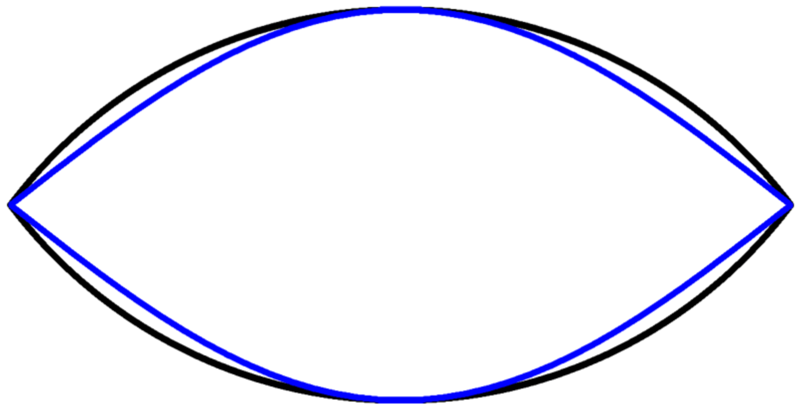
Six months after writing the original draft of this document I found for the first time a mathematical definition given for the beach ball panel's curve and design and it happened to match my theory (see

Appendix I). It wasn't until 11 months after that (February, 2014) that I found any more. The two people whose articles I found at that time gave alternate theories. One of them, "Geoff 42", uses an elliptical curve and even provides a table of pre-calculated pattern dimensions and a pattern dimension calculator[26], but it only provides calculations for 6-panel balls, and Geoff does not explain the derivation of the design or give the underlying formulas. The other, Matt Hirsch, suggests building a curve based on calculations of the circumference of a sphere at different latitudes[27]. Hirsch's method was linked to from a web article about building a large-scale beach ball[28], but the author of that article did not use the suggested method. Following is Hirsch's description of the method.

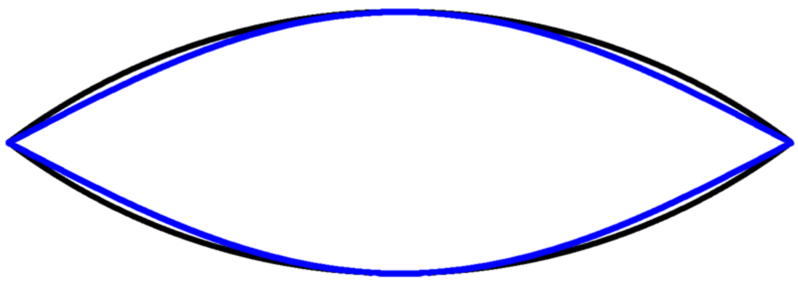
> Think of each section [or beach ball panel] as a constant angle slice of the surface of the sphere. So to draw a section, find the distance subtended on the surface of the sphere at that given latitude for the given number of sections. For example,
>
> Diameter (D) = 9
>
> Circumference (C) = 28.274
>
> 8 arcs. Each arc is 45 degrees (probably a little big), and so covers 45/360ths = 1/8th of the circumference, so it is 3.534 at the middle, and 0 at the ends. Now to find the width of the section at other locations, find the circumference of the circular slice of the sphere at that height. As you go from 0 to 90 degrees from the pole to the equator, the diameter of one slice of the sphere is d = D * sin(theta). Pi*d is the circumference at that latitude. 1/8th of that is the width of the slice at that angle.

This method produces a non-circular curve that is steeper near the ends than the circular curve. Below are comparisons using the four-panel and six-panel shapes. The shapes in blue are the result of the latitudinal circumference method with the width calculated every 5° (producing 36 segments from end to end). (I would have included Geoff's elliptical design, but I have no way to draw it on the computer, and drawing it manually using thumbtacks for the curve foci and a string to create the curve would be tedious. Geoff's computer-drawn illustration that accompanies the calculator, which I would have used, does not have the same shape as the photos of his actual template, having ends that are much more rounded than the pointed ones in the photos. I think the computer-drawn illustration is simply a true ellipse whereas the design method effectively produces two ellipses that overlap by roughly 80% and whose intersection forms the panel shape. When I overlap the computer-drawn diagram with itself according to how (I think) it should be based on the locations of the foci, I still do not get a shape that looks like the photo, leading me to believe that the ellipse in the diagram is not drawn strictly according to Geoff's method, but is only an approximated visual aid.)

[26] http://g42.org/xwiki/bin/view/Main/HowToSewASphere; calculator located at http://g42.org/xwiki/bin/view/Main/HowToSewASphereDerivation
[27] http://fab.cba.mit.edu/classes/863.12/people/catherine/matt.txt
[28] http://fab.cba.mit.edu/classes/863.12/people/catherine/makesomethingbig.html



Four-panel shape: my design method in black and the latitudinal circumference method inside it in blue.



Six-panel shape: my design method in black and the latitudinal circumference method inside it in blue.

I have seen patterns that have this type of shape (more acute and with less curvature near the ends) and now I wonder if they were created using the latitudinal circumference method. My initial assumption based on the above comparison was that this method would produce somewhat sharp and protruding vertices on the bag much like my earliest, failed attempt in the 1990s. The circular design produces what appear to be perfectly round seams on the finished bag with no vertex prominence at all, and I wouldn't want that to change.

Several months after writing this design comparison, I got around to actually constructing a 4-panel bag based on the latitudinal circumference method. The result confirmed my assumption. The bag's panel-face profile has a very slight lemon or football shape. The sharper curvature of the middle portion of the panels' curve (the waist) results in seams that appear somewhat acute rather than circular and therefore the seam profile of the bag has as much prominence and acuteness at the waist as it does at the vertices, giving that profile almost a square shape with the beach ball's two vertices and two equatorial peaks forming the four corners. I had greater difficulty in flattening the seam allowances out due to this sharper curvature which caused more severe puckering of the fabric.

Additionally, drawing the circular design, even by hand if you have a compass, is simple and easy. Drawing the latitudinal circumference method is very tedious and requires many complicated calculations and precise measurements to get a good curve, or a computer program that can draw the curve based on equations.

Dodecahedron

This is my second design. In the 1998 or 1999 Bright, Indiana volunteer fire department festival, I played one of the games and won an el cheapo, green, vinyl-coated hacky sack with the BP (British Petroleum) logo on it (BP had just built a gas station in Bright and was trying to make itself known). I liked the way it was designed: twelve pentagonal panels sewn together to make an almost perfect sphere. It was much rounder and more attractive than my four-panel design, so I decided to try to emulate this new design in my juggling bags.

As I discovered later, aside from looking more elegant, this design also has the benefit of allowing me to be much more creative with color arrangements than the four-panel design. Four is only divisible by 1, 2 and 4 and so I can use only these multiples of colors and produce a balanced look (actually, I came up with a very attractive 3-color arrangement during the writing of these formal instructions). I also have very few color arrangement options. Twelve, on the other hand, is divisible by 1, 2, 3, 4, 6, and 12 and this design also offers numerous ways to arrange the colors on the bag.

However, before I could make a twelve-panel bean bag, I had to figure out how to draw a perfect equilateral pentagon. (Keep in mind that at this time our family either did not have an internet connection or I did not yet know how to make much use of the internet, so I had to figure everything out myself.) I tried and failed to come up with a simpler shape (triangle, square, etc) around which a pentagon could be formed using the sides and corners of the inner shape as guides (similar to my stick skeleton idea for the four-panel design). I then knew that I would have to draw the pentagon without any guide. This meant that I would have to draw a side, measure out the angles at its ends with a protractor, and then draw the adjacent sides (there are ways to draw a pentagon using circles, but I did not know about these and I still do not understand them well enough to use them). So I had to figure out what the measure of the angles should be. I first thought that since a square has a total of 360°, a pentagon might as well. I actually had to draw the first angle before I realized that $^{1}/_{5}$ of 360° is an acute angle which an equilateral pentagon cannot have. I then tried 360° + 180° = 540°. This made each angle 108° which was perfect.

The second problem was what size to make the panels so that the finished bag was the most comfortable size for juggling. This was more difficult than in the case of the four-panel design because there were many more panel sides and heights to add up to get the circumference. Also, I did not know, nor could I come up with, any formula to tell me the distance from a side to the opposite corner (i.e., the height) of a pentagon. I had to figure it out the crude way: by drawing the pentagon (or at least half of it) and then measuring it with a ruler. After arriving at a reasonable circumference I would make a bean bag and, as with the four-panel bags, decide over a period of days or weeks if it fit my hands just right. I started out making panels with $^{3}/_{4}$ inch sides. This made the bag a little too small, so I moved up to $^{7}/_{8}$ inch. I was almost satisfied with this size, but I decided to try 1 inch. This size, at last, seemed perfect and I loved the fact that the sides were such a simple length.

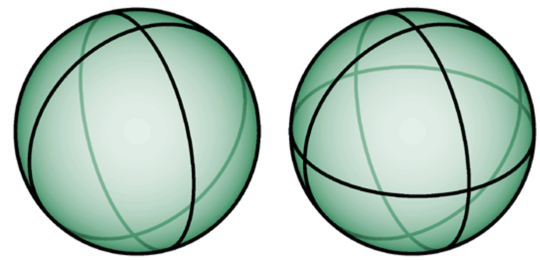
I later learned from Dr. Lee Sanders at Miami University's Hamilton, OH campus that the shape of this bag is called a dodecahedron. It has 12 faces, 30 edges and 20 corners. I learned during the research for this document that the dodecahedron is one of five Platonic solids (see http://en.wikipedia.org/wiki/Platonic_solids for more information).

Spherical octahedron

I developed this design in August, 2012 while writing this document. I found a website by someone named Peter Billam[29] that had patterns for several polyhedral beanbag designs using bulged polygons

[29] http://www.pjb.com.au/jug/leatherballs.html

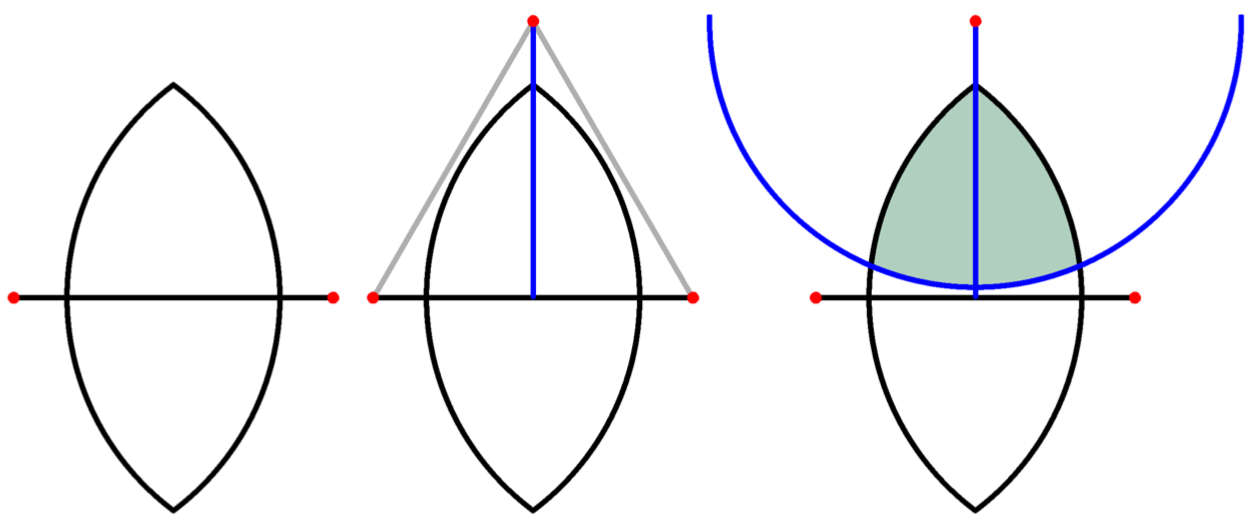
for the panel shapes, and these included an octahedron. The website did not give the definitions for the shapes, however, and so I did not know what curve radius they used (I later measured the octahedron panel template with a ruler and tested it with a compass and it appeared to be an angular form of a Reuleaux triangle).



However, I saw that, roughly, a spherical octahedron is my four-panel beach ball design with each panel cut in half widthwise and the two new edges rounded to match the adjacent edges. The panel would have to be based on an equilateral triangle because that is what forms the faces of a regular octahedron, and its sides should be rounded using the same curve radius as the beach ball's lemon curve panel because the octahedral bag has the same seam structure of two perpendicular, circumscribing seams but with one additional, identical seam around the equator. So I set about constructing an equilateral triangle with sides curved using that radius.

It took some time to figure out what size I needed the guide triangle to be and how to best use it to form the circular triangle. I initially experienced some lapses of logic and designed the panel by using a radius equal to the height of the triangle which resulted in the circular triangle meeting each of the guide triangle's sides. I later realized my mistake and redesigned it so that the ratio of the curve radius to the guide triangle's sides is the same as the ratio of the beach ball's curve radius to the distance between its two compass points. My thinking was that the compass points of the beach ball panel could be thought of as two corners of the triangle, and then I can create a third corner and draw the same curve from that as from the first two. This makes the ratio of the compass radius to the length of the guide triangle's sides 5:6, that is, the radius is 83.33% of the length of the side.

The illustrations below depict the design method. The red dots are the compass points or circle centers that produce the arcs. I start with the beach ball panel and its two compass points, I complete an imaginary equilateral triangle to get the third corner, and then I draw the same arc from that corner.



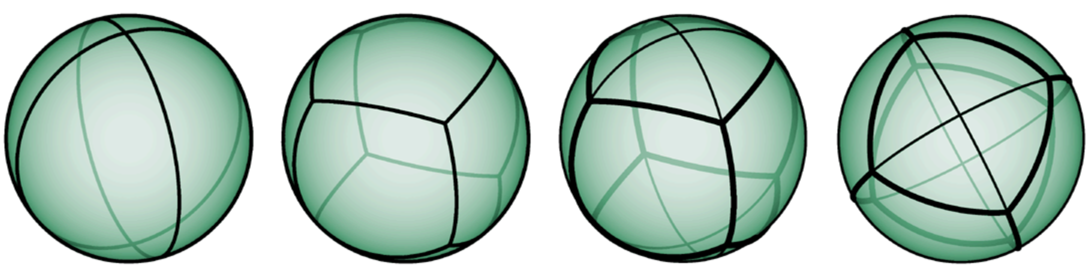
Spherical cube

I developed this design a few days after the octahedron and got the idea from the same website (http://www.pjb.com.au/jug/leatherballs.html) by Peter Billam. I had made a standard six-panel cubic beanbag years ago as a novelty, but I had little interest in making another or in including it in these instructions. The idea of a spherical cube seemed worth trying, however. It turned out so well that I consider it to be as worthwhile a design as any of the others and so I wrote out the formal instructions for it.

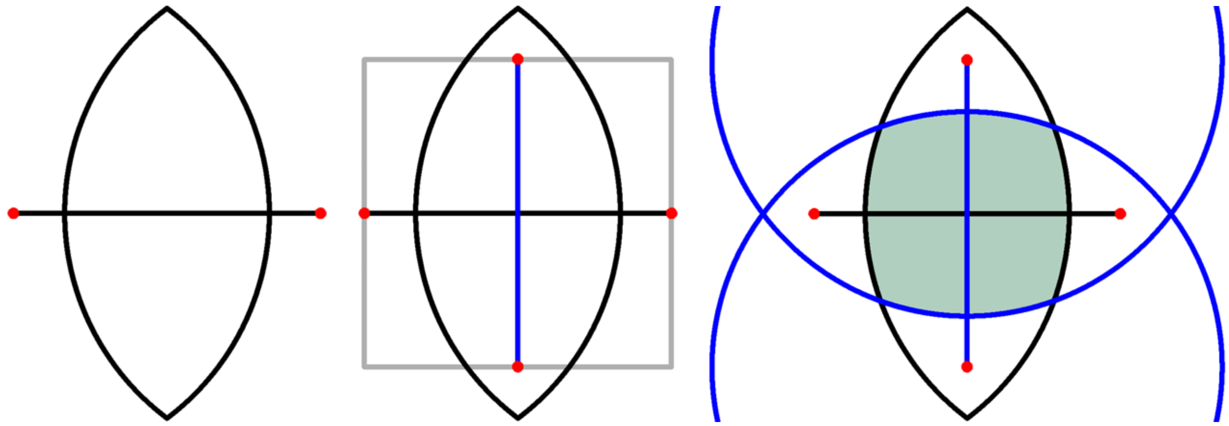
I designed the cube using the same assumption about the curve radius of the panel edges as I did for the octahedral bag; that is, I assumed it should match the curve of the four-panel beach ball design. At first this assumption was based on intuition and a small amount of reasoning – the panels each form a quarter of the bag's circumference as they do around the "equator" of the beach ball and are arranged in a similar fashion, so the curve be the same.

After I made the proof of concept bag, I began to think that I might have made an incorrect assumption about the curve. Perhaps a curve radius equal to the width of the panel (as opposed to width × 1.25) would make more sense and also be simpler to construct.

I almost got to the point of constructing a bag according to this design, and then began to understand my initial assumption more clearly and realized that it did make more sense because the cube is basically the same as the four-panel beach ball design except that the four panel tips at each pole have been exchanged for a new panel that matches the equatorial portions. Around the equator both designs are the same. The polyhedral spheres below that I constructed in SketchUp illustrate this. Just as the circular triangle of the octahedral bag is the end half of the lemon panel, the circular square is 50% taken from its middle and is arranged on the bag in the same positions as the lemon panels. It seemed logical that all three designs should be related to each other in this way and be based on the same curve. I also thought that if the prominence of the corners was reduced any more, the vertices of the cube would become slightly concave.



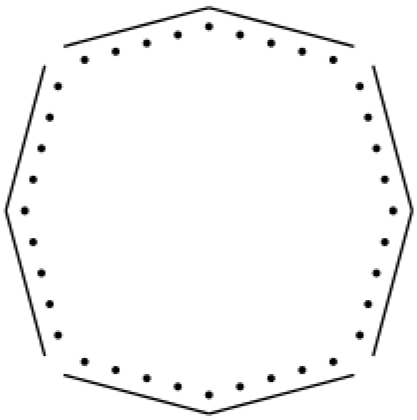
The illustrations below depict my design method. The red dots are the compass points or circle centers that produce the arcs. I start with the beach ball panel and its two compass points, I draw a vertical line through the horizontal line with a matching length to form an imaginary square and get the two vertical compass points, and then I draw the same arcs from those two points as from the original two. A short way to describe this is that the panel shape is formed by the intersection of two perpendicular beach ball panels.



A few days later I began designing a spherical tetrahedron using curves based on a three-panel beach ball instead of those for the Reuleaux triangle as Peter Billam calls for (according to my measurements of his panel shape), and the thought process involved in designing a three-panel beach ball is what caused me to discover my lapses in logic in designing the spherical octahedron (the error was not great) and to realize that I had made some very foolish assumptions. This led me to rethink all of my designs, including the cube.

I measured Peter Billam's bulged cube panel shape and found that the angled bulge he uses, when transformed into a curve, happened to have (whether by design or not I do not know) a radius whose ratio to the square's width is the same as the ratio of a square's width to its diagonal (0.7071). This was a much steeper curve than I used (my radius was width × 1.25). I thought that a steeper curve might, after all, work better because, while the curves of my beach ball meet at a four-corner vertex and the seams join to form continuous, circumscribing seams, the curves of the cube, after a short span, meet at a three-corner vertex and each seam runs into the face of a flat panel rather than another curved seam. A steeper curve might join more smoothly with the flat panel and produce a rounder bag. I decided that I could not be happy until I had determined whether my curve or this much steeper curve made a better bag.

I constructed templates and sewed a bag using 5cm squares rounded using a 3.5cm curve radius. The result confirmed what I had assumed before: The corners were pulled inward too far and were not only concave, but caused sharp puckering of each edge of the cube between them so that the bag effectively had twelve "corners" instead of eight. It was a cool shape (sort of a stubby, polyhedral star), but not at all what I wanted. I don't know how Peter Billam made this design work, but it does not work for me. With this experience, and looking again at the first cube I made, I think I can see that the same effect is nearly beginning to happen to it. It is not happening, but I think, as I did before, that if the corners were pulled inward any farther, it would be too much. Perhaps with a thin and stretchy fabric the steeper curves would work, but I want a more universally viable design.



Edit from 3/2013: I have since realized that the puckering of the edges of my cube may have been due to my using a curve instead of Billam's original angled bulges (Billam's bulged square pattern is shown on the left). The curve effectively increases the angle of the corners, making it too large for three such angles to meet and form a flat vertex. Based on an article I read months later on spherical geometry[30], the angled bulges should be constructed in such a way as to widen the square's corners from 90° to 120° so that the sum of the three that meet at each vertex of the cube will be 360°. I measured the angles with PhotoShop and they do appear to be 120°. However, the new, secondary vertices caused by the angled bulges will have a sum of only 300°. I think this would cause rather prominent vertices where my design has circular curves.

So I still do not feel convinced that Billam's design is much better than mine, but maybe I'll try it someday and find out. He does seem very well educated and he also claims that his bulged cube is spherical enough for contact juggling. Mine may be spherical enough for that, especially if made with a stretchy material and filled tightly, but I am not a contact juggler and so I can't judge that very well. Billam's panel shape would probably be more difficult to draw either by hand or with SketchUp because it is a semiregular octagon and I don't know of a simple way to construct it. Curved sides make more sense to me anyway since my aim is to form a sphere, not a polyhedron.

Equidistant cuboctahedron

Photos from (respectively) http://www.renegadejuggling.com/14-Panel-Suede-Leather-Ball-p115.html, http://www.higginsbrothers.com/html/juggling_balls.html, and http://www.flyingclipper.com/home/fly/page_256_94/dirtbag_14_footbag.html

I designed this beanbag in April, 2013, eight months after I wrote this document. I often saw beanbags made according to the design shown above and I liked it so I originally included it in the *Other Designs and Variations* chapter. I learned recently that Flying Clipper holds a patent on this design (I don't know if it is mathematically identical to the others or to mine) and implies that the goal for the design was to "produce the visual of a soccer ball" (see the third photo above, the "Dirt Bag"; the URL is below the photos). All I could determine about the design was that it appeared to be a cuboctahedron with the triangles' corners truncated slightly, turning them into semiregular hexagons. I had always assumed

[30] http://euler.slu.edu/escher/index.php/Spherical_Geometry

that the amount of truncation was arbitrary and only intended to decrease the prominence of the vertices of the beanbag. I had no interest in designing it both because of its apparent arbitrary nature and because I preferred the idea of curved edges to truncated corners. I occasionally tried to figure out the curves necessary to construct a spherical cuboctahedron, but I never succeeded. I always liked the look of these bags, though, and I couldn't entirely give up the thought of making a cuboctahedron-based bag.

This truncated design, I later learned, is kind of a combination or perhaps a halfway point between a cuboctahedron and a truncated octahedron, the latter of which is composed of squares and regular hexagons (http://en.wikipedia.org/wiki/Truncated_octahedron). Flying Clipper refers to the truncated triangles as truncated hexagons, so they may have come from the opposite direction from me and used the truncated octahedron as the basis for their design.

I recently found a true cuboctahedral beanbag (shown below), which is the only one I have ever seen and is made by Juggling Thingies. I don't like the look of the true cuboctahedron as much as the modified one, though.

True cuboctahedral bags from http://jugglingthingies.homestead.com/files/index.htm

Except for its visual beauty, the true cuboctahedron (without curved edges) never seemed to me to be as good a design as the dodecahedron for three reasons. First, the angles that form each vertex have a sum of 300° while the dodecahedron's have a sum of 324°, so the cuboctahedron has sharper, more prominent vertices. Second, the edges of the cuboctahedron are longer than those of the dodecahedron by about 40% for the same circumference. These two properties are due to, and can be summarized by, the fact that triangles and squares with three and four sides are farther from being round than the pentagon with five and so, except in much larger numbers, cannot produce as good a sphere. Another way to look at it is that a dodecahedron has 30 edges and 20 vertices while a cuboctahedron has only 24 edges and 12 vertices, so the latter must have longer edges and sharper vertices to complete the solid. As a result, I think the cuboctahedral beanbag will feel less smoothly spherical than the dodecahedral even though it has two more faces. That brings me to the third reason: The cuboctahedron has two more panels to cut out and sew.

In April, 2013, I became interested in figuring out an optimal form of the cuboctahedron concept for a juggling beanbag. I wanted to discover if there is a non-arbitrary choice for the amount of truncation applied to the triangles and a way to get the aesthetic attributes of the cuboctahedron without losing the geometric benefits of the dodecahedron. The process of designing this polyhedron was by far the most mathematically intensive of all my designs up to this time and of all projects of any kind I've done since college. Some of my failed attempts involved formulas of ridiculous complexity with inverse trig functions nested within trig functions nested within radicals within sums of fractions within radicals all part of huge fractions.

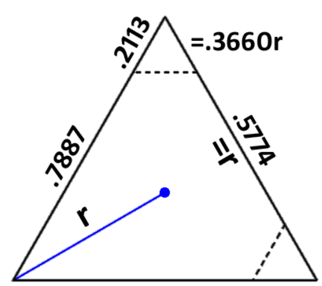
I began modifying the cuboctahedron simplistically by merely truncating the triangles in 2D based on fractions of the triangle's sides. I knew this was an arbitrary choice but I hoped it would lead to some kind of discovery about the design (and it did). I began with $^1/_6$ of the side (giving it a vague connection to a hexagon), making the short side of the resulting hexagon $^1/_4$ the length of the long. I compared this visually to the beanbag photos and it looked roughly the same.

I then began constructing cuboctahedrons in SketchUp and I tried to construct one that had the same truncation, and I also observed the effect this had on the surface area of the faces. I began hoping for a design that had the same surface area for both face shapes as calculated by SketchUp (the square is normally much larger than the triangle). This still seemed somewhat arbitrary, but it at least made a bit more sense as an approach to designing a juggling bag with a uniform surface. I did not know enough math to construct these designs directly and had to do it partially by trial-and-error. I did not achieve precision.

At some point it occurred to me that by constructing cuboctahedrons with truncated triangles, I was bringing the triangular/hexagonal faces closer together in relation to the squares. I measured the true cuboctahedron in SketchUp and found that the distance between the triangles is 15.47% greater than between the squares. This gave me the idea that I stuck with, which was to construct a cuboctahedron in which both face types were the same distance from each other, giving the resulting ball a more constant diameter. I had to do hours of thinking and trying to make SketchUp do what I wanted, as well as a lot of calculations of angles and distances of various aspects of the necessary polyhedron just to get it drawn, but I had made use of SketchUp measurements of some of the dimensions along the way. Crude construction and measurements of the faces' dimensions weren't good enough for me; I wanted a mathematical definition.

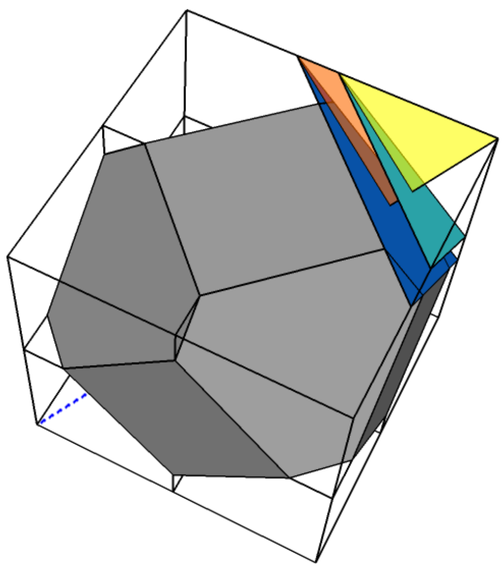
I spent another couple of days trying to find a formula that would define the hexagon shape. The eventual result was a ratio between the radius of the standard cuboctahedron triangle and the radius of my larger triangle (before truncation). This ratio can also be applied to the sides of the triangle so that it tells how much of each side to cut off. The ratio is

$$\frac{1}{3-\sqrt{3}} = \mathbf{0.7886751346} \rightarrow$$



I cannot yet prove it mathematically, but through measurement of multiple iterations of this panel shape, I found that the radius of the full triangle is equal (to as many decimal places as I bothered to test it) to the length of the long side of the derived hexagon and (obviously) to the square's width. There is probably a fascinating geometrical reason why this occurs exactly and only at the point at which the polyhedron attains equal widths between all opposing faces, but I don't yet see it and I haven't felt up to the task of discovering it. However, it makes me feel that I've designed an important polyhedron and so I like it all the more.

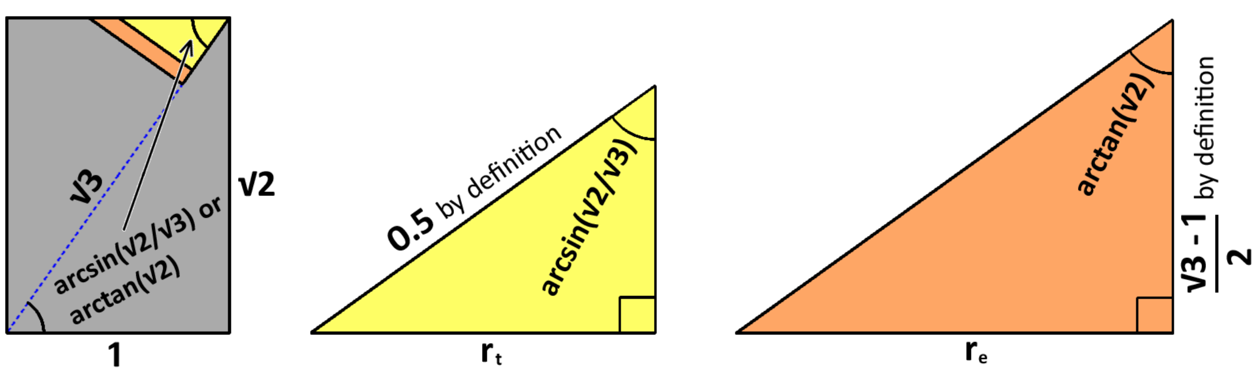
I later attempted to find out if my design is the same as the Renegade brand 14-panel bags, which I really like the look of (the other brands probably use an identical design), by measuring a high-resolution photo of them in PhotoShop. My measurements tell me that the long side of the hexagon is a little under 2.75x the length of the short side. Dividing .5774 by .2113 yields 2.73, so I would say the two designs are probably the same.

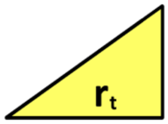


The method I came up with to find the definition of the hexagon shape is illustrated on the left. I use a cube to construct my cuboctahedrons (an octahedron can also be used as a starting point, but a cube is easier to draw). I formed a hypothetical right triangle (illustrated in yellow) by drawing an imaginary line from the corner of the cube to the center of the triangular face of the true cuboctahedron, shown in teal blue (this line, if continued, would extend through the center of the cube to the opposite corner as shown by the blue dotted line on the lower left), drawing another line from the center of the triangular face to its corner which intersects the center of the cube edge (this is the triangular face's radius), and then using the cube edge as the third side. I was able to find the angles of this triangle and the length of the hypotenuse (which is half the length of the cube edge) all in the form of ratios of a unit cube.

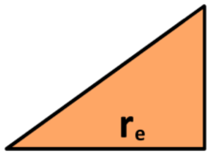
I then extended this triangle inward (orange) to where the proposed new triangular face (blue) would be which is defined such that its distance from the opposite triangular face is the same as the cube's width. I was able to calculate this triangle's dimensions because the angles are the same as the first and the side that is perpendicular to the plane of the triangular face has a length, by definition, equal to the diagonal of the cube minus the width of the cube, all divided by two.

Below is some of my work and the derivation of the ratio formula. I included this both for the benefit of anyone who wants to understand it or check my work, and so that when I forget what I did and question whether I did it right, I can recheck my work. (My brain tends to be very faulty and I can't fully trust it.) The grey rectangle is the diagonal cross section of a unit cube (the short sides are the edges of the cube and the long sides are the diagonals of the cube's faces). Its dimensions can be found using the Pythagorean theorem. The yellow and orange triangles are the ones from the diagram above. r_t and r_e are the radii of the triangular faces for the true cuboctahedron and the equidistant version, respectively. I used two different expressions of the same angle to make the formulas simplify more easily (a trig function and its inverse cancel each other out).





$$\frac{r_t}{0.5} = \sin\left(\arcsin\left(\sqrt{2}/\sqrt{3}\right)\right) = \frac{\sqrt{2}}{\sqrt{3}} \xrightarrow{\text{solve}} \mathbf{r_t} = \frac{\mathbf{\sqrt{2}}}{\mathbf{2\sqrt{3}}}$$

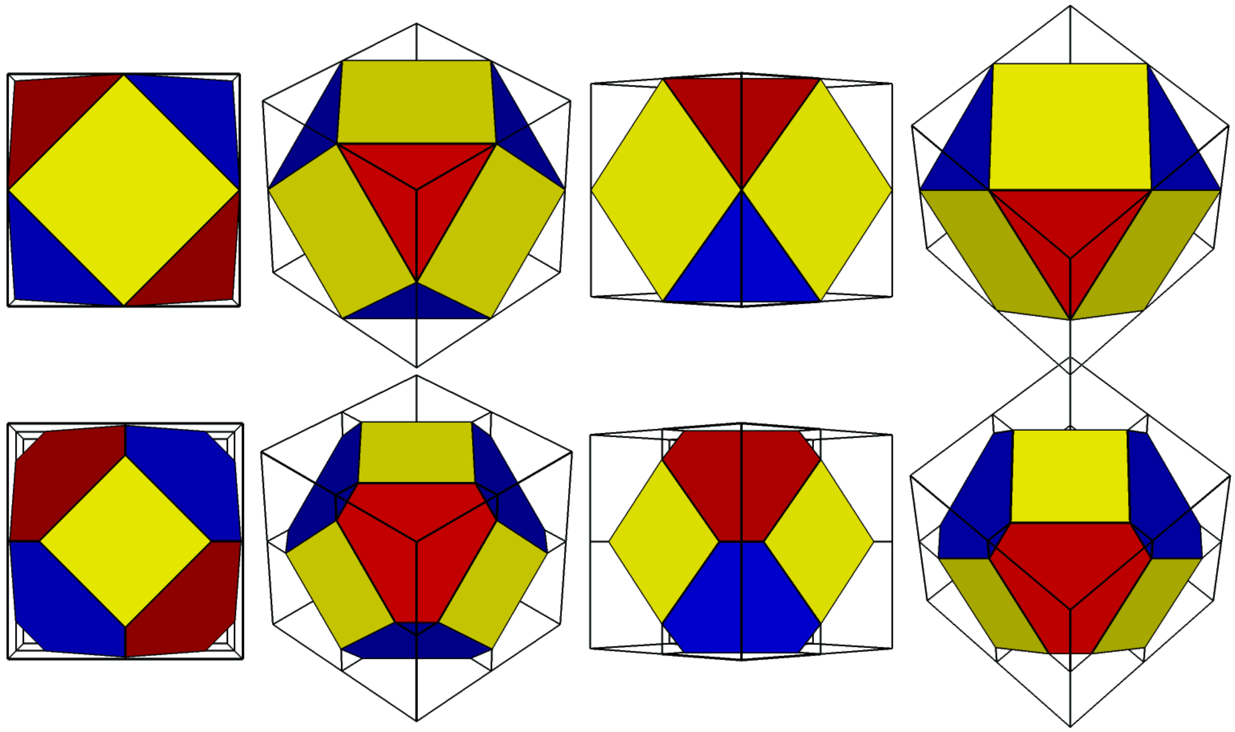


$$\frac{r_e}{\frac{\sqrt{3}-1}{2}} = \tan\left(\arctan\left(\sqrt{2}/1\right)\right) = \sqrt{2} \xrightarrow{\textit{solve}} \mathbf{r_e} = \frac{\mathbf{\sqrt{2}(\sqrt{3}-1)}}{\mathbf{2}}$$

$$\frac{\mathbf{r_t}}{\mathbf{r_e}} = \frac{\cancel{2\sqrt{2}}}{\cancel{2}\sqrt{3}\left(\cancel{\sqrt{2}}(\sqrt{3}-1)\right)} = \frac{1}{\sqrt{3}(\sqrt{3}-1)} = \frac{\mathbf{1}}{\mathbf{3-\sqrt{3}}}$$

Because the truncation of the larger triangular face is due to its intersection with the adjacent face, which occurs at a point in line with the center of the cube edge which is where the corner of the smaller triangular face intersects, the larger triangular (now hexagonal) face's new height will be that of the smaller face. Because the reduction in height will cause the same ratio of reduction to the other measurements (including the radius), I can infer that the ratio of the two triangles, yellow and orange, (which are based on the radii of the two triangular faces) gives me the ratio of this truncation. I sometimes doubt the validity of this reasoning, but my original, crudely but correctly constructed version yielded the same ratio (by using SketchUp measurements), so that gives me some confirmation.

Below is a comparison of the two cuboctahedron versions. The top row is the true cuboctahedron and the bottom row is my modified version. Each one is within the original cube framework from which I formed it. The equal width between all opposing faces can be seen in the third column. In those two images the top and bottom edges are square faces and the diagonal edges are triangular/hexagonal faces. Observe that in the true cuboctahedron the diagonal faces are farther from each other than the top and bottom faces.



A true cuboctahedron has 24 edges and 12 vertices, and mine has 24 long edges, 12 short edges, and 24 vertices. Basically, each vertex of the cuboctahedron has been turned into two vertices and a new edge.

To construct the equidistant cuboctahedron from a cube, the corners of the cuboctahedron's square faces must be offset from their normal points (the centers of each cube edge) by 0.1340 of the cube's edge length, and so must the corners of the triangles before truncation. That value is the hypotenuse of the orange triangle minus the hypotenuse of the yellow triangle which simplifies to

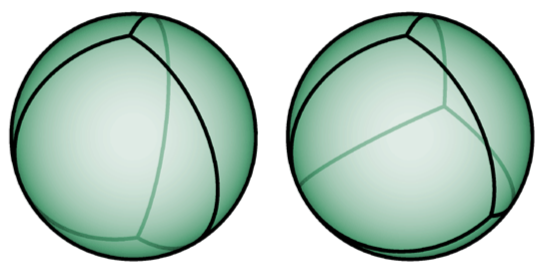
$$\frac{3-\sqrt{3}}{2} - 0.5 = \frac{2-\sqrt{3}}{2}$$

The truncation of the triangles helps to solve the first two problems I mentioned concerning the cuboctahedron as a beanbag design leaving only the third (two more panels to sew) as the price to pay for the beauty of the cuboctahedron. The new vertices are 330° as opposed to 300° (and 324° for the dodecahedron), and the edges are only 18% longer than the dodecahedron's as opposed to 40%. The triangles are now hexagons and so have a roundness more nearly that of the pentagon, and the solid now has more edges and vertices than the dodecahedron. There is even the benefit of the two face shapes being more nearly the same size than before, which was one of my early design goals and will improve the uniform feel of the bag. The resulting beanbag is not only visually gorgeous, but also luxuriously spherical and uniform. I love it.

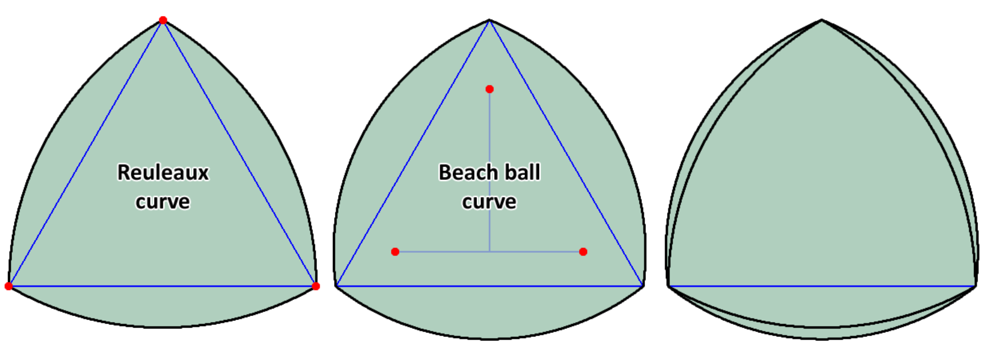
Spherical tetrahedron

http://www.renegadejuggling.com/4-Panel-Suede-Ball-p116.html

Peter Billam's website (from which I got the inspiration for my spherical cube and spherical octahedron designs – see http://www.pjb.com.au/jug/leatherballs.html) also has a rounded tetrahedron pattern. Through measurement and experimentation I determined that the panel shape Billam used for this pattern is a Reuleaux triangle. I tried making a Reuleaux tetrahedron beanbag and it was so non-spherical that I did not deem it worth including formal instructions for in this document. As Billam puts it, "it has a shape a bit like an egg with four ends." It was a cool-looking beanbag, but not as easy to catch as a spherical bag.



I later realized that a better, and probably the correct, way to make a spherical tetrahedron is to use the same method I used to design my octahedron. For the octahedron I used the curve from the 4-panel beach ball design and applied it to the equilateral triangles because the two designs are closely related. For the tetrahedron, which is also composed of equilateral triangles, I would use the curve from a 3-panel beach ball since both solids have three-corner vertices and I want all four vertices on the tetrahedron to match those of the beach ball. The figures on the right compare the two panel structures. I had always wondered why the Renegade brand tetrahedral bags look so round (see examples above). It is probably because they use a much steeper curve – perhaps even this curve. The diagrams below show the difference between the two panel shapes (the red dots are the compass points/circle centers).



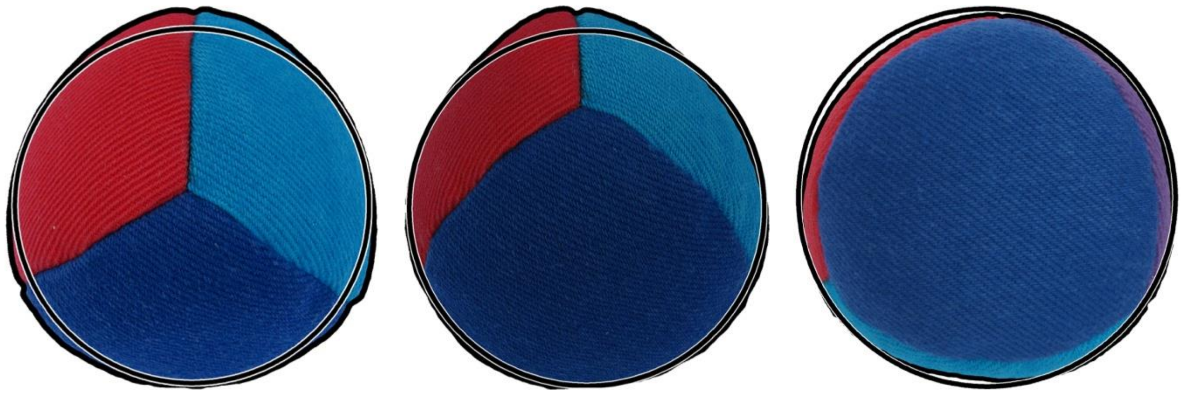
I realized this possible design improvement in late 2012 but only on April 28, 2013 did I bother to give it much formal thought and write about it. When I did, I became interested in actually making one to see how good the design was. I just finished a denim tetrahedral beanbag using the 3-panel beach ball curve and it is indeed much more nearly spherical and more jugglable than the Reuleaux version. It is still much less spherical than the 4-panel beach ball, though. It still has that "egg with four ends" feel to it. Renegade's suede leather is perhaps stretchier than my denim. I think if the panel's curve were much steeper, it would hardly have a corner and would have too wide an angle for three such angles to meet without buckling. I once made a rounded cube with too steep a curve and it caused severe distortion in the bean bag (see the "Spherical cube" section in this chapter). Also, I think the steepness of the curve would cause a lot of puckering of the seams, which is happening a bit even in my bean bag. In any case, I think the beach ball is a better 4-panel design than the tetrahedron.

Here is a comparison of the two tetrahedron designs. The top row is the Reuleaux version and the bottom row is the beach ball curve version. They are both filled to the same firmness, roughly equally broken-in, and I rolled them both around in my palms before photographing them to bring them to their best shape. Using the right curve makes a big difference.

Update from the next day: After tossing my new tetrahedron around and kneading it a lot, and breaking it in further by dampening it and putting it in a tumble dryer, I can say that it does round out pretty well and is very jugglable, though it still does not have quite the consistent body and roundness of the beach ball (catching it around a corner or edge feels a bit thin), and I can feel the vertices, though they are pleasantly round. Having a vertex opposite each face, it also lacks the symmetrical feel of the beach ball.

I juggled with the new tetrahedron along with two 4-panel beach balls and I could easily feel the greater angularity of the tetrahedron when I gripped it, especially when it landed with a vertex against my palm, but that is partly due to the stiff seams of the denim, and to the fact that it is less broken-in than the others; with further use it might become more nearly as round as they are. It does mix very well with the others and is roughly as easy to catch, even when clawing.

Below are photos of my broken-in tetrahedron with circles superimposed for reference. Though I rolled the bag around in my palm before photographing it to bring it to its best shape, you can see that it is still a bit acute near the middle of each seam, giving it a slight triangular shape when viewed from a vertex and a slight egg shape when viewed from a lower angle.



I definitely like the look and feel of this bag. It is visually attractive and interesting and has a nicely rounded angularity, and, being a relatively asymmetrical polyhedron compared to the other designs in this document, it is fun to juggle with as a novelty. Though I still think the beach ball is a better use of four panels due to the fact that it is more nearly spherical even before being broken in, the difference is not great (and would probably be much less with a stretchy fabric), and I recommend the tetrahedron design for someone who wants a fun novelty. (Another drawback of the tetrahedron is that it has fewer good color arrangement options. It has only two that I can think of: each panel a different color or each of two colors on a pair of panels.)

Equidistant icosidodecahedron

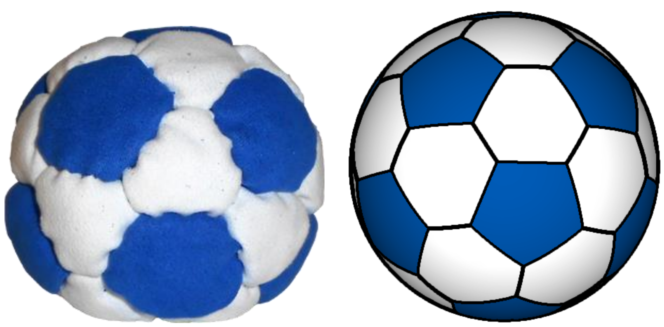


Photo from http://www.flyingclipper.com/home/fly/page_1024_134/tossaball_phat_tyre_32_panel_juggle_ball.html. CG Equidistant Icosidodecahedron drawn by me in SketchUp.

In April, 2013 I discovered the Tossaball Phat Tyre 32-panel juggling bag made by Flying Clipper (photo above). This type of panel structure is commonly used for footbags (Hacky Sacks), but this is the first time I have seen it used for a juggling bag. The geometric shape of this structure is somewhere between the icosidodecahedron which has 12 pentagons and 20 triangles and the truncated icosahedron which is the polyhedron soccer balls are based on and has 12 pentagons and 20 equilateral hexagons. The truncated triangles or semiregular hexagons used here look a lot like the ones in my equidistant cuboctahedron, but I think these are more truncated (closer to a regular hexagon, less triangular). From what I have read, Flying Clipper holds a patent on the footbag version of this design.

For months I wondered if an equidistant version of the icosidodecahedron would produce those truncated triangles. In case you haven't read the equidistant cuboctahedron section, I mean by "equidistant" that all faces are the same distance from the center or, to put it another way, opposing faces of both shapes are the same distance from each other. The distance between opposing triangular faces of the icosidodecahedron is normally 10.3% greater than between the pentagons.

On July 29, 2013 I finally felt ambitious enough to do the calculations and crack the Equidistant Icosidodecahedron. A big step forward was when I realized that the dodecahedron can be used as the basis for the icosidodecahedron (I should have realized this from the name) just as a cube (or octahedron) can be used to build a cuboctahedron. An icosidodecahedron is really a fully truncated dodecahedron, or, as it is officially called, a "rectified" dodecahedron[31]. I did not know before this how to build an icosidodecahedron. I later realized that the same is true of an icosahedron, and starting with an icosahedron is a much easier way to construct an icosidodecahedron and greatly reduces the number of calculations needed to design the hex panel shape. The method I learned for constructing an icosahedron is also easier than for the dodecahedron.

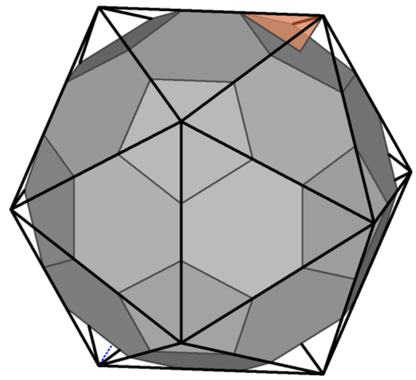
It turns out that the equidistant version has hexes that are more nearly equilateral (less triangular) than Flying Clipper's appear to be (compare the photo and the CG equidistant version above). The short side is 69.2% the length of the long. I tried building a version that uses the hexes from the Equidistant Cuboctahedron (in which the radius of the guide triangle equals the long side of the hex). That hex has a short side that is 36.6% the length of the long. I think Flying Clipper's hexes are somewhere in between. The benefit of the equidistant version, aside from its equidistance which is not of great importance in terms of roundness with so many panels, is that the two panel shapes are nearly the same size which will improve the look of color arrangements that do not distinguish between them. See the *32-Panel Equidistant Icosidodecahedron* chapter for more on this.

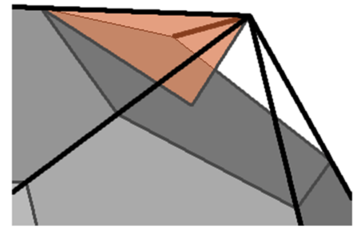
To solve the Equidistant Icosidodecahedron, I used a method similar to the one I used for the Equidistant Cuboctahedron. First, some measurements of icosahedrons from http://en.wikipedia.org/wiki/Icosahedron:

> If the edge length of a regular icosahedron is *a*, the radius of a circumscribed sphere (one that touches the icosahedron at all vertices) is
>
> $$r_c = \frac{a}{4}\sqrt{10 + 2\sqrt{5}} = a\sin\left(\frac{2\pi}{5}\right) \approx 0.9510565163 \cdot a$$
>
> and the radius of an inscribed sphere (tangent to each of the icosahedron's faces) is
>
> $$r_i = \frac{\sqrt{3}}{12}\left(3 + \sqrt{5}\right)a \approx 0.7557613141 \cdot a$$

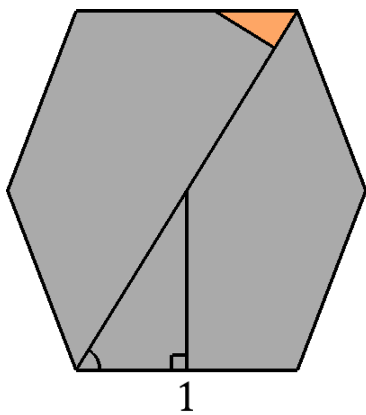
31 http://en.wikipedia.org/wiki/Rectification_%28geometry%29

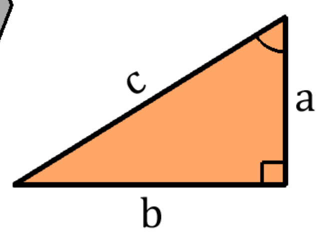
I began by forming a hypothetical triangle (illustrated in orange) to define the position of the equidistant pentagonal face in relation to the vertex of the icosahedron. The short leg of the triangle runs from the vertex to the center of the proposed pentagonal face (it would continue to the diagonal opposite vertex). The long leg is the radius of the pentagonal face.





The grey hexagon below is a 2D view of the above diagram. (It is a cross section of a unit icosahedron. The top and bottom edges are the edges of the triangular faces and the diagonal edges are the heights of the triangular faces). The diagonal line that runs through it is the diameter from corner to corner of the icosahedron, which has a length of $2r_c$.





The marked angles are equal to $\cos^{-1}\left(\frac{1}{2r_c}\right) \approx 58.28252559°$

This is derived by using the triangle in the grey cross section diagram (which has the same angles as the orange triangle) and knowing that the adjacent side has a length of 0.5 and the hypotenuse is the circumscribed radius r_c.

Side *a* has a length, by definition, equal to $r_c - r_i \approx 0.1952952022$

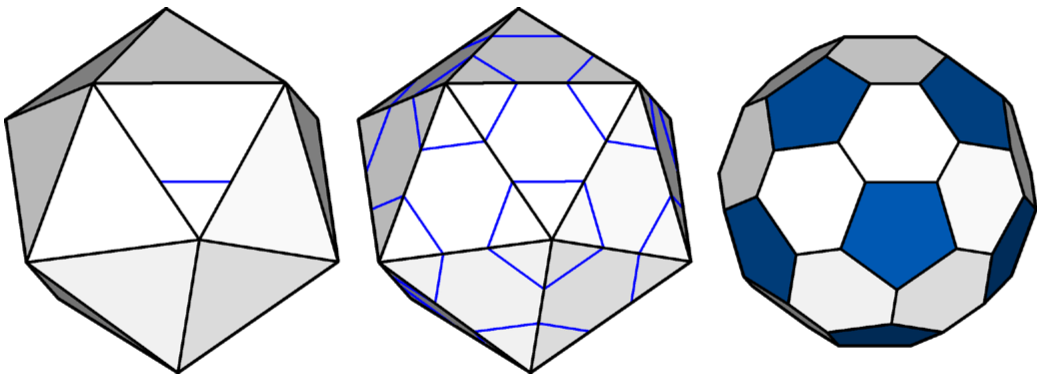
The reasoning for this is that the inscribed radius gives me the pentagonal face's target distance from the center and the circumscribed radius gives me the distance from center to corner. The difference between them is the distance from the corner at which the pent faces must be positioned to be equally distant from the center as the hex faces.

Side *b* has a length equal to $\tan\left(\cos^{-1}\left(\frac{1}{2r_c}\right)\right)(r_c - r_i) \approx 0.315994275$

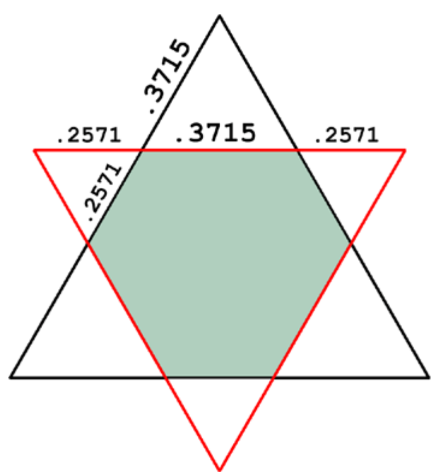
The hypotenuse, *c*, has a length equal to

$$\frac{r_c - r_i}{\cos\left(\cos^{-1}\left(\frac{1}{2r_c}\right)\right)} = \frac{r_c - r_i}{\frac{1}{2r_c}} = (r_c - r_i)2r_c \approx 0.3714735493$$

The length of the hypotenuse allows me to build the Equidistant Icosidodecahedron from an icosahedron as shown below. The initial blue line in the first step intersects the edges of the triangular face at a point that is 0.3715 or 37.15% distant from the shared corner. I draw those around every vertex and then truncate the vertices into pentagonal faces.



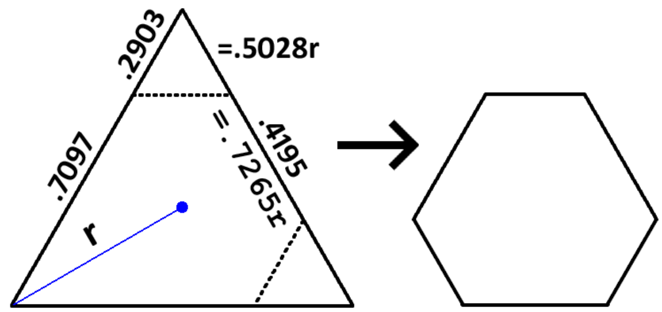
The 0.3715 truncation allows me to draw the panel shape (shown in green below) from an equilateral triangle, but it is the long truncation meaning that the hexagon will be oriented with its long sides facing the triangle's corners as shown by the black triangle below. I would prefer (though it doesn't matter greatly) to truncate my triangles using the short truncation as depicted by the red triangle.



The side of the red triangle measures (0.2571 × 2) + 0.3715 = 0.8855793521. That value is the ratio of the length of the red triangle's side to that of the black triangle, so if I divide the hex's short side, 0.2571, by 0.8856 I will get the percentage of the short truncation on the red triangle's side.

The truncation, then, to construct the hex panel shape from an equilateral triangle is

$$\frac{0.2570529041}{0.8855793521} = \mathbf{0.2902652378}$$

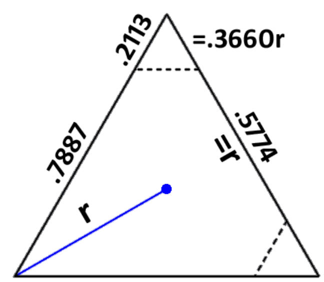


How to draw the equidistant icosidodecahedron and variations

To truncate an icosahedron to produce hex faces matching my semiregular hexes, you must reverse the conversion I did above which redefines the hex according to a triangle with the corners at the short sides. This is so that you can truncate according to the hex's long side proportion, but in relation to the icosahedron's edge length. Here is the formula that expresses the truncation as a percentage of the icosahedron's edge length. Let L be the hex's long side proportion (in this case 0.4195) and S be the short side proportion (0.2903).

$$\text{Icosahedron Truncation Percentage} = \frac{L}{2L+S}$$

For my hex above this results in the 0.3715 or 37.15% truncation I used to create the equidistant polyhedron. If you wanted to create the polyhedron based on the hex from my 14-panel design (right) the formula would look as follows:

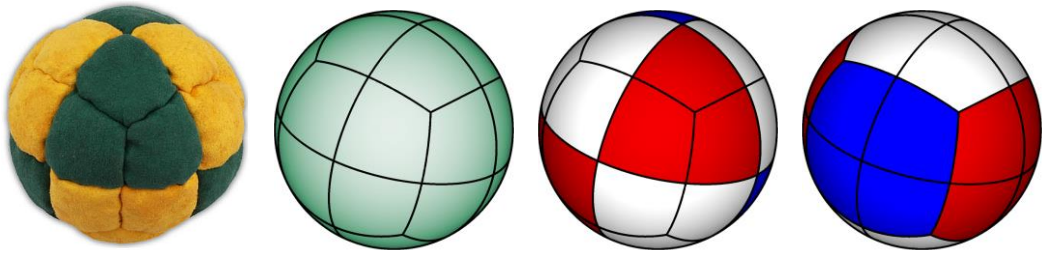


$$\frac{0.5774}{2(0.5774) + 0.2113} = 0.4227 = 42.27\%$$

This makes the truncation on the icosahedron 42.27% distant from its vertices.

To create a true Truncated Icosahedron (or soccer ball) which has equilateral hexagons and pentagons, the truncation is one third of the icosahedron's edge length. To create a true Icosidodecahedron which has equilateral triangles and pentagons, the truncation is half the edge length. Basically, you just make one truncation across the centers of each edge.

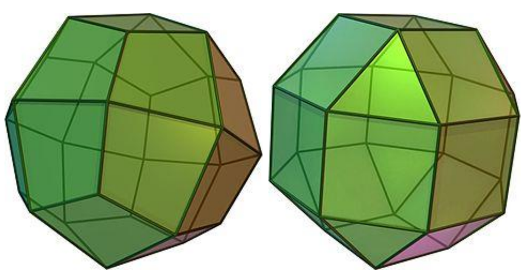
Custom deltoidal icositetrahedron



Footbag photo from http://www.expo-star.com/lview.asp?mainid=12&Subid=0&pid=52. CG illustrations made in SketchUp by me.

I developed this design in October, 2013. In my lineup of footbag panel structures in the *Other Designs and Variations* chapter you can see examples of 24-panel footbags (one of which is duplicated above). When I first looked at it I saw it as an octahedron with each triangle divided into three "kites" (definition: http://en.wikipedia.org/wiki/Kite_%28geometry%29). But weeks later I was examining it and thinking about how I might design the kites because I liked the look of this structure, and I realized that I also saw a cube pattern within it. Excited, I took my spherical cube and octahedron Sketchup models, overlaid them on a sphere, and indeed this formed the 24-panel shape! This is illustrated above. That means that this structure supports all the color arrangements of the cube and octahedron (and the 4-panel beach ball) in addition to its own unique arrangements.

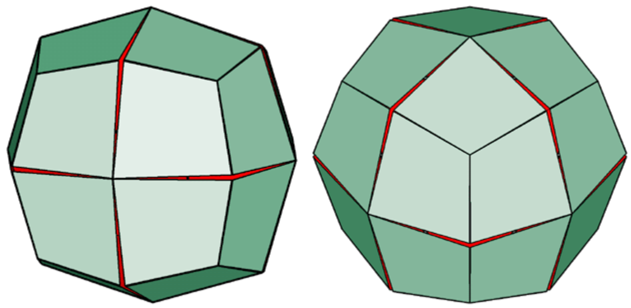
After attempting and failing to figure out how to draw an angular form of this shape, I did some web research and learned that this is (or is similar to) a named solid, the "deltoidal icositetrahedron". This is a Catalan solid or Archimedean dual, being the dual of the rhombicuboctahedron, which is a 26-face Archimedean solid also used as a footbag design (see the footbag lineup for an example).



Deltoidal icositetrahedron and Rhombicuboctahedron images from Wikipedia (http://en.wikipedia.org/wiki/Deltoidal_icositetrahedron and http://en.wikipedia.org/wiki/Rhombicuboctahedron)

Lacking knowledge of how to draw a deltoidal icositetrahedron and finding no help on the web, I tried to design an ideal variation of it by designing the kite shape in 2D. My goal was uniform vertices on the polyhedron. I would have liked to design a spherical form of the solid (with faces having circular edges) instead of a true polyhedron, but other than the fact that two of the edges should be rounded using the curve from the octahedron and the opposite two with the curve from the cube, I don't know yet how I would draw it.

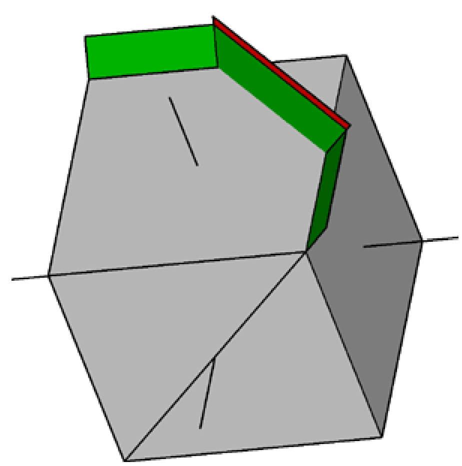
By trial-and-error I settled on a shape whose obtuse angle was 111° and whose acute angles were all 83°. The cardboard model I made looks great, but when I assembled the face shape into a polyhedron in SketchUp (which I did not figure out how to do until a day or so later), the faces did not fit together. The illustrations below show this version of the solid, which I redesigned by forming it around a cube and using trial-and-error to make the angles almost exactly 111° and 83°. The red is the gaps between the faces.



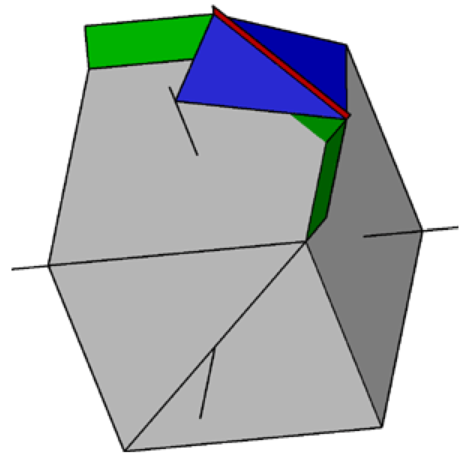
Except for the fact that my kite shape does not quite form a closed polyhedron, it is theoretically ideal. The 4-way vertices are 83°×4 = 332° and the 3-way vertices are 111°×3 = 333°. The ratio of the circumferences as calculated by the kite width (between the two opposite acute angles, 6 of which circumscribe the polyhedron), the long side of the kite (8 of which circumscribe the polyhedron), and the kite length (obtuse angle to opposite corner, 4 of which plus 4 short sides circumscribe the polyhedron) is 1.0061 : 1.0 : 1.0042. So it has almost perfectly uniform circumferences and vertices.

I have tried many different and more sophisticated methods of designing a polyhedron that retains as nearly as possible these attributes but has a face shape that fits together. I lacked sufficient math and primarily used trial-and-error. I even tried constructing a true deltoidal icositetrahedron by first building a rhombicuboctahedron (which took me a long time to figure out how to draw) and connecting the centers of the faces. The four points for each face did not lie on the same plane, however, and so resulted in a 48-face polyhedron. I don't know what I did wrong there.

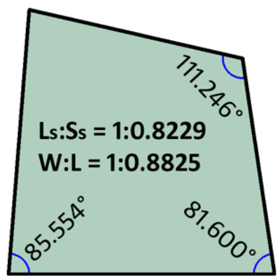
Over the course of four days and many hours of experimentation I formed what I thought was an optimized polyhedron, adjusted it, rethought it and corrected it, arrived on what I thought was a final design, returned to thinking my original was best, and rethought it again. I finally settled on one of my earlier designs in which I had corrected my originally flawed method of forming the shape around a cube. There are two ways (that I know of) to form the panel shape on a cube so that it will fit together into a polyhedron, and these are depicted below.

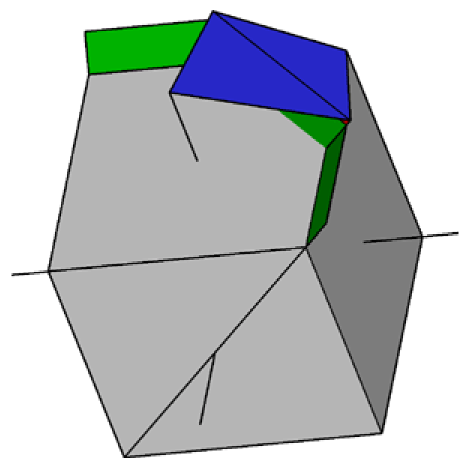


Since I want the vertices and the dimensions to be as uniform as possible, I first drew guides on the cube to mark uniform distances from the center. The lines protruding from each face have the length of the cube's diagonal and represent an equal distance from the center as the cube's corners. This is like overlaying an octahedron on the cube, which makes sense as the icositetrahedron is related to both the cube and the octahedron. The tops of the green walls on the edges are also the same distance from the center. The walls extend to the centers of each edge and I connected them across the top face of the cube. The diagonal line on the foremost cube face shows how I constructed the walls. The entire wall is not actually needed, but only a stick in the center of each edge as I used for the faces. But I needed the wall during my experimentation and I left it there. The red strip will be explained later.

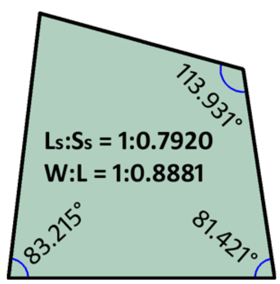


One way to fit a kite onto this framework is to make its corners meet the corner of the cube and the top of my green walls (at the point that marks the cube edge centers). This results in the icositetrahedron having equal diameters at those vertices. But as you can see it does not meet the octahedron vertex. The kite below shows the resulting dimensions (Ls and Ss stand for Long Side and Short Side, and W and L stand for the kite's Width and Length).





The other way to form the kite is to make it meet the octahedron vertex and the cube's corner. This configuration causes it to lie above the green walls at the top of the red strip (which I made to provide a reference for this version of the kite). This results in the kite dimensions below.



To determine the altitude of this red strip, extend the central wall to a higher altitude than you know you'll need, draw a line from the center stick to the cube's corner which will intersect the wall, and draw a line from the intersection point to each edge of the wall, making sure it is parallel to the wall's top/bottom. That line forms the top of my red strip.

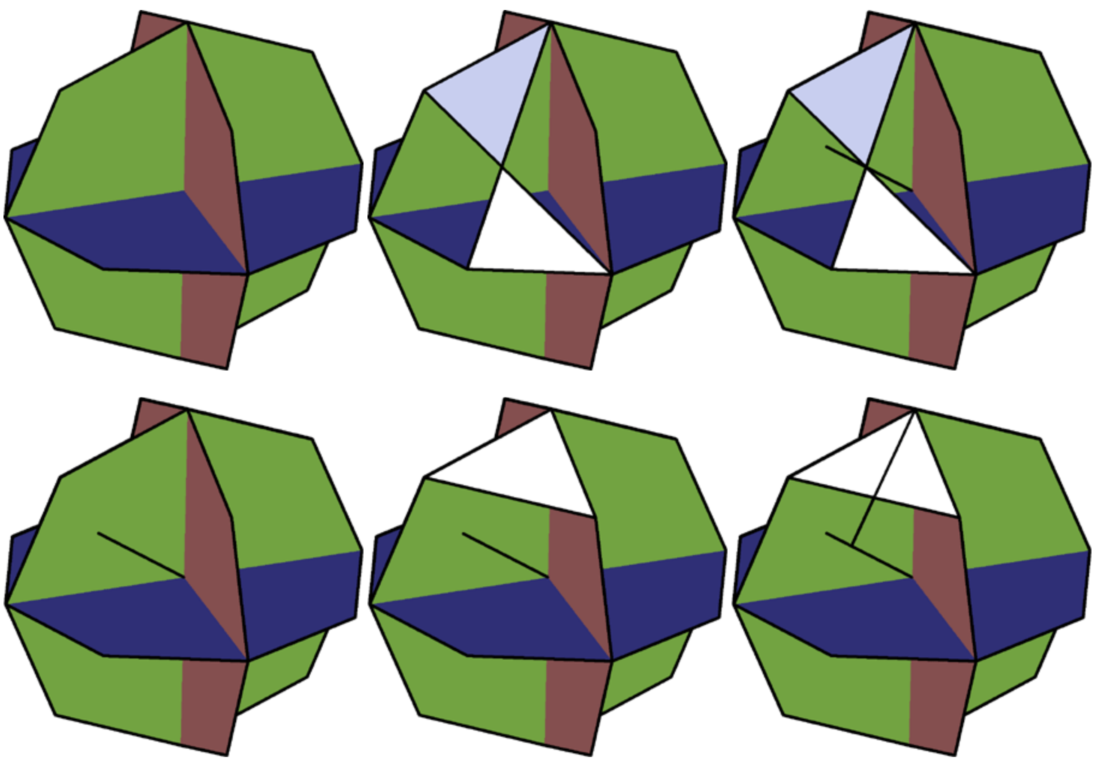
I also tried making the kite meet the octahedron vertex and the edge points and this causes the end of the kite at the corner of the cube to fall beneath the surface of the cube and it no longer fits together as a polyhedron.

If you want to draw this shape yourself, you don't necessarily need to draw the walls as I drew them, which were a result of inexperience and design experiments. What you need are simply the reference points, for which lines like the ones on the cube faces are sufficient. (I left the full walls in place for the illustrations because I thought they would make the structure's design easier to understand.) Once you have drawn the first face, rotate copies of it or of the entire model until you have formed the entire icositetrahedron around the cube.

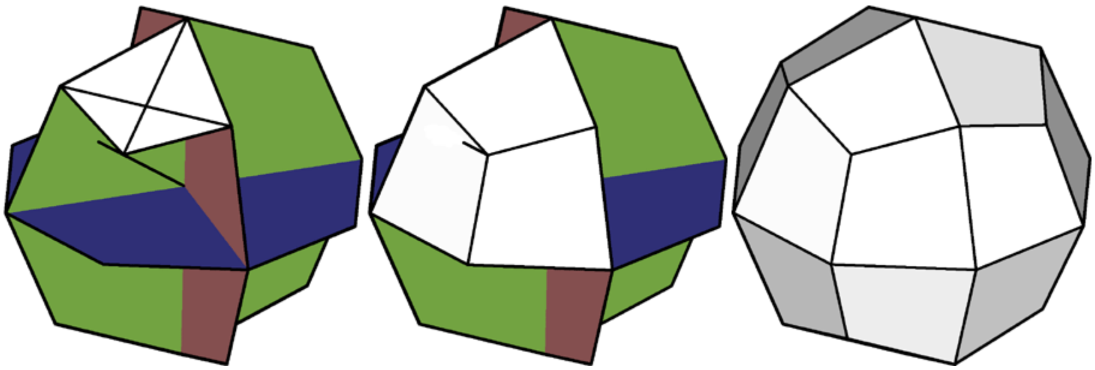
I have not been able to find any information on the web telling me what the angles on the deltoidal icositetrahedron kite are supposed to be, but my high-res PhotoShop measurements of a net I downloaded from Wolfram MathWorld are 115.3° and 81.6° (all three acute angles match). The ratio of the long side to the short side is supposed to be approximately 1:0.7735 according to Wikipedia and Wolfram MathWorld[32].

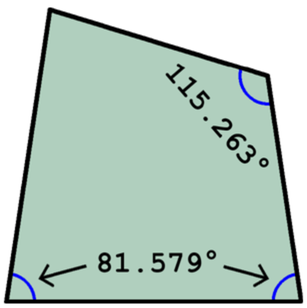
Then a few days later I finally realized the correct way to draw the shape and it produces a kite that matches my PhotoShop measurements of Wolfram's kite. My method is illustrated below.

First, draw an octagon and rotate two copies of it (both centered on the first) so that there is one aligned with each axis. Then draw a line from the center of the figure (easier to find if you draw the figure centered at the origin) that extends through the center of the space between the three octagons and make it longer than you know you'll need (to find the center, draw two lines from corner to opposite corner as shown below; their intersection marks the center). The kite will meet the octagon corners as shown and will meet the line at a point that lies on the same plane as the three other kite corners (approx. 0.9473 of the octagon radius). To find that point, first form the upper triangle of the kite, then draw a line through the triangle's center from the octagons' top vertex, making sure it is parallel to the plane of the triangle, and extend it to the center line. Then complete the kite. Draw another line to complete the trio of kites. Then rotate copies of the figure until you have completed the solid



[32] http://mathworld.wolfram.com/DeltoidalIcositetrahedron.html

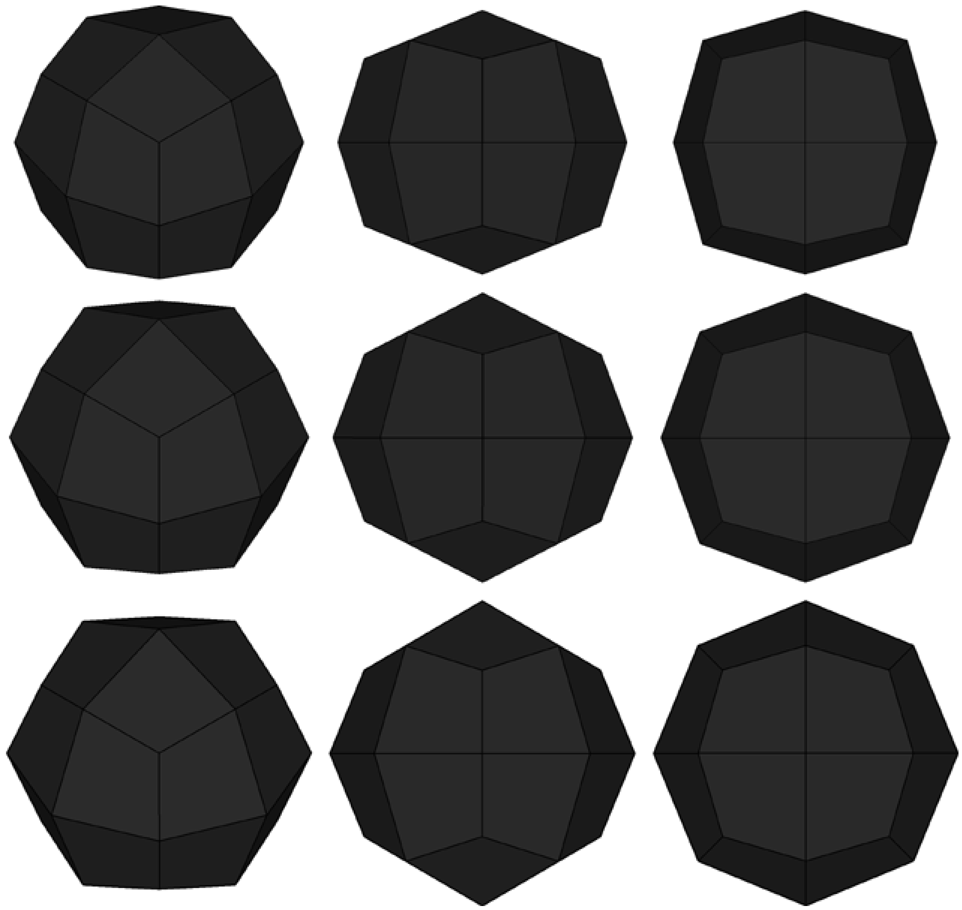




Though I have not been able to find any source that will tell me the angles of the kite face, I have found the ratio of the short and long side and the ratio of the length and width[33]. My method produces a kite shape that matches these ratios. Also, as I said, this kite matches the angles of Wolfram's kite as measured in PhotoShop.

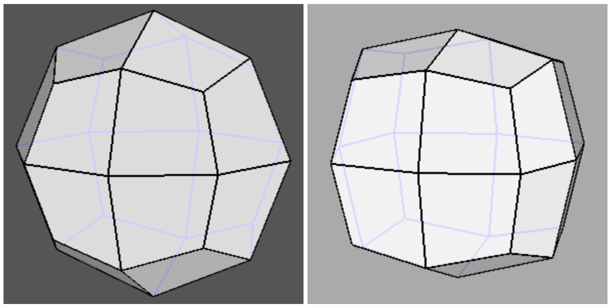
Below are three different profiles of each version of the icositetrahedron. The first two are the versions I formed using the cube (in the same order as I presented them before) and the third is the octagon version. Except in the left-most profile, which looks a little rounder in the top version, the middle and bottom versions make a much more uniform shape. The bottom shape forms a uniform octagon in the right-most profile, but has sharper vertical vertices in the middle profile.

[33] http://en.wikipedia.org/wiki/Deltoidal_icositetrahedron#Dimensions, http://mathworld.wolfram.com/DeltoidalIcositetrahedron.html, http://dmccooey.com/polyhedra/DeltoidalIcositetrahedron.html



The second version down (my chosen version) has the following vertices: 341.8° × 8, 332.9° × 6, and 325.7° × 12. The true version has 345.9° × 8 and 326.4° × 18. My vertices are slightly more uniform, but it's not a huge difference. Visually, however, the top and bottom vertices, those corresponding to the cube faces, are significantly sharper in the true version. I made a beanbag according to both designs and the true version did seem to have slightly greater bulges on those vertices when loosely filled. It was hardly noticeable, though.

Back near the beginning of my research I found a web page titled *Platonic and Catalan Polyhedra as Archetypes of Forms Belonging to the Cubic and Icosahedral Systems* that showed (among many other polyhedra) two forms of the deltoidal icositetrahedron (shown below). The one on the left with the sharper vertices is the true Catalan solid while the other is some alternate version that looks more cubic. I liked the more cubic version for a beanbag because of its blunter vertices and it influenced my work. My first version above is probably that one or very close. My second version is about halfway between the two, which is preferable because I don't want the beanbag to be too cubic.



Deltoidal icositetrahedron images from
http://www.mi.sanu.ac.rs/vismath/zefiro2009april/__cubic&icosahedral_forms_from_archetypal_Platonic&Catalan_solids.htm

In my chosen version the horizontal diameter (between the "cube edges") is 3.53% greater than the vertical diameter (between the "cube faces"). The diameter between the 3-way vertices (the "cube corners") matches the "cube face" diameter. In the third version down (the true version) the "cube face" and "cube edge" diameters match, but are 5.56% greater than the "cube corner" diameter.

Below is a comparison of the second and third version in terms of their circumference uniformity. There are three simple ways to measure the circumference of this polyhedron:

Long Side × 8

Kite Width × 6

Kite Length × 4 + Short Side × 4

I prefer the first method because it is the easiest to use in sizing the panel, so I used it as the basis of comparison. As you can see, my icositetrahedron has a more uniform circumference.

	My chosen version	**True version**
Long side circumference	1	1
Kite width circumference	0.9960	0.9799
Kite length/short side circumference	0.9851	0.9723

Based on my analysis, the cardboard models I constructed, and on the beanbags I made of each design, I think my design is slightly better than the true icositetrahedron, but the difference is so small that it is hard to decide.

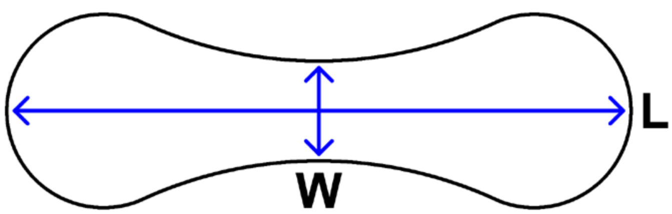
Baseball


Sources respectively: http://www.ebay.com/itm/221263945612?_trksid=p2048036, http://modified.in/footbag/viewtopic.php?t=14125, http://en.wikipedia.org/wiki/File:Baseball_%28crop%29.jpg

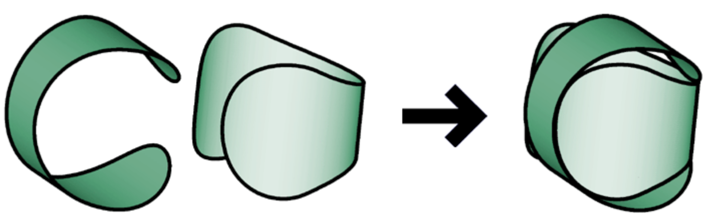
This is the panel layout used for baseballs and it was used for the original Hacky Sack by Wham-O (shown above on the left). It is also similar to the shape of the groove on tennis balls and, if you divide the panels into quarters, you will get a structure similar to that of basketballs (but basketballs use different curves and panel proportions).

Until January, 2014 (17 months after the first edition of this document) I had no interest in this design because I thought it would be greatly inferior to the other designs in terms of roundness and uniformity. Also, I had no understanding of how to design the panel shape. Then I read a PDF scan of a hand-written article from 1979 by John Diebold entitled *Designing patterns for juggling beanbags*[34] (copied in Appendix II). This article gave a technical discussion of the design with illustrations. Some important aspects of the design were missing from the article and I had to figure them out, but it was a great start and it made me interested in trying the design.

As Diebold states, "Sphericity is approximated in the two-piece beanbag by ensuring that the circumference in three orthogonal directions is the same." Diebold defines two basic dimensions of the panel shape as waist (W) and length (L). The circumference of the bag is W+L.

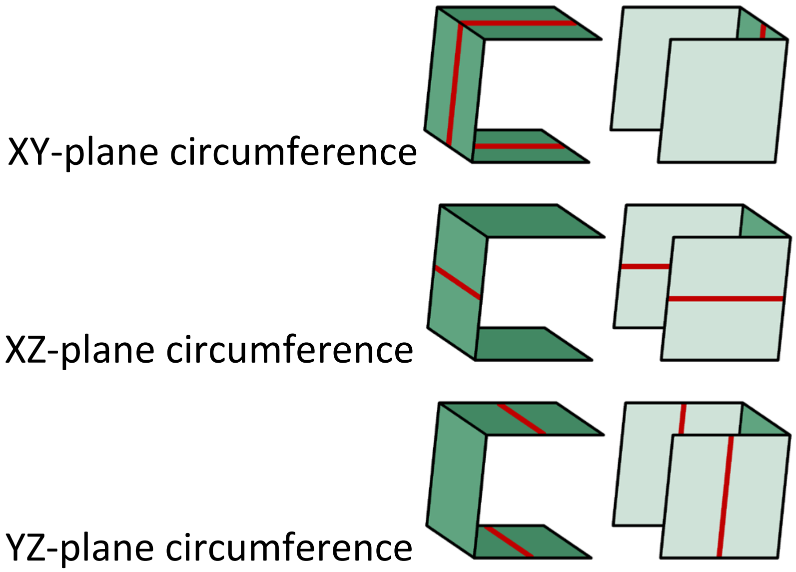


The panels fit together as shown below.

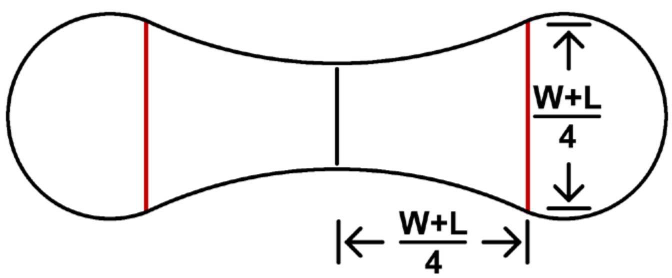


[34] http://www.ldeo.columbia.edu/~johnd/John_D_juggling_beanbags.pdf

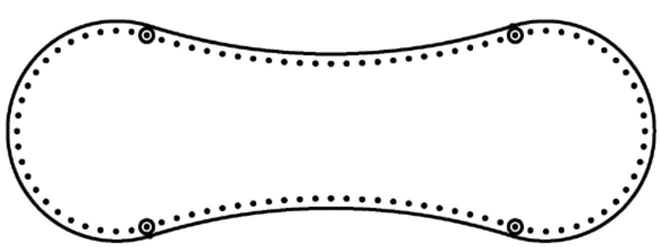
The panels can be approximated as shown below to make the three orthogonal circumferences easier to understand. The circumferences are shown by the red lines.



For the third circumference (YZ-plane), the red lines are located one quarter of the circumference distant from the middle of the panels and are one quarter of the circumference in length. I call these the “shoulders”. For the actual panel, the shoulders are located as shown below. Note that one quarter of the circumference is greater than one quarter of the panel length and so the lines are off-center within the panel halves.



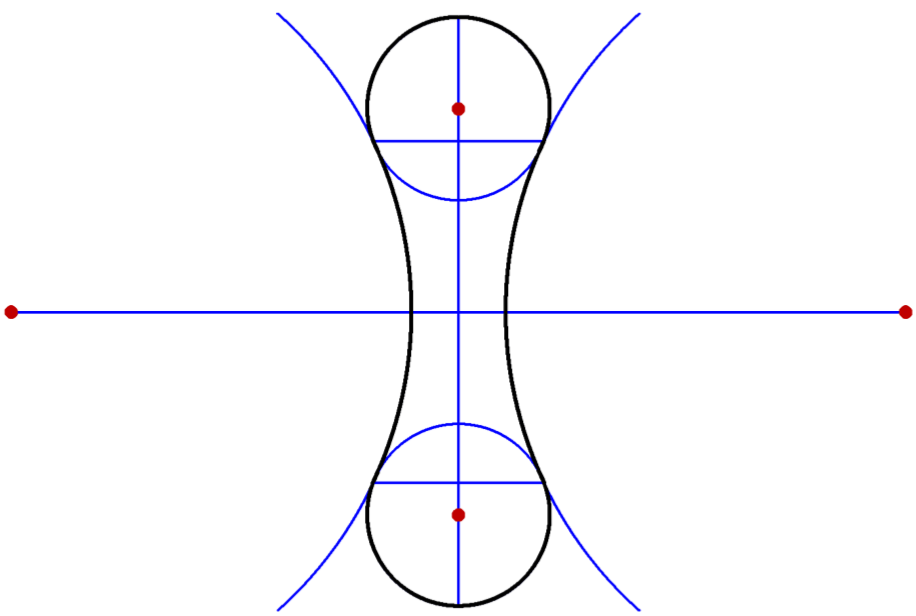
One important aspect of the panel shape is the ratio of the waist to the length. Diebold says that the best results are obtained when the length is about 6W. He gives no reason for this, however, and the example with which he immediately follows that statement has L = 6.85W. He then shows several more examples with other ratios. Peter Billam, whom I’ve mentioned before, has patterns for 2-panel beanbags[35] and they use L = 4.675W (measured in PhotoShop). Billam’s design is shown below.



[35] http://www.pjb.com.au/jug/leatherballs.html

I spent a day or two trying to decide if there is any non-arbitrary choice for the ratio. I finally realized that one missing aspect of Diebold's discussion was the length of the arcs. The convex arcs on the ends of the panels (beyond the red shoulder lines) must be the same length as the concave arcs at the waist so that when the panels are sewn together the shoulders will line up and form the correct circumference. I found (using an arc length formula[36] and trial and error) that these arcs, when circular, match only when L = 6.25W. (Actually, the waist arcs are shorter by 0.036%, but that's close enough. Using further experimentation and SketchUp measurements I narrowed the exact ratio down to between 6.23 and 6.24.) L = 6W makes the waist arcs longer by 0.84%, which is close enough for a beanbag, but not quite enough for a perfectionist, and while the mismatch in arc length is insignificant, the change in ratio will perhaps make a slightly significant difference in the look of the bag. L = 4.675W, incidentally, makes the waist arc longer by 7.46% which is difference of 7.3mm in a tennis ball-sized bag. So, the arc length consideration yields the non-arbitrary choice I needed. My illustrations use the 6.25 ratio.

A second missing aspect of the discussion is the nature of the curves. I used circular curves by default because they are simple and I had no reason yet to use any other, nor any good way to draw a different type of curve. Billam appears to have used circular curves as well. But Diebold appears to have used hand-drawn curves, most of which do not look very circular. An elliptical or function-based curve would change the arc lengths and possibly allow a different W:L ratio to work. I don't know what affect they would have on the shape of the bag, though. Below is my design framework with the circles. The red dots are the circle centers.



The circles intersect at the ends of the shoulder lines. The formula to find the circle radii (found on Wikipedia[37]) is as follows (C = chord or width of the arc, H = height of the arc from chord to apex):

$$R = \frac{C^2}{8H} + \frac{H}{2}$$

[36] $2r(\sin^{-1}(c/2r))$, where r = radius and c = chord length. I used an automatic arc calculator, though: http://www.handymath.com/cgi-bin/arc18.cgi?submit=Entry.
[37] http://en.wikipedia.org/wiki/Arc_%28geometry%29

For the waist curve the chord is the distance between the shoulders and the height equals (shoulder length - waist) / 2. For the end curves the chord is the length of the shoulders and the height is the distance from the shoulder to the end of the panel. For all the calculations, see the instructional chapter in the section on altering the bag size.

Now that I have made a 2-panel beanbag I find I actually prefer it to the 4-panel beach ball design, except that it is harder to make. It feels better in my hand and is more spherical and has a more uniform seam structure. It also looks more attractive to me. When made with two different colors it reminds me of the yin-yang symbol, particularly at the angle shown in the third photo below. Each of the photos below is a different angle of the same bag. As you can see, it's very round except near the shoulders of the panels. The squareness of the forth photo is about how the beach ball looks from the poles when made with stiff fabric. I photographed the bag the day after I made it, though, so it is not broken in. Maybe with enough use it would round out more. I also wonder if a different type of curve could eliminate the angularity at the shoulders.

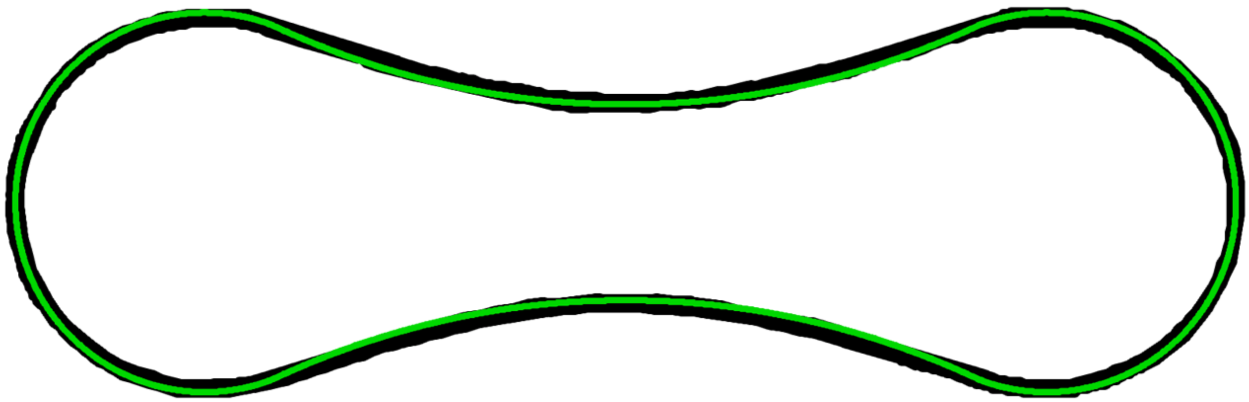
I have not yet been able to find an official definition of the baseball panel shape. Just today (2/10/2014, over a week after designing my bag) I found an article by Richard B. Thompson from 1998 entitled *Designing a Baseball Cover*[38] (which I have not fully read or analyzed) that gives an in-depth, theoretical, mathematical analysis of the baseball panel shape. Following is a quote from the first page of the article.

> In the 1860s C. H. Jackson patented a pen and ink drawing of a plane shape that could be used to form the cover of a baseball. This shape is still in use today on all major league baseballs. According to Bill Deane, a Senior Research Associate with the Baseball Hall of Fame, Mr. Jackson's design was produced by "trial and error." In practical terms, he wanted a piece of leather that could be sewn to an identical piece and then stretched to cover the yarn-wound core of a ball.

Wolfram MathWorld says of this article, "Thompson's attribution of the current design to trial and error development by C. H. Jackson in the 1860s is apparently unsubstantiated, as discovered by George Bart." [39] Anyway, after the above paragraph Thompson includes what are supposedly Jackson's sketches of the baseball panel shape for the patent. I took a magnified screenshot of one of the panels pictured, touched it up a bit in PhotoShop and then overlaid my panel shape to compare them. Following are the two shapes, with mine in green over Jackson's.

[38] http://math.arizona.edu/~rbt/baseball.PDF

[39] http://mathworld.wolfram.com/BaseballCover.html



C. H. Jackson's panel shape in black with mine overlaid in green.

My shape and Jackson's are nearly identical and have the same waist:length ratio, but the curves are slightly different. Jackson's curve looks smoother and more level across the shoulders, which is where my bag has the sharpest angularity. That is where the two different curves of my design meet and cause a somewhat abrupt change in curve. When I look at actual baseballs, this difference in the shape of the curve appears even more pronounced. The ends of the panels seem to be slightly elongated and more level across the shoulders compared to mine. The few photos and videos I've been able to find that show an unwrapped baseball panel confirm this. The waist curve is shaped slightly like a wide V and the end curves look somewhat elliptical. Below are the best depictions of this I could find.

Baseball from http://www.freshnessmag.com/2012/04/12/supreme-x-rawlings-official-league-baseball-player-preferred-glove-available-now/. Baseball panel from http://thedevilishdish.blogspot.com/2013/07/baseball-bracelet-how-to.html.

Discovery Channel's "How It's Made" episode on baseballs (https://www.youtube.com/watch?v=phCBDOXexfs). Frame captured @ 1:06.

When I learned that actual baseball panels have such a different shape from mine, I began to think that I might not have created as good a design as I thought. This was in March, 2014, well after I had done all the write-up for the design explanation and instructions and felt mostly settled in my mind about it. I wondered if the squareness of the bag was due to poor design rather than the limitations of forming a sphere out of two flat panels. I even considered demoting the baseball design into the *Other Designs and Variations* chapter until I figured out how to correct it, or at least making note of the fact that my design was not excellent but merely an inferior, approximated version of the baseball concept. This was rather disappointing.

I decided that I needed to think of a way to confirm whether or not my panel shape could fit together around a true sphere. The method I chose was to fit paper panels around a tennis ball. I measured a tennis ball I have and drew a panel that was sized exactly to that circumference. I printed and cut out two panels and taped their ends to the ball. I then carefully molded the panels to the surface of the ball, creating uniform creases around the edges. At that point, when I tried to match the edges of the panels to each other, they seemed not to match very well around the shoulders and seemed to confirm that the true baseball panel shape would fit better. But to be sure I was fitting them together correctly and not shifting the panels while pressing the edges down, I taped the edges of the panels together at small intervals around the entire seam. At the end, to my great relief, the panel edges fit together roughly perfectly with uniform snugness around the tennis ball and with the shoulders perfectly aligned. This assures me that my panel shape does form an excellent ball, allowing for the creasing or stretching distortion needed to make the panels spherical.

Going back to Richard Thompson's article, on page 58 under the heading "Real Baseballs" the author discusses measurements taken from a freshly cut leather panel of the shape used by Rawlings for their

National League baseballs, and the W:L ratio was 1:6.21 (1.196” x 7.426”). So that is further confirmation that I am at least using the official ratio.

I don’t know yet how to define the true baseball panel shape. I think it would require a function-based curve, but perhaps it would be possible to approximate it with several different circular curves joined together. Unless I find a simple way to define and draw a better curve, I’m willing to accept a possibly slightly inferior one for the sake of design simplicity.

APPENDIX I – SUPPORT FOR MY BEACH BALL DESIGN CONCEPT

In March, 2013 (6 months after writing this document) I found support for my theory for designing the panels of the beach ball. My theory consists of two parts: First, that the length and width are related to each other and to the circumference of the ball such that 2 × length = width × number of panels = ball circumference; and second, that the curve is circular (as opposed to elliptical or function-based). A blogger called "The Shishi Girl"[40] has a page[41] dedicated to making six-panel beach ball style felt balls, which she uses as Christmas tree ornaments. She calls the panels "peels" and provides a document showing some of the math involved in designing the peels. Her theory matches mine. I included her document on the following page (it is an image file) in case it gets removed from her site.

This was a significant discovery for me because prior to this I had only my own intuition and very limited experience to tell me this is how the panel should be designed, and yet I extended the theory for use in the spherical cube and spherical octahedron panels (see the *How I Developed My Designs* chapter), and assumed that it would apply to any number of panels even though I had only constructed the four-panel ball (it wasn't until July, 2014 that I got around to making an eight-panel beach ball). The first part of the theory (the dimensions of the panel) is obvious and I rarely felt any doubt about it. The second part, however, was unproven and that made me feel insecure, especially when assuming that it would apply to other panel multiples. It felt right somehow, but I didn't really understand it and nobody (until now) that I could find on the internet discussed this design in any depth or said that the curve should be circular. Some people say to draw the curve by hand, some provide PDF patterns that are obviously hand-drawn, and some provide computer-drawn patterns, but even they do not state that they used circular curves or provide any of the math behind the curve. Some of those patterns appear to use non-circular curves and I later found alternate theories for the curve design (I added a couple of examples to the end of the beach ball section of the *How I Developed My Designs* chapter). Even though I still do not have mathematical proof of the theory, it's encouraging to have some outside support for it.

The Shishi Girl does not provide a method of calculating the curve radius, but only states that the curve is circular. Fortunately, one her readers left a comment in which she provided her own formula. Her formula, $r = (w^2 + l^2) \div 4w$, seems to be correct; I tested it by using it to draw the panel shapes for the two through eight-panel configurations. Before I knew this formula I used a tedious, trial-and-error method. I first drew a stick skeleton of the panel (two perpendicular lines that crossed at their centers) whose length was half the desired circumference and whose width was equal to the circumference divided by the number of panels. I then found through trial and error a circle radius that produced an arc that met the top, middle, and bottom points of one side of this skeleton. I'm proud to say I managed to arrive at the correct width:radius ratio using this method for 4, 6, and 8 panel configurations (1:1.25, 1:2.5, 1:4.25), and for the 4-panel ball I found it using only paper and manual tools.

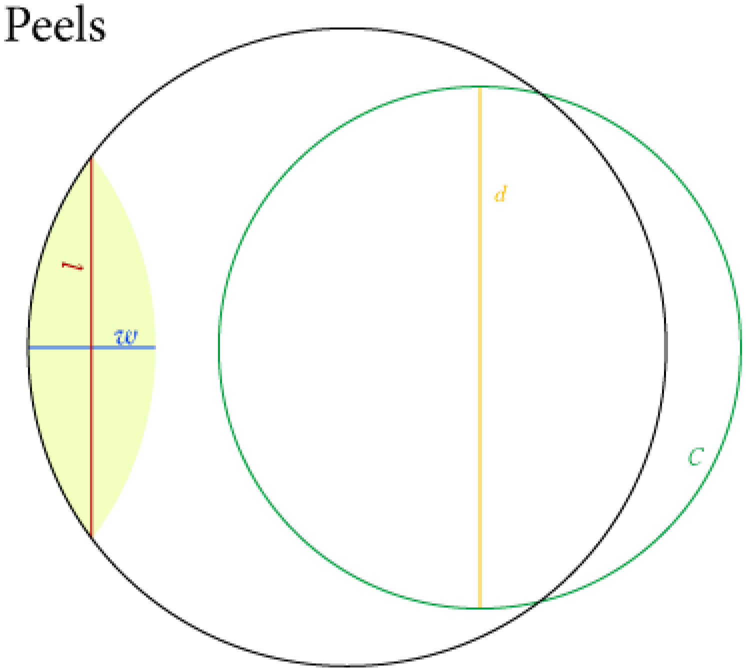
[40] http://shishigirl.blogspot.com/

[41] http://shishigirl.blogspot.com/2008/12/basic-geometry-ii.html

Construction of a Ball From Flat Peels

the mathematics of a correct sphere

To build a ball from "peels" you must make pieces such that the width (w) multiplied by the number of peels (p) is equivalent to the circumference (C) of the ball; also, the length of each peel (l) must be equivalent to $0.5(C)$.



For a six-peel ball, the equations are:
$w = C/6$ and $l = 0.5(C)$

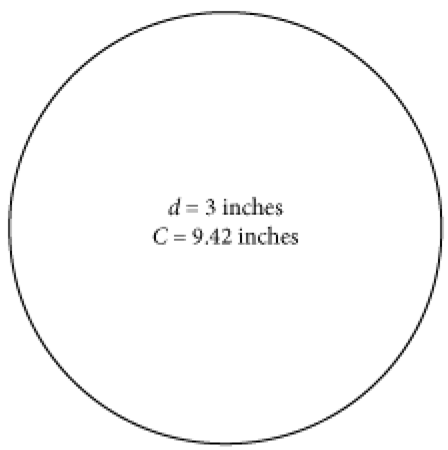
Additionally, the arc of the peel's edge must be equivalent to the natural arc of a circle.

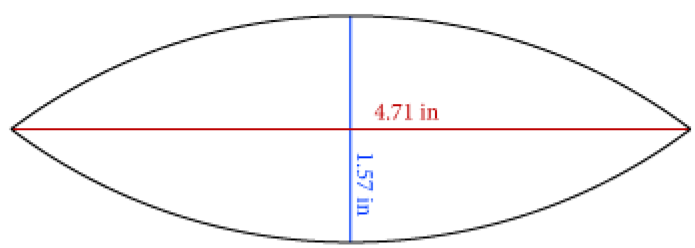
For a ball of diameter 3 inches:

$w = [(3)(3.14)]/6$
$w = (9.42)/6$
$w = 1.57$ inches

and

$l = 0.5(9.42)$
$l = 4.71$ inches





the shishi girl

3-inch diameter circle

"peel" pattern for a 6-piece ball with a diameter of 3 inches

softie tutorial pictures and additional help can be found at shishigirl.blogspot.com

APPENDIX II – JOHN DIEBOLD'S 2-PANEL DESIGN DISCUSSION

Source: http://www.ldeo.columbia.edu/~johnd/John_D_juggling_beanbags.pdf

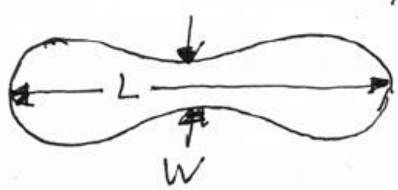
Designing patterns for juggling beanbags

24/12/79

by John Diebold

Sphericity is approximated in the two-piece beanbag by ensuring that the circumference in three orthogonal directions is the same. This type of beanbag is made of two identical pieces, like a baseball.

We define the two basic dimensions as W and L, for waist (width) and length, respectively:

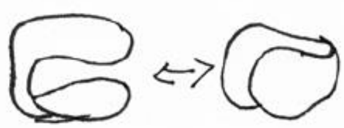


The circumference should be L+W.

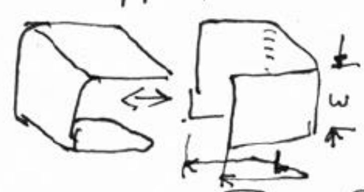
The usual diameters and circumferences are:

DIA	L+W
2.25"	7.07"
2.38"	7.48"
2.50"	7.85"

These pieces fit together so:



and can be approximated by:



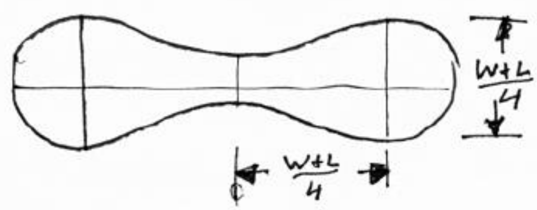
The similarity of the pieces ensures that two directions, shown by dotted lines, are equal in circumference

The other dimensions which must add up to L+W are, obviously, these:

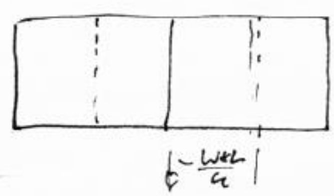
and these lines should fall near the midpoint of the quadrant's perimeter.

24/12/79 ②

in The pattern, The location of These lines is determined Thus:

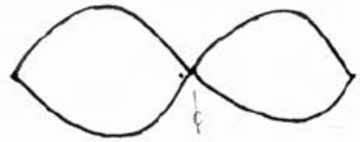


shapes can be quite variable. one extreme is when L=3W, The cube.

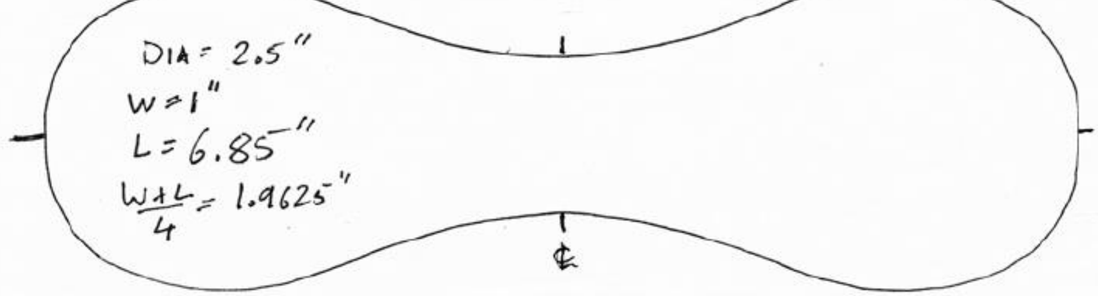


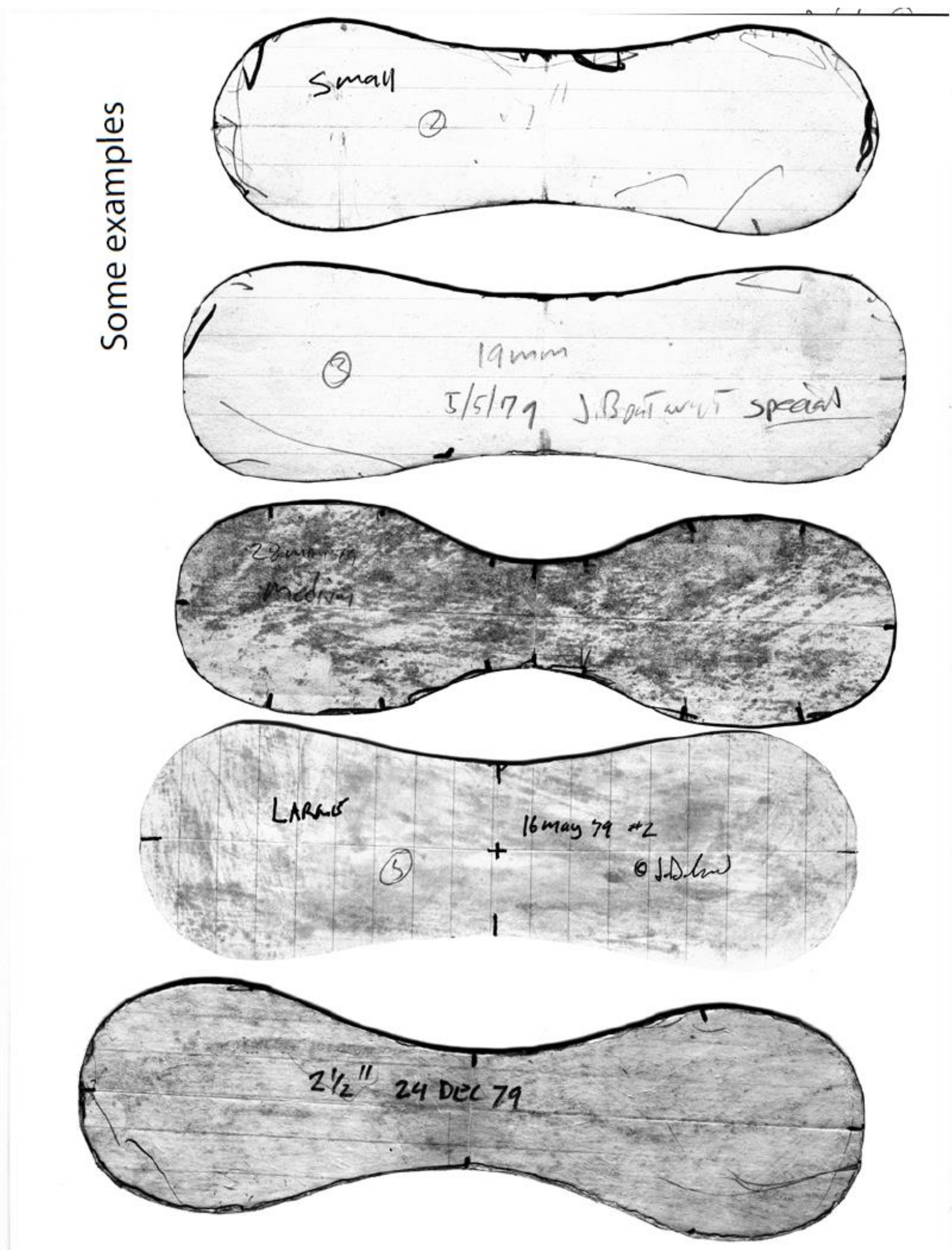
Another is L = ∞W, or W = 0, The "orange peel"

max accuracy ⇒ max. segments
See: mandolins + Cameron Powers' Lady guitars



each of These produces squarish edges. The best results (for 2 pieces) are obtained with a ratio of about L=6W

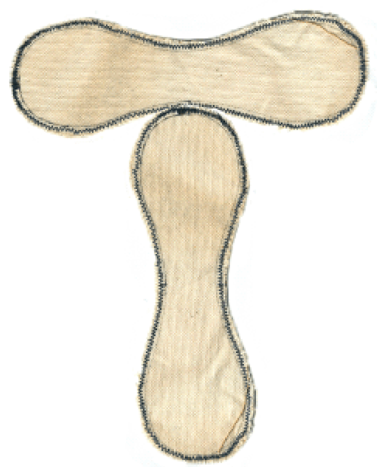



Some examples
Small
19mm
LARGE
16 may 79 #2
2½" 24 DEC 79

how to make the juggling beanbag

Trace the pattern six times onto heavy canvas duck or other suitable fabric. Remember to make four marks where the two centerlines intersect the sewing line. Cut the pieces out, leaving about 1/4" of fabric outside the outlines. use a sewing machine and run a zigzag stitch around the 1/4" extra strip, to prevent future fraying:

For each ball, put the two halves together like this, with centerline marks aligned, and start sewing along the outlines. using a large strong crewel needle and good heavy button thread or light twine. Make sure the centerline marks line up as the sewing progresses:



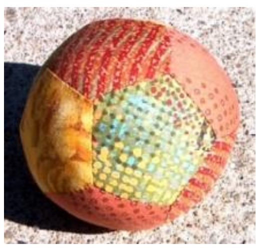
Continue sewing until there is a remaining gap of about 3/4". Leave the threaded needle hanging, and use a dowel, ballpoint pen or other tool to turn the nearly-finished ball right side out. Using a funnel, fill the ball with something - bran works well, rice is possible. Then using the "invisible stitch", finish sewing the ball. If a somewhat floppy ball is desired, you're done. If you want to create a taut ball, go over the seam again, from the outside, using the baseball stitch. This will cinch up any looseness - this added step is the key to making the once-famous Diebold juggling beanbag. Sew two more balls, and juggle.

APPENDIX III – FABRIC BALL PROJECT IDEAS

There are many ideas for fabric ball projects on the web. Following are some of the best ideas and patterns I have found. Most or all of these can be found by searching Google Images for "fabric ball". These balls are stuffed with soft stuffing, not pellets. Some of these balls use types of polyhedra I do not provide definitions for in this document, but they can be found on the internet.

Child-safe Christmas ornaments by "The Shishi Girl" from http://shishigirl.blogspot.com/2008/12/basic-geometry-ii.html. This is a six-panel beach ball design made with felt. I provide instructions for a four-panel beach ball and an explanation for using other panel multiples (See the *4-Panel Beach Ball Style Instructions* chapter and the *Other Designs and Variations* chapter for more information).

Icosahedron



Dodecahedron

Truncated Octahedron

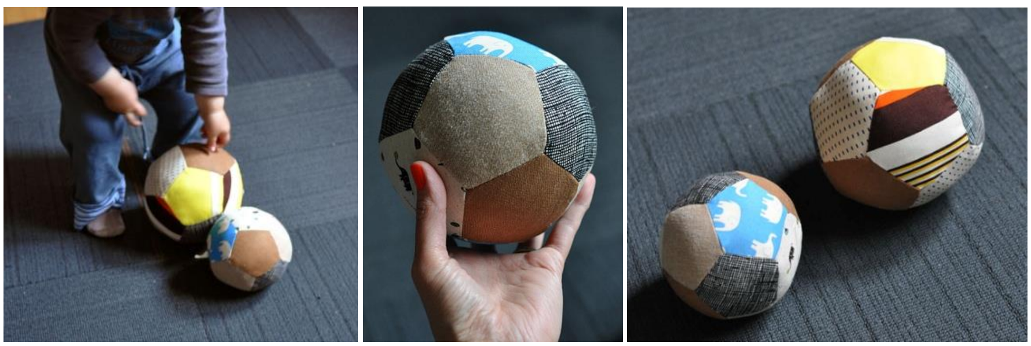
Icosidodecahedron

Rhombicuboctahedron

Rhombic triacontahedron

Balls inspired by the book *Patchwork Puzzle Balls* by Jinny Beyer (images from http://www.personal.psu.edu/axd2/quilt/qlt36.html). These balls are all three to five inches in diameter and I assume they are intended to be used as children's toys or perhaps as decorative centerpieces. I define the dodecahedron and icosidodecahedron in this document, but not the others.

Large, corduroy toy balls by Crafty Panties from http://www.craftster.org/forum/index.php?topic=269264.0 (patterns from *Patchwork Puzzle Balls* by Jinny Beyer). These are Icosidodecahedrons, which I give some information about in the 32-Panel chapter.



Random fabric scrap balls by Jennifer Murphy from http://andothersillythings.blogspot.com/2011/05/pentagon-colorful-fabric-balls-tutorial.html. These are dodecahedrons, which I define in this document.

Activity Ball for babies by ElmaRi from http://www.lovilee.co.za/2012/diy-activity-ball/.






Here are some very unusual polyhedron or geometry-based fabric ball designs.

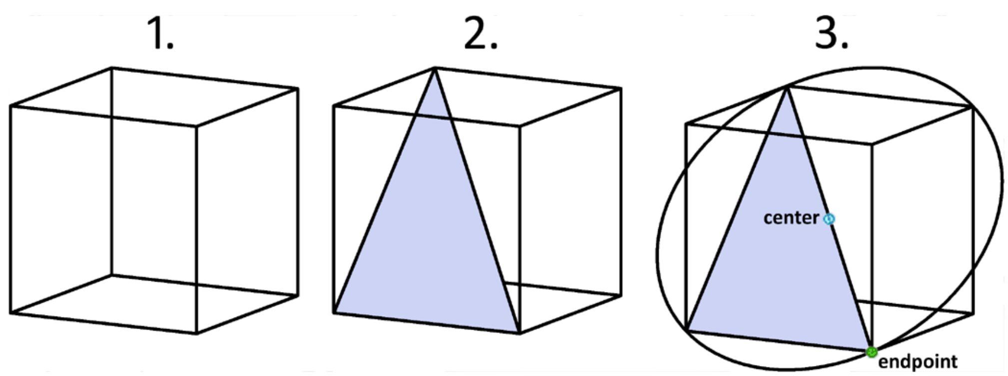
APPENDIX IV – HOW I DRAW SPHERICAL POLYHEDRONS

I had to figure out for myself how to draw spherical polyhedrons because I could find no information on the web for doing this. I decided to include instructions here for anyone else who wants to draw them. The beach ball and spherical octahedron are composed simply of circles. To draw them, draw a circle, rotate a copy of it to create the framework of the polyhedron, and then position its center at the center of a sphere of the same size. (Finding the center of the sphere is easier if you draw it at the origin.)

To create spherical versions of polyhedrons lacking circumscribing edges, follow the directions below. These types of polyhedrons cannot be painted on a face-by-face basis in SketchUp (at least I don't know how) because the "circular" edges of the polyhedron, being in fact composed of many small facets, do not precisely match the faceted surface of the sphere and so the entire surface of the sphere is considered one face and will always be a single color. I color these using PhotoShop by taking screenshots of the sphere with each color and then cutting out (using the Magic Wand Tool) the panels I want of each color and positioning them together. I also use PhotoShop to make the edges thicker by copying just the black edges into a new layer and using the Stroke attribute of the layer to thicken them.

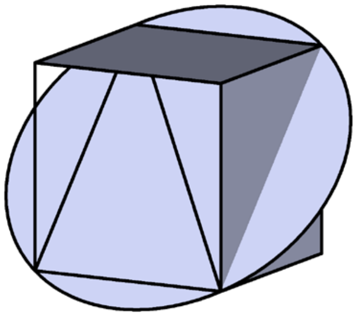
For any spherical polyhedron, I recommend using circles with a large number of sides so you get a good curve, but if you use too many sides, the sphere will take a long time to generate unless you have a very fast CPU. I use 90 (360/4).

(I was not able to use SketchUp to draw the baseball design. This is currently too advanced a design for me to draw in 3D. I used a combination of my beanbag photo, MS Paint to overlay curves onto the bag's seams, SketchUp to obtain 3D shaded, colored spheres for the fill color, and PhotoShop to bring it all together and pretty it up.)



1. Draw the polyhedron (instructions for drawing common polyhedrons can be found online). (Make sure it is fairly large because later you will have to draw a matching sphere and SketchUp has (or at least had) trouble rendering very small spheres when I set the circle's side count very high, leaving holes at each pole.) Erase the face color, leaving just the edges.
2. Draw a line from one corner to the diagonal opposite corner through the center of the solid and then draw a second line to complete a triangle. This temporary face is just for aligning the Circle Tool.
3. Select the Circle Tool, align it with the triangular face, and draw a circle from the center of the diagonal line (which is the center of the solid) to any of the four corners on the same plane with the circle.

4.



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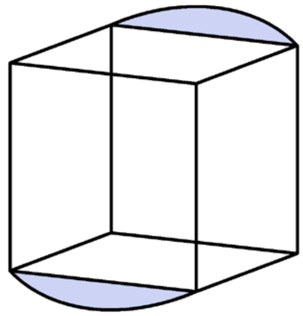
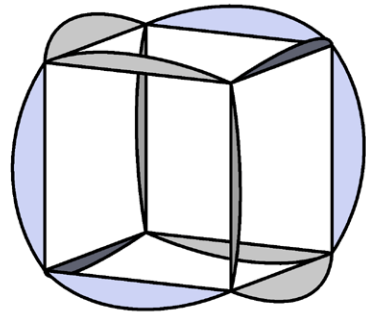
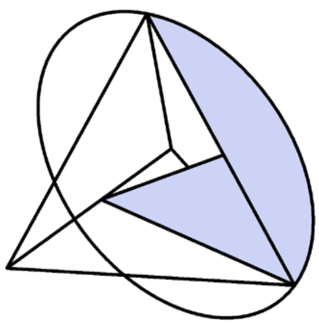


6.

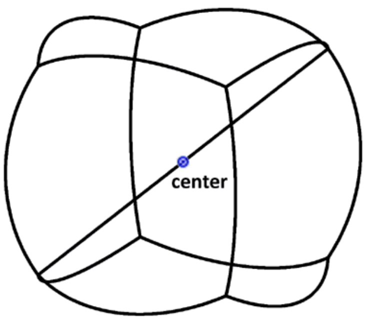


4. When you draw this circle you will get several unintended faces as shown in figure 4.
5. Erase the superfluous faces, excess circle portions, and the temporary triangle, resulting in figure 5.
6. Repeat these steps for each pair of opposing edges until you have something like figure 6.

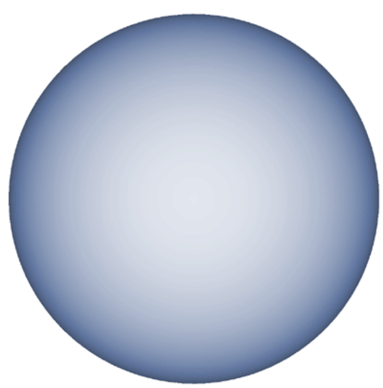


For an asymmetrical polyhedron such as the tetrahedron, draw the first line from edge center to opposite edge center as shown on the left and then complete the triangle to form a face. In this case you will create only one edge arc instead of two and the rest of the circle will have to be erased.

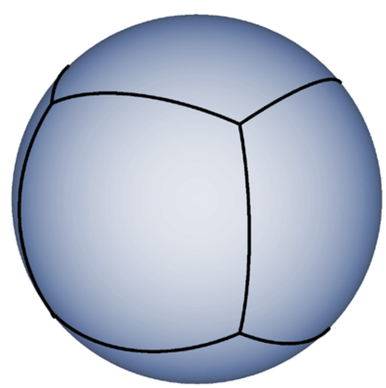
7.



8.



9.



7. Draw a line through the center of the solid as in Step 2 (or leave the last one you drew) so that you can find the center of the solid. You may, but it is not necessary, erase the straight edges of the original polyhedron leaving only the spherical version behind as shown in figure 7. Draw the line before erasing the original polyhedron because otherwise it will be difficult or even impossible to find the exact corners or edge centers.

8. Draw a sphere at the origin that has the same radius as the spherical polyhedron (half the length of the line from the previous step). There are instructions online for drawing spheres.

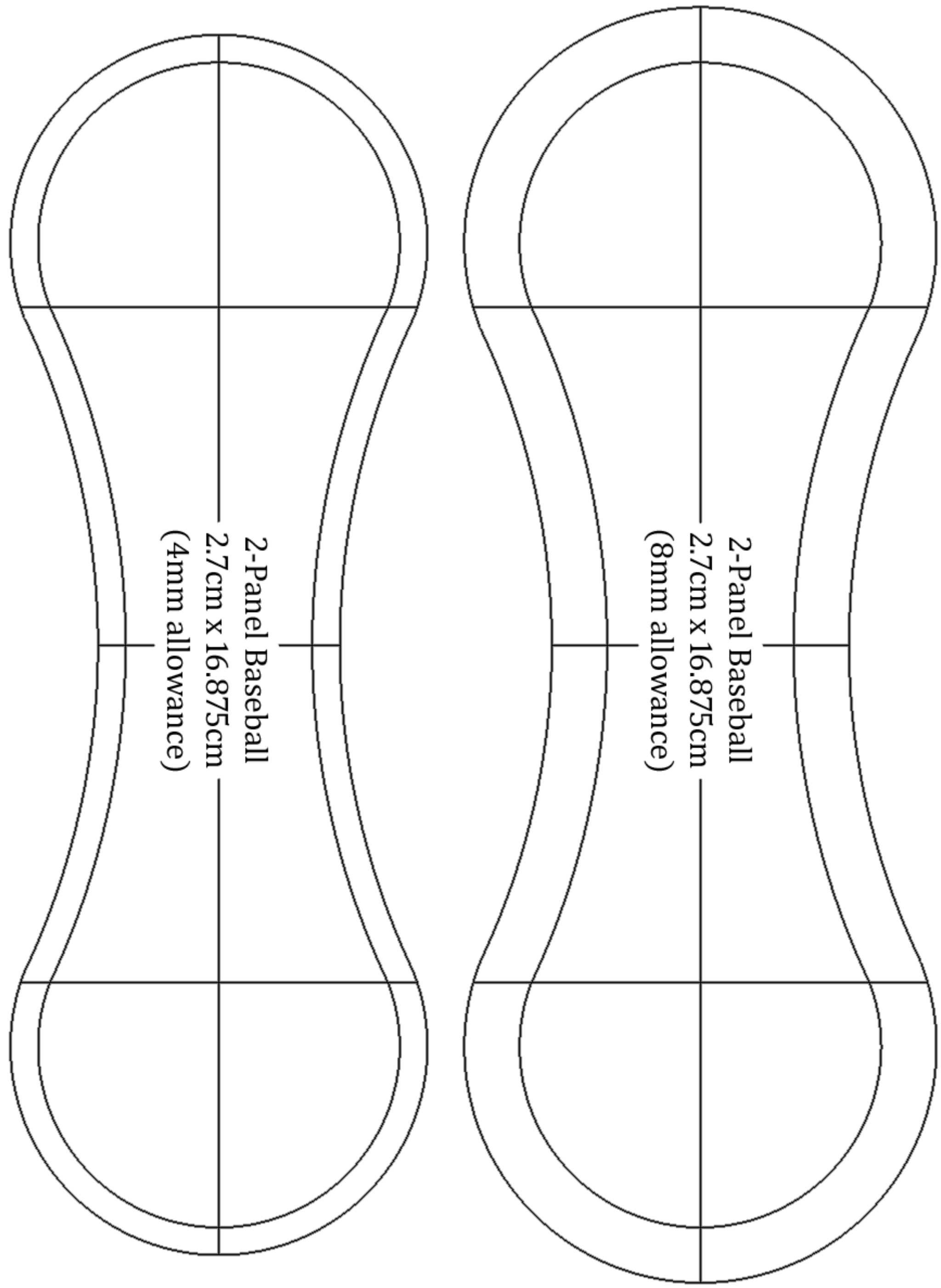
9. Position the center of your spherical polyhedron at the origin so it is exactly centered within the sphere. You'll have to hide the sphere's faces first so SketchUp will allow you to position the polyhedron at the origin rather than on the surface of the sphere (**View** menu -> **Face Style** -> **Wireframe**). This should result in something like figure 9. If you have drawn everything correctly and positioned the polyhedron framework in the exact center of the sphere and yet the lines of the polyhedron are partially hidden within the sphere, this is probably because the sphere is a tiny bit larger than the polyhedron due to rounding. In this case make the sphere slightly smaller.

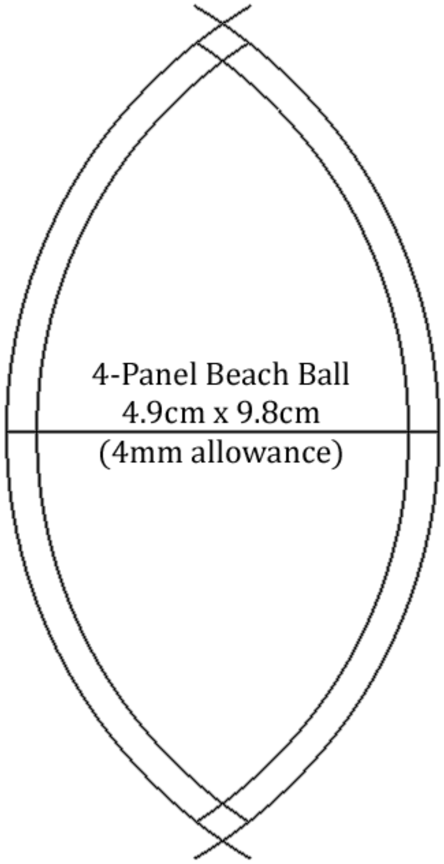
APPENDIX V – READY-TO-PRINT PATTERNS

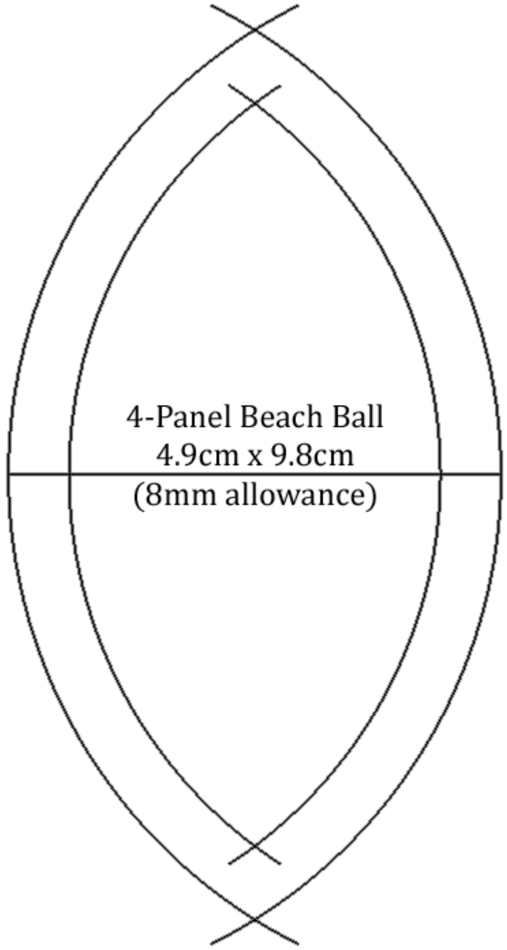
For the convenience of readers who are in a hurry or who don't have the necessary tools I have included all the patterns from the "How to draw the panel shape for a tennis ball sized bag" sections of the instructional chapters, plus the pocket-sized, 2 inch diameter octahedral pattern I mention in a few places (the one with a 6cm guide triangle side length). Most of these patterns include both a 4mm and an 8mm seam allowance, and I set up the patterns having two shapes in such a way that you can make a dual template (one having both shapes side-by-side). Except for the pocket-sized octahedron, these patterns produce tennis ball sized bags ($2^5/_8$" or 67mm diameter) when firmly filled and made with my denim and manufacturing practices. When printing, be sure to enter the desired pattern's page number in the Print Dialog so you don't print the entire document

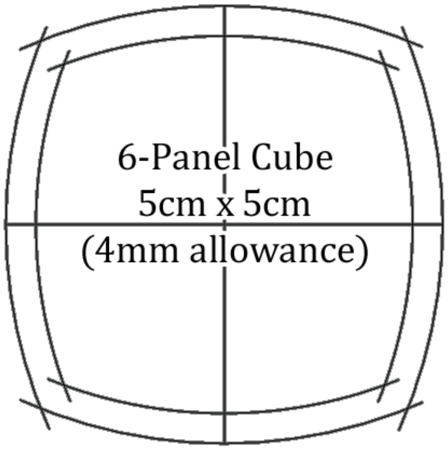
If you're viewing this as a PDF with Adobe Reader, you can scale the print to create patterns for other bag sizes. (Microsoft Word does not have a percentage scaling option, but you can use it to convert the document to PDF by using the **Save As** dialog and selecting PDF as the file format.) In Adobe Reader, click **File** -> **Print**, then in the Print dialog under *Page Sizing & Handling* click the **Poster** button. Beneath that will appear a box labeled *Tile Scale*. Enter your scaling percent in that. For example, if you want a 2-inch bag, divide 2 by $2^5/_8$ which yields 0.7619 or 76.19%. Print the pattern at that percentage and the resulting bag should be about 2 inches in diameter. Scaling the patterns will also change the seam allowance, so you may want to compensate for that when you cut out the patterns.

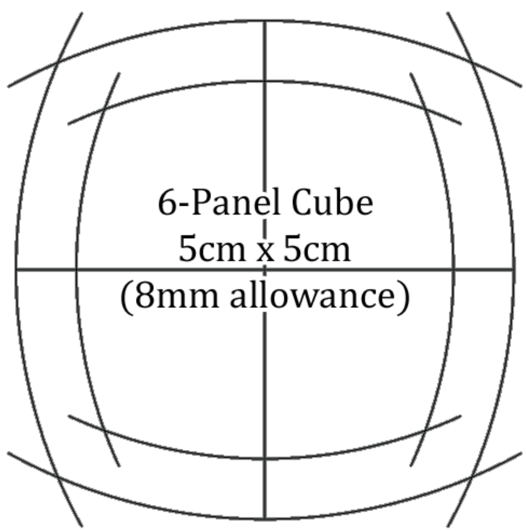
For patterns with curved sides meeting at vertices I include excess curve at each vertex to help you guide your scissors into the curve.


2-Panel Baseball
2.7cm x 16.875cm
(4mm allowance)
2-Panel Baseball
2.7cm x 16.875cm
(8mm allowance)

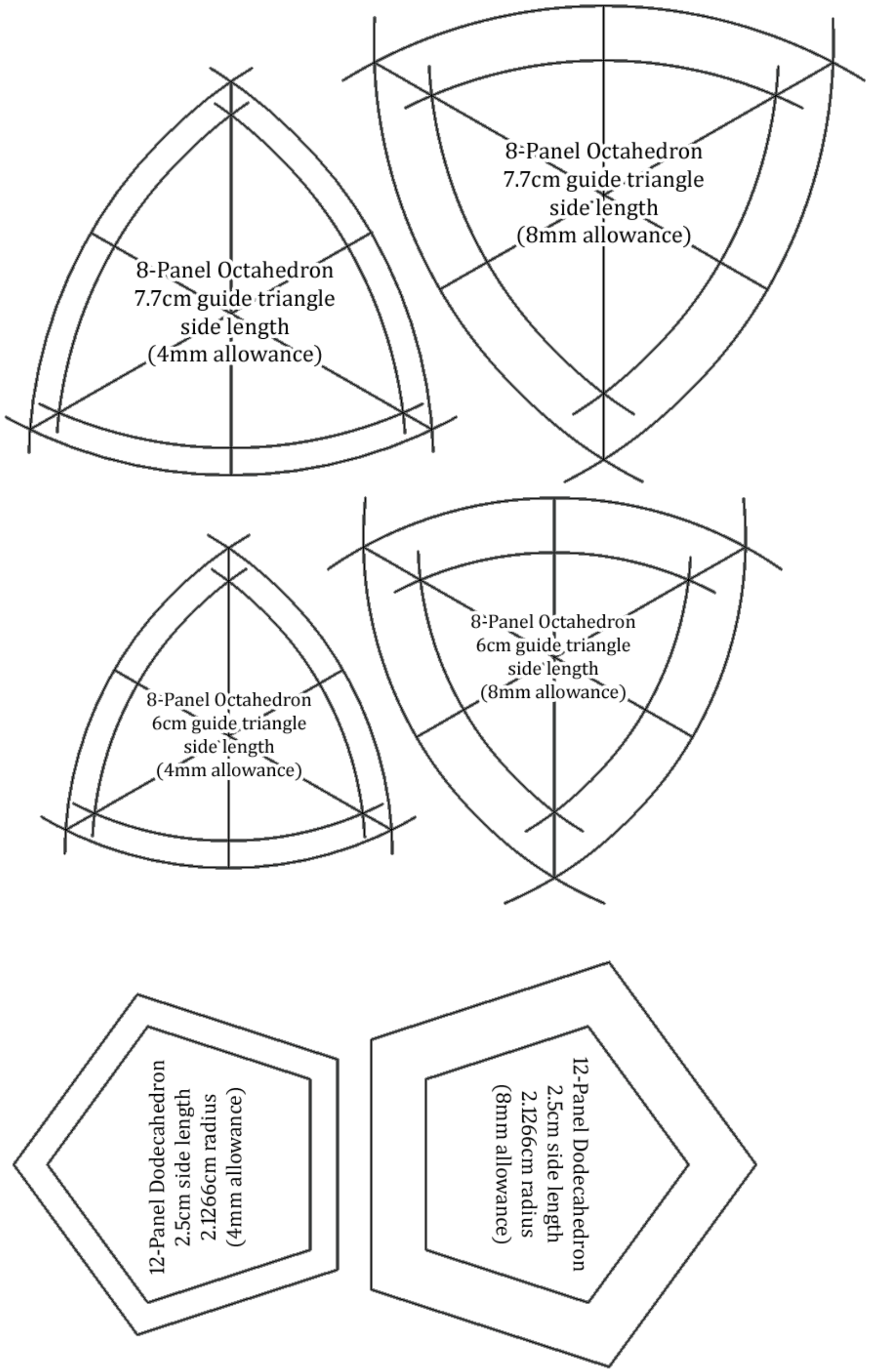

4-Panel Beach Ball
4.9cm x 9.8cm
(4mm allowance)


4-Panel Beach Ball
4.9cm x 9.8cm
(8mm allowance)

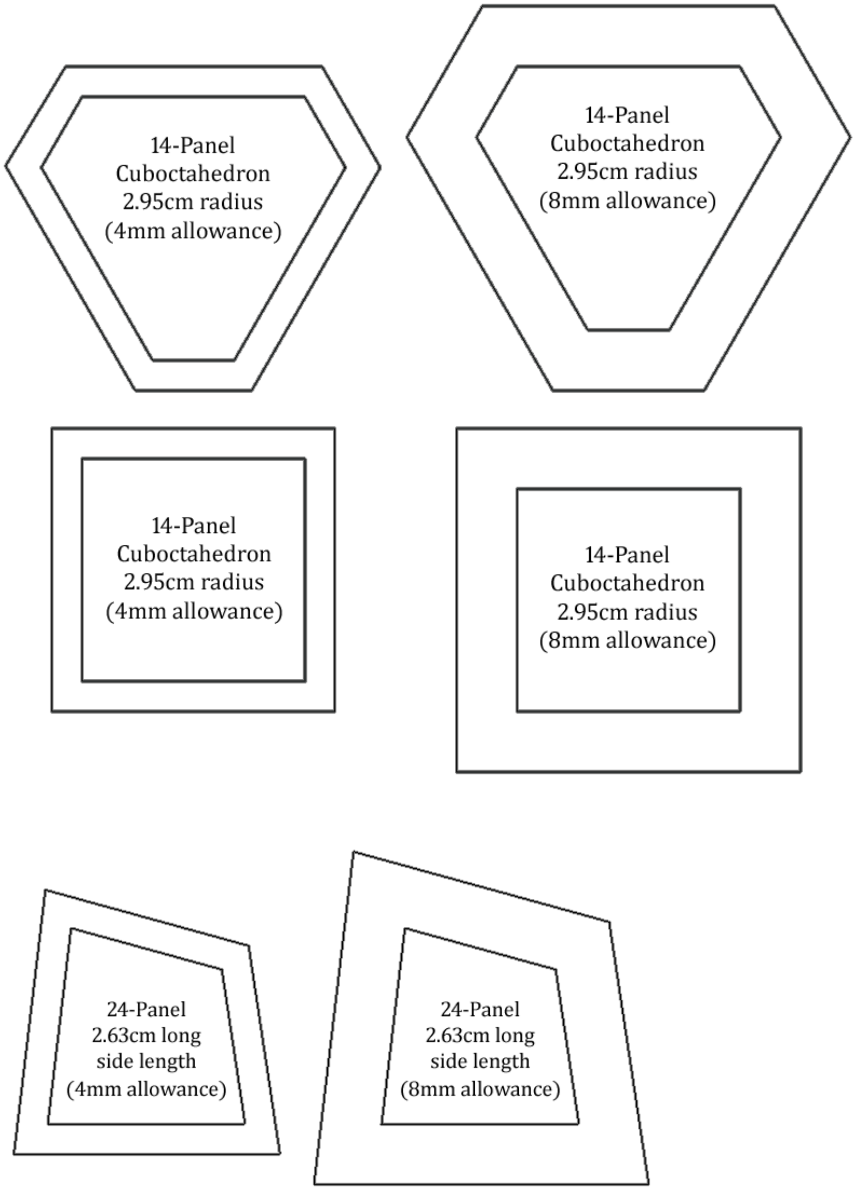

6-Panel Cube
5cm x 5cm
(4mm allowance)

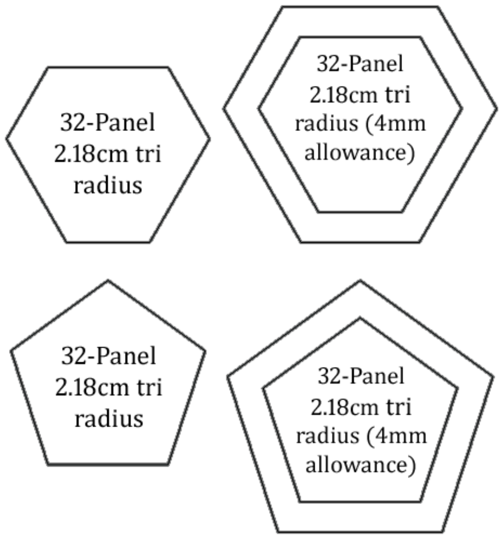

6-Panel Cube
5cm x 5cm
(8mm allowance)

Appendix V – Ready-To-Print Patterns



Appendix V – Ready-To-Print Patterns




32-Panel
2.18cm tri
radius
32-Panel
2.18cm tri
radius (4mm
allowance)
32-Panel
2.18cm tri
radius
32-Panel
2.18cm tri
radius (4mm
allowance)