The Complete Homemade Juggling Beanbag Guide

32-Panel Equidistant Trunc Icosahedron (and variations) Chapter

By Joshua Clifton
Contact me on Reddit under Joshuacliftojm
or at juggle@joshuaclifton.com

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This is part of a multi-document guide. Hyperlinks with the icon[1] open other guide documents[2], if they are saved to the same folder (**CTRL+Click** opens them in a new tab).

Visit my website to download those, and check back occasionally for revisions and corrections:

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Compare the Last Edited date above on the right with the one on the web page to see if I have submitted changes.

Please contact me with your thoughts! Feedback on this project would be helpful and encouraging. You may also request custom patterns or other help with your project.

If this guide is useful to you, please **consider donating at my website** linked on the left. I am not monetizing the guide, and I am in need of income.

My website also provides blank **color arrangement diagrams** for experimenting with new arrangements in an image editor.

1 Icon from https://freesvg.org/vector-illustration-of-external-link-icon

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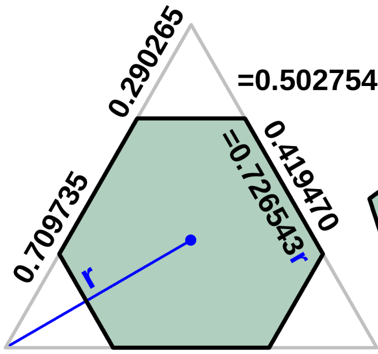
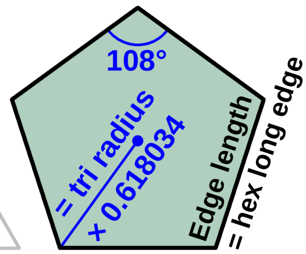
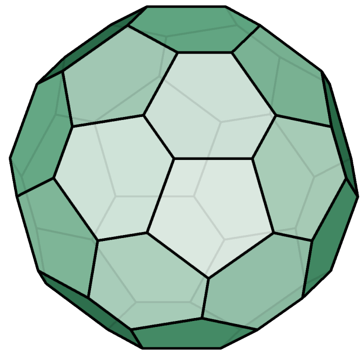
32-PANEL EQUIDISTANT TRUNCATED ICOSAHEDRON (AND VARIATIONS) INSTRUCTIONS







“Tesselation of Shuriken” arrangement (4 colors)

“Four-Color Patchwork Pents with Cube Hexes” arrangement (6 colors)

My original denim bag (“Cube Hexes/Blur” arrangement)

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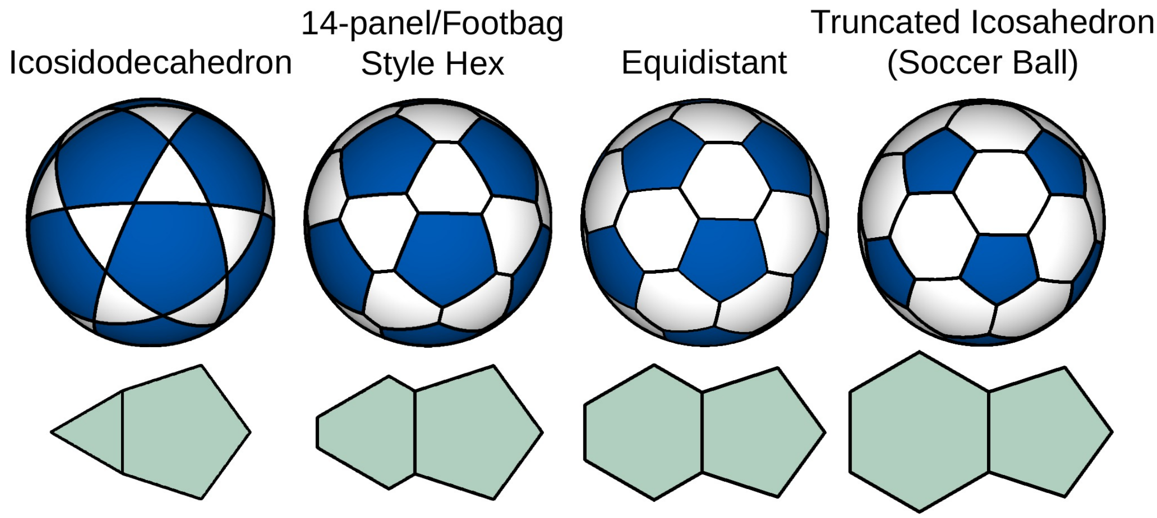
Design Notes

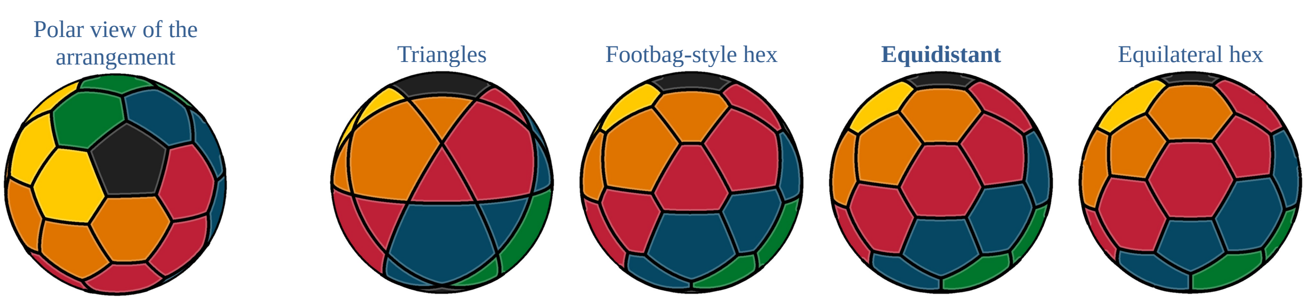


The four 32-panel designs in this chapter. I provide patterns and mathematical definitions of each.

This design is composed of 12 pentagons and 20 hexagons (or triangles if you choose that version). I provide formulas and patterns for four 32-panel variations (shown above). The first three each has advantages in how it supports color arrangements (and the second has the hex shape commonly used for footbags). The fourth is just here for completeness. **The patterns for all variations are in the Ready-to-Print Patterns section, and there are sections on how to size and draw the patterns for each design. The color arrangement diagrams are in a separate document titled "32-Panel Color Arrangements".**

The four variations form a progression from an icosidodecahedron to a truncated icosahedron, both of which are Archimedean solids, meaning (among other things) that they are composed of equilateral polygonal faces. My modified hex designs form two transition points at roughly ⅓ intervals between the two Archimedean solids and have hexes that are only semiregular.

My main, Equidistant hex shape accomplishes two things. First, but not importantly, it makes the solid equidistant, meaning that both face shapes are the same distance from each other, giving the solid a **more constant diameter**. Second, it **makes the two face shapes approximately the same size**. This is important because it improves the look of color arrangements that do not distinguish between them such as the “Five Swirls” arrangement below. Because the swirls are composed of both pents and hexes (or triangles), they have a more uniform width and better appearance when the two panel shapes are the same size. The smaller, footbag-style hexes and the larger, equilateral hexes both produce poorer swirls, and the triangles do not really work at all.



The swirl arrangements, among others, look better with the Equidistant hex that is the same size as the pent.

For an explanation of how I designed the Equidistant Truncated Icosahedron, read the "How I Developed This Design" section.

The 14-Panel/Footbag Hex variant uses the typical footbag hex shape and works better for color arrangements that form **creature and character faces**, or that **emphasize the pentagons and use the hexagons as a background** (see the 32-Panel Color Arrangements document for examples). The hex shape came from my 14-panel design. The short edge is 0.366 of the long edge instead of the 0.692 proportion of my Equidistant hex.

The icosidodecahedron, which has triangles instead of hexagons, **allows for very different-looking color arrangements than the hex variants and is very versatile**. My color arrangements document has many examples of the designs that can be created. **The true, Archimedean truncated icosahedron has no particular advantage** that I know of except that it makes a **correct-looking soccer ball**.

This is a rather tedious design to make, but it is beautiful and fun to show off. I use a 4mm seam allowance (but I cut the panels generously, adding about 1mm). The denim I used for my first one was rather frizzy to work with and not ideal for this design with so many crowded panels. The corduroy was much better, but a thin, non-fraying fabric like Ultrasuede would probably be best.

If you are particular about the size of the beanbag, remember to be precise in making the templates and in sewing along the lines, and on the correct side of the lines, where the edges of the template were (inside the lines for exterior templates, outside them for stencil templates). **The large number of panels means a tiny change in size in the template or stitching path amounts to a large difference in beanbag circumference. For footbags with gathered seams, I suggest trying two sizes (½″) or 25% larger than your target size.**

Supplies

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- **For the templates**
  - Cardboard or Template Plastic, X-Acto Knife or Scissors, Glue Stick or Adhesive Tape (to attach the patterns to the template material). **For drawing the patterns by hand:** Paper, Protractor, metric Ruler, fine-point Pencil.
- **For the beanbag**
  - Fabric, Needle and durable Thread, Scissors, Fabric Marker or soft Pencil, beanbag Filler, Funnel.
- **For your information**
  - Unless you are experienced with this sort of thing, I recommend that you browse through the **General Information and Techniques** chapter (in the ***01 – Homemade Juggling Beanbag Guide – Index & Supplementary Chapters*** document) before starting. You may find some tips there that will improve your experience and your beanbags.

Printing and Drawing the Patterns, and Color Arrangement Ideas

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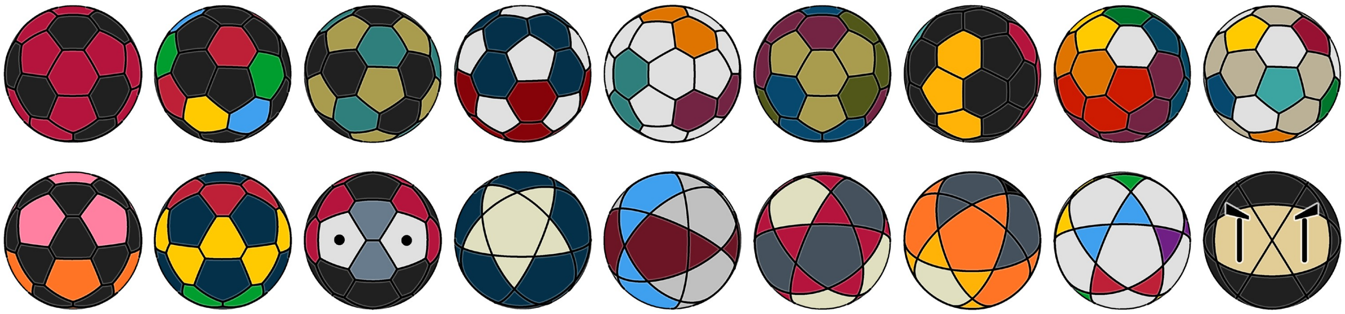
Later in this chapter I provide ready-to-print patterns for all of the design variations. (When printing them, be sure to tell the Print Dialog to print only the page(s) you want so you don't print the entire document.) For footbags with gathered seams, I suggest trying two sizes (½″) or 25% larger than your target size. Each design variation also has its own section with pattern sizing formulas (and the mathematics behind them), pre-calculated pattern measurements, and drawing instructions.



The color arrangement diagrams for the 32-panel designs are in a separate document titled "32-Panel Color Arrangements". Above are examples of the ball illustrations. In the next subsection, "Cube Hexes (aka "Blur") arrangement explanation", I explain the relationship between the 32-panel structure and the cube and octahedron, and how that relationship can be used to create color arrangements.

The 32-panel structure allows for **great freedom of creativity in arranging colors**. The color arrangement document has over 80 arrangements that I have found or invented so far (including ball and assembly layout diagrams), and photos of footbags made by other people.

I provide printable blank color arrangement diagrams for the ball views and the assembly layouts for all variations of this design except the truncated icosahedron (soccer ball). You can use those to experiment with color arrangements without having to make a beanbag or build a 3D model.

I used the 14-panel hex variation of the 32-panel design to create a "Turtle Ball". I wrote an essay about it at the end of this chapter, in the section titled "Turtle Ball". The essay includes detailed photos and assembly diagrams.

I had fun figuring out color arrangements and creating the illustrations. I hadn't done that for my first edition. I started with a small collection of arrangements, and then as I worked on other parts of the document I kept finding or imagining new ideas or variations and adding them.

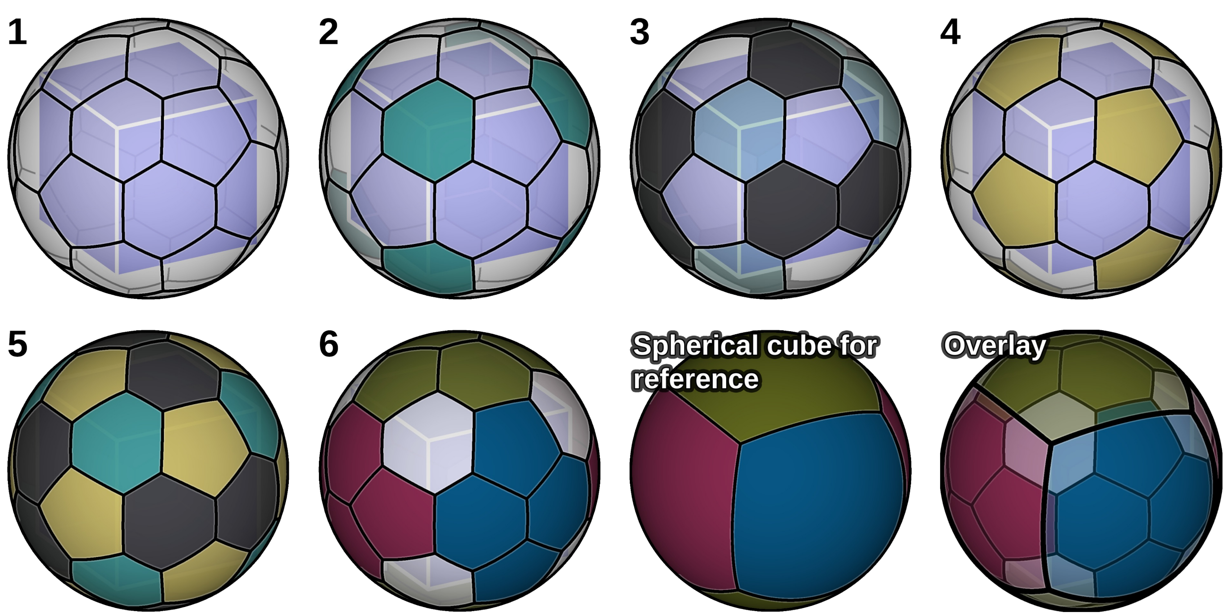
To aid me in this task, **I stuck colored thumbtacks into an all-white 32-panel bag** I made using my design-testing fabric. I recommend this as a way to design new arrangements or to use as a reference to aid you in correctly assembling the bags. You could also draw a truncated icosahedron or icosidodecahedron in a CAD program and color its faces, but I prefer the hands-on, thumbtack method.

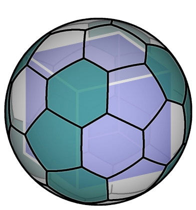
One thing to consider about this polyhedron is that there are **12 pentagonal faces**, and they are arranged in the same way as on the dodecahedron. This means that **any of the dodecahedron arrangements can be used for those faces**. Another color arrangement consideration is that **the hex panels have a correspondence to the cube and can be arranged as the six faces and eight vertices of the cube**. This is explained and illustrated in the next subsection. The eight cube vertices also correspond to the faces of an octahedron since the two polyhedra are duals of each other, and so the corresponding **hex panels can also be arranged according to the octahedron design**.

Cube Hexes (aka “Blur”) arrangement explanation

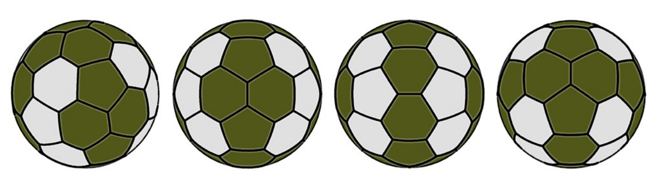
There is a correspondence between the hexagon panels of the 32-panel design and a cube’s vertices and faces. The Cube Hexes arrangement (my name for it, but called “Blur” by others) is a popular arrangement and exemplifies this correspondence. Quite a few other arrangements also relate to this concept, so, after trying and failing to explain this to my sisters, I decided to illustrate it for anyone else who doesn’t understand it or see its significance in creating color arrangements.

1. The first illustration below shows a reference cube inside a 32-panel ball, aligned so that its eight corners align with the centers of eight hex panels.
2. I color those eight hexes teal. I call this arrangement “Cube Corners”. Those hexes also correspond to the faces of the octahedron (the cube and octahedron are duals of each other), and so can be colored according to the octahedron arrangements.
3. Each face of the cube is aligned, center to center, with a pair of hexes. I color those pairs black. Each hex pair is perpendicular to those adjacent to it and parallel to the one opposite it. I call this arrangement of six hex pairs “Cube Pairs”. All hexes have now been colored.
4. I color all the pentagons a single color. Incidentally, as I explained before, they correspond to the twelve faces of the dodecahedron. If I placed a dodecahedron inside the ball, the pent panels would align with its faces. For that reason, any dodecahedron arrangement can be applied to those panels.
5. Finally, I combine the three partial arrangements to get the arrangement I call “Cube Hexes”.
6. Illustration #6 shows a related arrangement I call “Cube Patches”, but which is called "Six Eggs" by others. The eight teal hexes are now white, and each pair of black hexes is combined with the two gold pents on either side of it to form six single-color patches.





The Claw Marks arrangement is formed by connecting pairs of corner hexes by the pent panel in the middle of each pair to create four roughly parallel stripes resembling slashes made by claws.



Cutting Out the Templates

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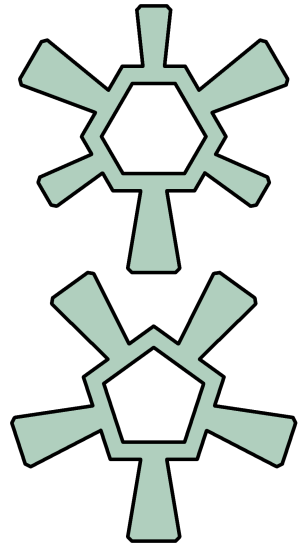

Because this design has so many panels, **I recommend making combo type templates as shown on the right,** or at least the stencil (interior) type (if you don't use cutting templates). The combo type includes the stitching template on the inside and the cutting template on the outside (actually just the corners of it because the tabs interrupt it). **Interior tracing is much faster and easier than exterior**, and for the cutting patterns you really only need the corners. My Ready-to-Print patterns are the combo type, but can be used to make any other type of template. An X-Acto knife and steel ruler are the best tools for cutting out the interiors.

If you use a thick marker to trace the patterns, **remember to stitch on the *outside* of stencil type patterns, where the edges of the template were (inside the lines for exterior templates**), so you don't change the size of the ball. If the marker soaks through the fabric you're using, however, you will need to stitch inside the patterns to **hide the lines within the seams**. In that case, cut out the templates' interiors slightly outside the lines, shifting the edges outward by the width of the marker lines. Then the edges of the patterns they produce will be correctly positioned for stitching inside them. For combo templates, shift the outer edges by the same amount to maintain the same seam allowance.

I recommend keeping the inner part that you cut out of stencil or combo templates for use in drawing the front stitching patterns. Step 2 of the Assembly instructions explains why.

Making the Panels

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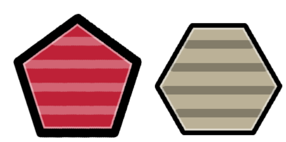
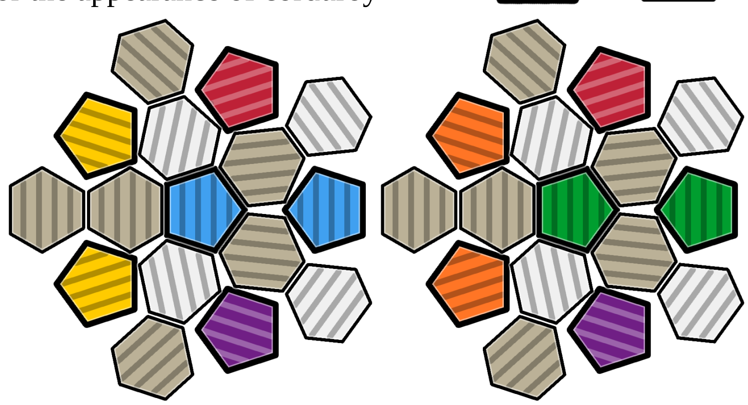
Note that if you are using separate stitching and cutting templates and they are not translucent, you must be careful which pattern, cutting or stitching, you trace first so that the **second template doesn't hide the lines of the first** and prevent you from aligning the two. **Do not necessarily use them in the sequence below**.

1. You will need **12 pentagons and 20 hexagons**, and **you will be tracing the patterns onto the back of the fabric (the side that will be inside the bag)**. If you use cutting templates, first trace those. If you are using the combo type like those illustrated above, trace the inside and outside of them and skip Step 2.

   To balance the stretch of woven fabric or the appearance of corduroy cords, you can either orient all patterns the same way in relation to the grain, or half one way and half the other, and then rotate them randomly when you lay them out for assembly, or you can use my suggestion illustrated on the right, with a long edge aligned with the grainline and the panels arranged as shown.

2. If you are using separate stitching and cutting templates, use the smaller, stitching templates to trace the stitching patterns within each cutting pattern, being sure to center them well (centering them allows you to align the patterns more easily as you sew, but is not otherwise important).

3. Cut out the panels.

Assembly

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Following is a method of assembling the panels I developed based on my 12 and 14-panel methods. I used this method for the two bags I made of this design and it works well. It is simple, easy to follow, and helps prevent me from losing track of my color arrangement.

The method consists of forming two separate hemispheres by sewing two rings of panels around a central pentagon, **and then sewing the hemispheres together around the equator**, leaving a few seams open through which to turn the bag out. Those seams are marked with dashed lines in the illustrations.

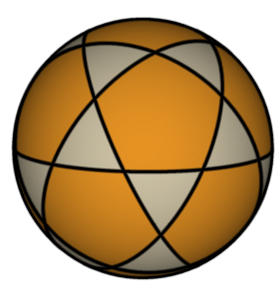
There are probably more efficient methods of assembly that require fewer seams to be double-stitched, but based on my few attempts to devise one, I think they would be rather more complicated, and more difficult to diagram and follow. So I will let the more advanced stitchers figure out their own improved methods. Remember that when you double-stitch a seam, you can make the stitches about twice as long if you are using the backstitch, and, provided you don't pull the stitches too tight, you will not get a puckered seam.

As an alternative to my method, **in case you prefer simply to add panels to a central pentagon** ring by ring until you end at the opposite pentagon, you can use that layout type from my color arrangement diagrams instead of the one in Illustration A. Then, after following the stitching paths in Illustrations B and C, use a similar path to continue adding rings of panels. Illustration D will be irrelevant in that case, and some of my written instructions will not apply.

The stitching pathways and techniques are a little different for the icosidodecahedron (pents and triangles). I discuss them in the next section.

Helpful Hints: While assembling the bag, remember the following points.

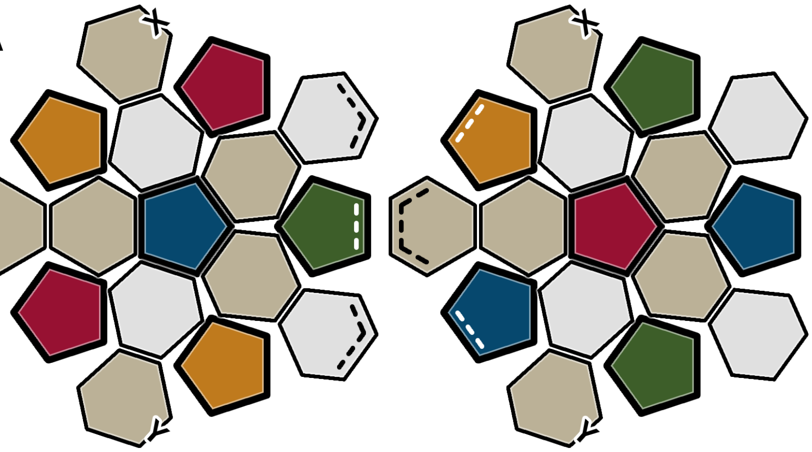
- **Every intersection will have three panel corners** (I have a few times accidentally joined four). Two will be hexes and one will be a pent. The exception is if you are using triangles instead of hexagons. In that case, every intersection will have four corners: two triangles and two pentagons, each shape opposite its match.



- **No pentagon will join to another pentagon**, but each pentagon corner will point to another, with a short hex pair seam in between (in the case of triangles, each pent corner will touch another pent corner).

- If you are using hexes with two different edge lengths, the **short edges will always join to another hex**, and the **long edges will always join to a pent**. If you are using triangles, **all triangle edges will join to pentagons**. Triangles will only touch corners with each other.

A



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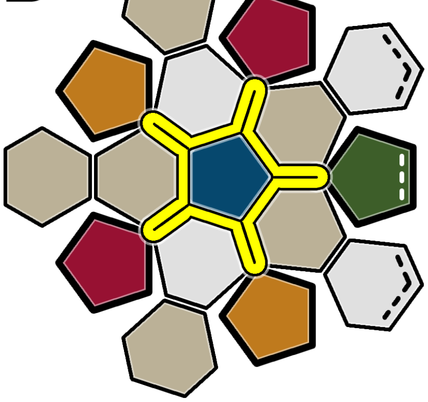
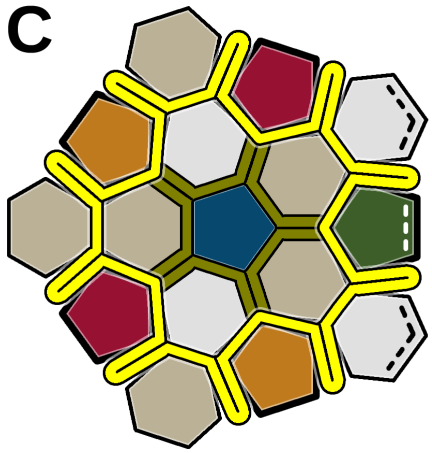


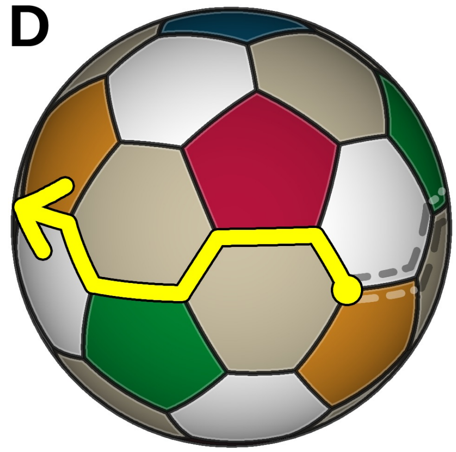
Hemisphere Inner Ring: Sew five hex panels to the central pentagon, sewing out and back in at each spoke seam.

C



Hemisphere Outer Ring: Sew the ring of alternating pents and hexes to the inner ring, again sewing out and back in at each spoke seam.

D



Sew the two hemispheres together around the equator. Note that the ball is inside out at this point, so the front stitching patterns (the dashed lines) will not actually be visible. I show them just for positional reference.

1. ***Illustration A:* Lay the panels out as shown** (I prefer to place them front face up, though placing them with their backs up will create a mirror-image version of chiral color arrangements) and **arrange them according to your color pattern**. The pairs of edges labeled X and Y will be joined in the finished bag (X to X and Y to Y). This may help you arrange your panels correctly. For woven fabric or something like corduroy, I recommend rotating the panels so that the grain of the fabric or corduroy cords are balanced and the ball has a good aesthetic and does not have a lopsided shape. You can either rotate the panels in random orientations, or use the suggestion in the "Making the Panels" section.

2. Use the stitching template to **draw stitching lines on the *fronts*** of the ten outer panel edges shown with dashed lines in Illustration A. My stitching pathway leaves these edges partially unsewn so the bag can be turned out between them. They will then be **sewn from the outside following the front stitching lines**. (If you use a thin fabric and don't need such a large opening, just skip marking a pair or two of panels.) Be sure to align the template as well as possible with the stitching patterns on the backs.

   If you want to **hide the stitching lines within the seams**, sketch them a millimeter or two nearer to the panel edges and stitch slightly inside them (toward the middle of the panels). **If you use a Stencil or Combo type template,** use the inner portion that you cut out of the template to draw these patterns, since the main template will cover the area near the edge.

I have found it helpful to **add marks along the front stitching lines for each stitch** so that I can more easily keep the exterior stitches even with each other and not get a skewed seam. I space the stitch marks $\frac{1}{8}$″ (3mm) apart. If you **make these marks on your template first**, you can more easily transfer them onto these and future panels.

3. ***Illustration B:* Start with a center pentagon and sew five hexes to it**. Each time you reach an intersection, **sew out and back in along the adjacent hexes' short seam** (the spoke seam) and then continue around. The duplicate stitches can be up to twice as long as the first stitches if you are using the backstitch and are careful how tightly you pull them (if you pucker the fabric, wiggle it straight again). **Attach the panels with their front faces together,** resulting in an inside-out bag.

4. ***Illustration C:*** When you complete the hub and spokes, begin following the **outer stitching path**, attaching the **outer ring of alternating pents and hexes**. Again, each time you reach a spoke seam, sew out and back in before continuing around. This step, as well as Step 6, **can be confusing**. Refer to my Helpful Hints at the beginning of this section to avoid mistakes. At the end **you should have completed a hemisphere of 16 panels**.

5. Sew the other hemisphere the same way.

6. ***Illustration D:* Join the two hemispheres** at one of the seams adjacent to the section with the front stitching lines. Be sure to align the seams correctly **so as to form your intended color pattern** and to make the front stitching lines on each half meet each other. Starting at that point, sew the two hemispheres together around the entire ball until you reach the other end of the front stitching lines. Joining the first couple of equatorial edges is **easier if you flip the hemispheres right side out** so the front faces of the panels are exposed and can be placed together.

   I recommend that you tie the thread a seam or two prior to the other end of the front stitching lines, and then continue, and use the same thread when you sew the final opening closed from the outside. This will allow you to loosen some of the stitches to aid in turning the bag out, and then you can tighten them again from the outside. **Tying the thread a couple seams back will prevent the stitches from loosening beyond that point**.

   When you reach each seam intersection as you proceed around the equator and must cross the spoke seams, use the method I describe in the "Stitching Techniques" section of the **General Information and Techniques** chapter under "Crossing seam intersections...".

7. **Sew a few starter stitches** at one end of the final seams to make it easier to continue from the outside. If you don't need the entire opening to turn the bag out, continue to sew as much as you don't need. To **reduce the number of stitches you need to make from the outside**, you can make extra stitches and then loosen them to allow the panels to spread enough to turn the bag out. Then you can pull them tight again from the outside. If you want to do this, or if you want to be able to loosen the last several stitches enough to push a funnel between them, **your final thread will need several inches of extra length**.

8. **Turn the bag right side out through the opening**. A good method for this is to use the back end of a pen or other slender tool to push the fabric through the opening from the opposite side and then either invert the bag around the tool, or remove the tool and work the bag through with your fingers. **Be gentle so as not to pop any stitches**.

9. **Pull out the last stitch so that the thread is on the outside** where you can get to it. Continue sewing the opening closed following the front stitching lines. For help, see the "Stitching Techniques" section of the **General Information and Techniques** chapter under "Backstitch from the exterior Approaches". Fill the bag at some point during this final sewing with a funnel. I find it helpful to **put some filler in first to prevent the bag from collapsing** while I sew, and to hold the seam allowances in place and give me something to push the needle against.

   You can sew the entire opening closed before fully filling the bag, which prevents the filler from spilling back out while you sew. Just loosen the last several stitches enough to push the funnel between them, or at least to push some filler in with your fingers. Then re-tighten the stitches (see "Tips on finishing the bag").

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Assembly Diagrams and Tips for the Triangle Variation

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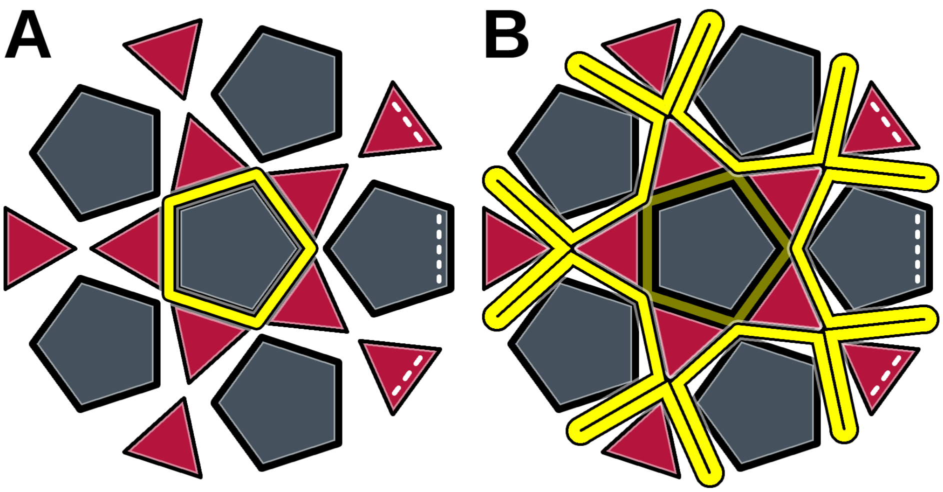
If you use my 2-hemisphere assembly method, the stitching pathways are a little different for this variation. Below are illustrations for them. The first path has no spoke seams. It just runs around the hub pent. The outer pathway does have spoke seams, requiring each spoke to be double-stitched.

When you add the fourth corner to each intersection and when you reach each intersection around the equator, I recommend following the suggestions in the **General Information and Techniques** chapter under "Stitching Techniques", subheading "Closing seam intersections tightly". **4-way and above intersections take a little extra care if you want them tightly closed and elegant-looking**. In short, **stitch each panel tip to the one diagonally opposite it (the thread will form an X across the intersection) and cinch them together**.

After both hemispheres are finished, sew them together around the circumference of the ball as for the main design. The seams of this design form a straight, continuous seam rather than a zig-zag one.

Because of the 4-way vertices, as you sew around the equator **you will repeatedly need to cross pairs of stitched seams**. To do this, use the method I describe in the "Stitching Techniques" section of the **General Information and Techniques** chapter under "Crossing seam intersections while sewing patches of multiple panels together". The retreating stitches I recommend at the intersections help to **tightly close the 4-way vertices**, which can open up a bit if they are not cinched shut well. **To really close all the intersections well, follow the advice above**.

With those points in mind, follow the main assembly instructions in the previous section.



Stitching paths for one hemisphere of the triangle variation of the 32-panel design

Ready-to-Print Patterns for All Variations

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The pattern pages are 8.27″×11″ (210mm×279mm) to fit both “Letter” and “A4” sizes. ***Make sure the print is not being scaled to fit the printer margins*** *(select Default/None scaling/Actual size/Ignore printer margins). To verify correct sizing,* ***compare the centimeter grid to a ruler*** *and adjust the next print if necessary. (Note that PDF viewers and printers can both contribute to slight size inaccuracy.)*

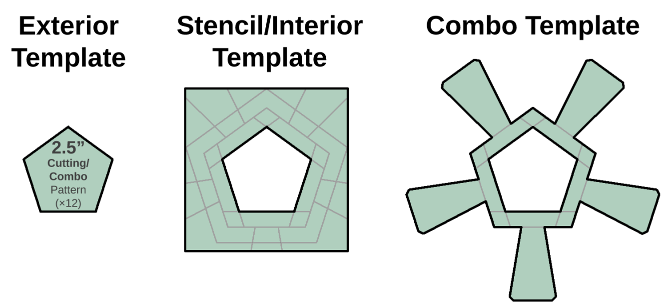
The **main, Equidistant design patterns** begin on the next page. Those are followed by the **Footbag Hex Variation**, the **Triangle Variation** (icosidodecahedron), and finally the **Equilateral Hex Variation** (truncated icosahedron/soccer ball).

The patterns are for beanbag diameters from 2″ – 3″ in ¼″ increments, with 7″ patterns for scaling to larger sizes. The Hex variant patterns are reduced by 3.8% and the Triangle variant by 3.0% from the mathematical calculations to account for the inflation in size I observed in my corduroy bag. **If you use a dense/stiff or completely non-stretch fabric, I recommend enlarging the pattern to about 105%.**

To make the templates, I recommend cutting out the portions of the paper with the patterns you want and gluing or taping them to your template material, and then cutting along the patterns.

The patterns are Combo patterns. They have the **stitching patterns on the inside (filled with gray)** and the **cutting patterns on the outside**, with **4mm, 6mm, and 8mm allowances** so you can choose the amount that works best for your fabric and preference (the lighter, 6mm pattern is a hair under ¼″). They also include **tabs to help you hold the templates down**. You may not need all the tabs with a rigid enough template material.

The examples on the right show the **three ways you can cut out the templates**. If you want separate stitching and cutting templates, you will need to print the patterns twice.



To produce other pattern sizes or correct an incorrectly sized beanbag, adjust the size scaling in the print dialog. For example, to reduce my 2.5″ pattern to the 2.3″ size recommended by the Juggling Store for small hands and numbers juggling, divide 2.3 by 2.5, multiply the result by 100, and that is your scale (92% in this case). If your beanbag ends up not being the expected size, see the **General Information and Techniques** chapter under “Adjusting/Scaling a Pattern to Produce an Accurate Ball Size” for help with correcting it.

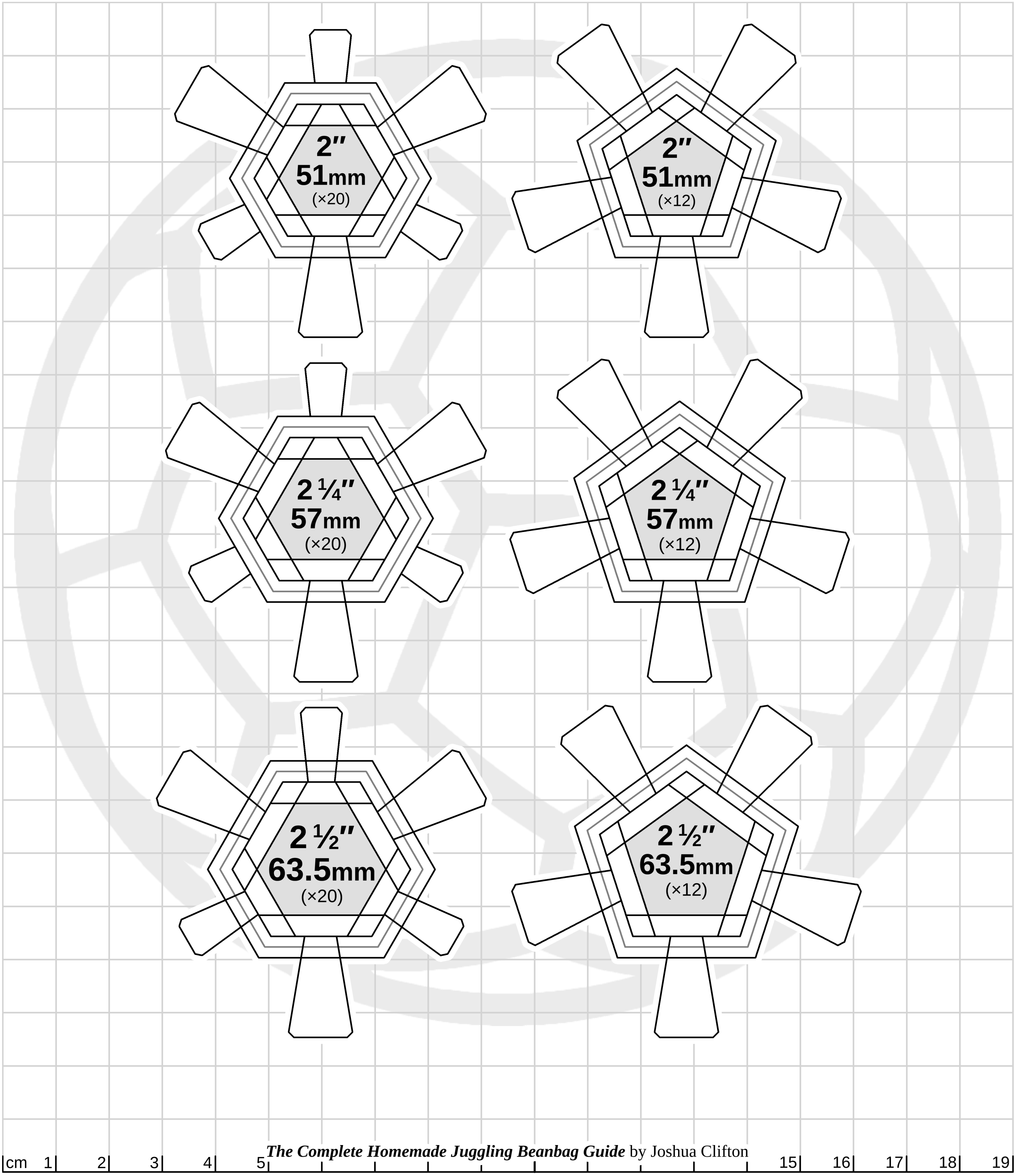
The table below provides the scaling for the ⅛″ increments between my ¼″ sizes. The centimeter grid can be used to verify correct scaling.

Target Diameter	Print this pattern size	At this scale
1¾″ (44.5mm)	2″	87.5%
1⅞″ (47.6mm)	2″	93.8%
2⅛″ (54.0mm)	2¼″	94.4%
2⅜″ (60.3mm)	2½″	95%
2⅝″ (66.7mm)	2¾″	95.4%
2⅞″ (73.0mm)	3″	95.8%

Equidistant Truncated Icosahedron (32 Panels)

Uses 20 hexagons, 12 pentagons

Pattern sizes are adjusted for corduroy and do not account for gathered seams.
For footbags with gathered seams, try two sizes (½″) or 25% larger than target diameter.



Equidistant Truncated Icosahedron (32 Panels)

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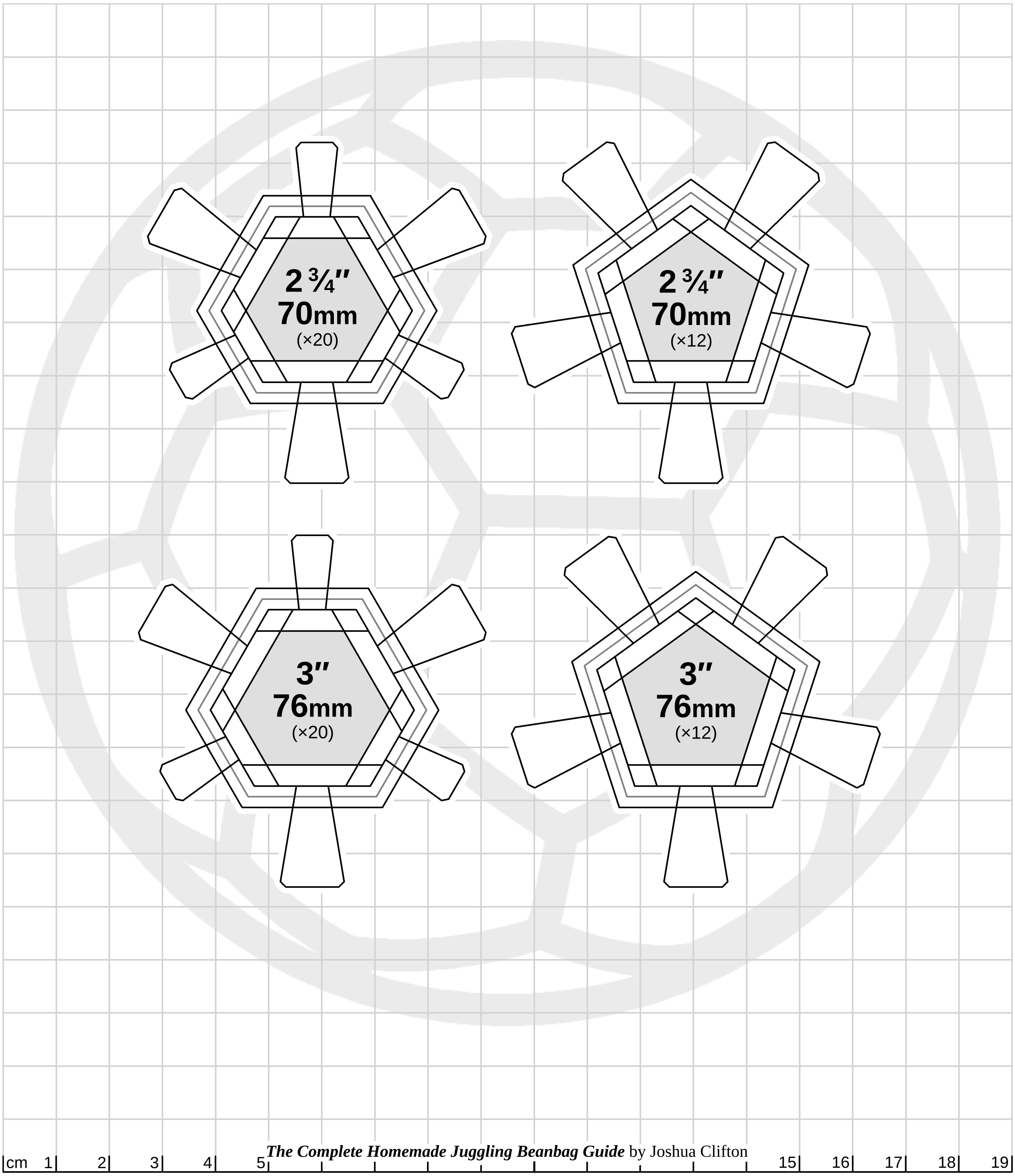

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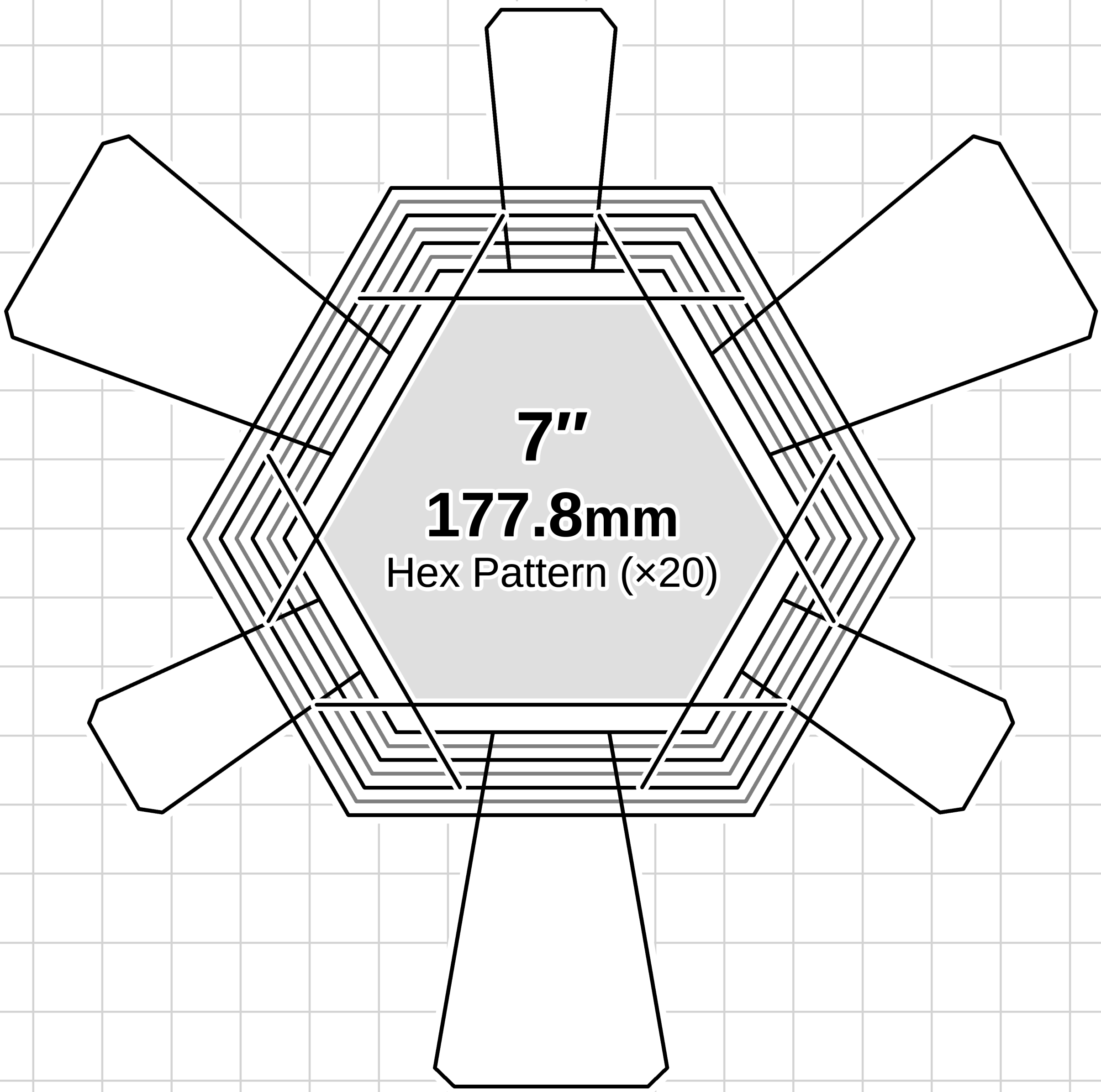
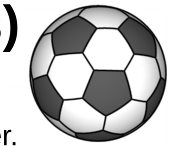


cm 1 2 3 4 5 15 16 17 18 19

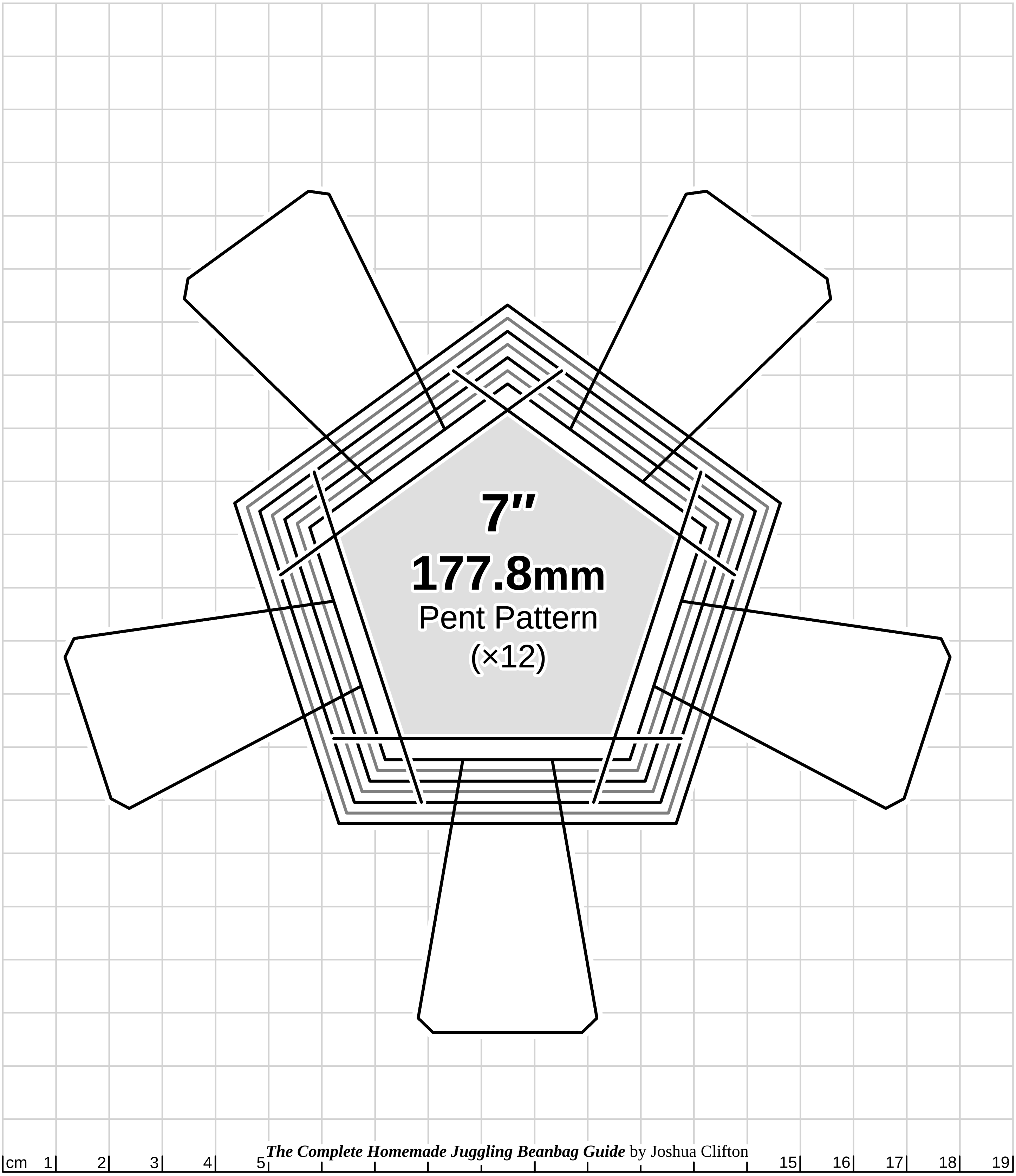
Equidistant Truncated Icosahedron (32 Panels)



Uses 20 hexagons, 12 pentagons

Pattern sizes are adjusted for corduroy and do not account for gathered seams.

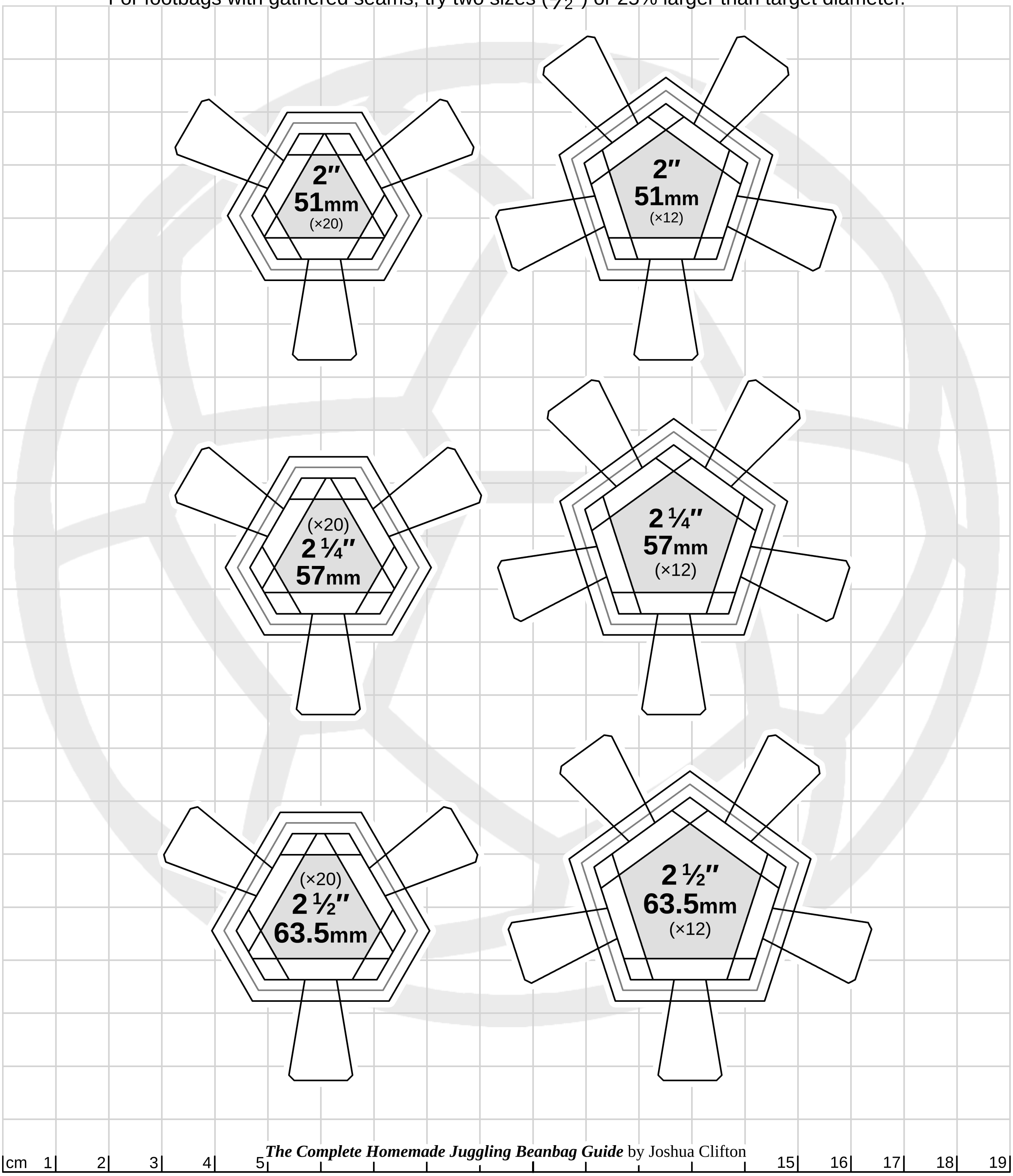
For footbags with gathered seams, try two sizes (½″) or 25% larger than target diameter.



Modified Icosidodecahedron (32 Panels)

14-Panel/Footbag-Style Hexes (Uses 20 hexagons, 12 pentagons)

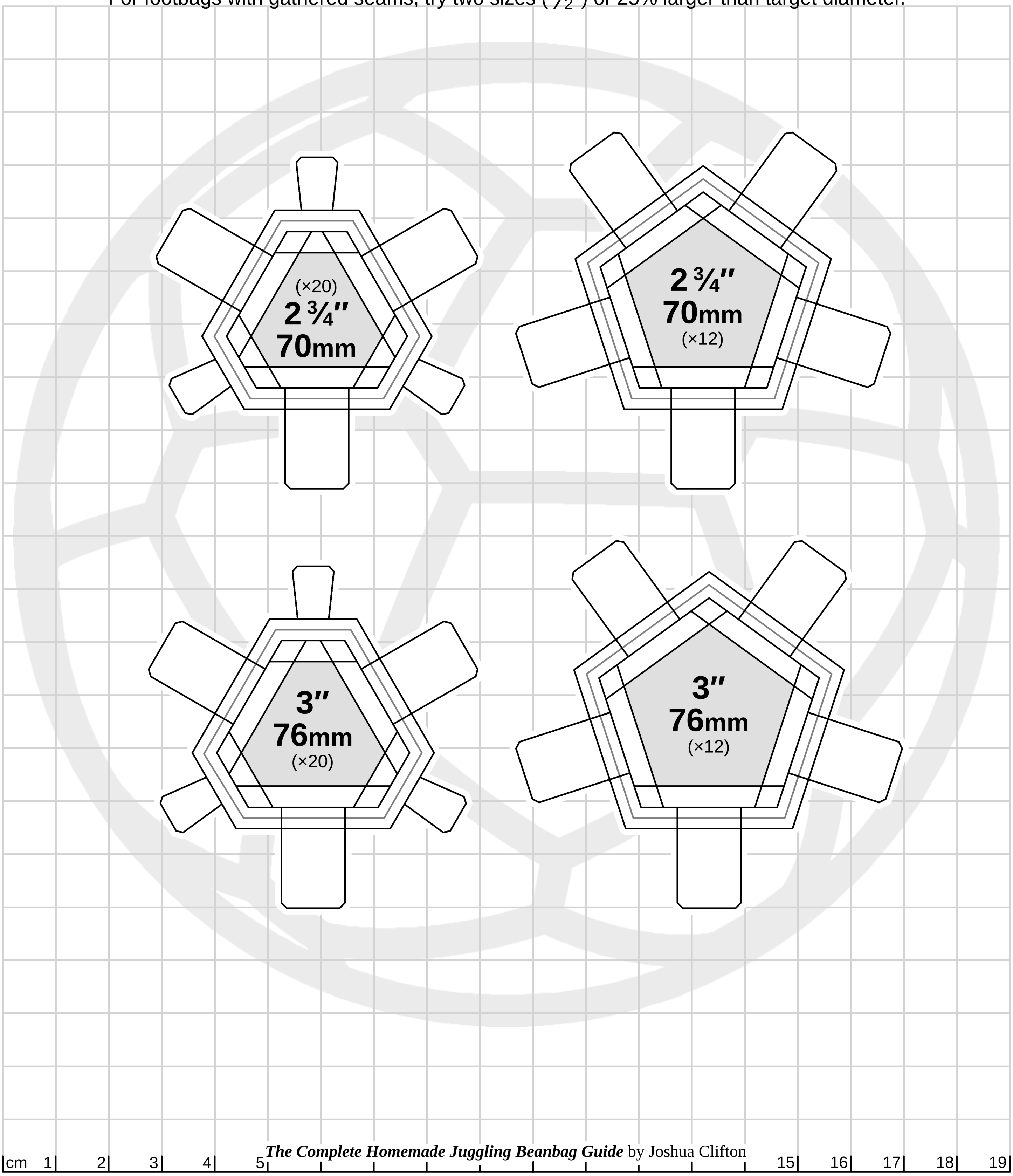
Pattern sizes are adjusted for corduroy and do not account for gathered seams.
For footbags with gathered seams, try two sizes (½″) or 25% larger than target diameter.



Modified Icosidodecahedron (32 Panels)

14-Panel/Footbag-Style Hexes (Uses 20 hexagons, 12 pentagons)

Pattern sizes are adjusted for corduroy and do not account for gathered seams.
For footbags with gathered seams, try two sizes (½″) or 25% larger than target diameter.



Modified Icosidodecahedron (32 Panels)

14-Panel/Footbag-Style Hexes (Uses 20 hexagons, 12 pentagons)

Pattern sizes are adjusted for corduroy and do not account for gathered seams.
For footbags with gathered seams, try two sizes (½″) or 25% larger than target diameter.

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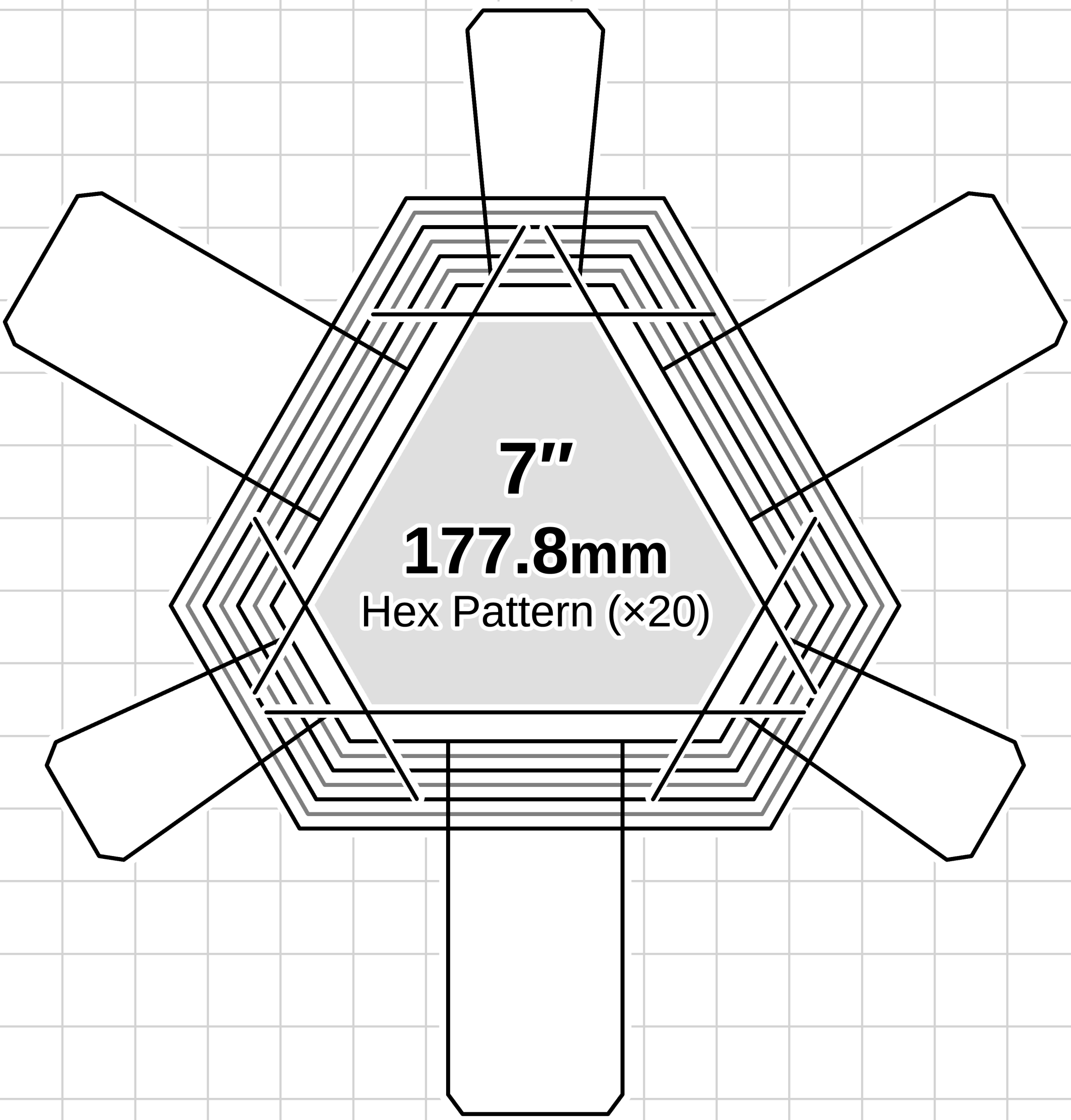

cm 1 2 3 4 5 15 16 17 18 19

Modified Icosidodecahedron (32 Panels)

14-Panel/Footbag-Style Hexes (Uses 20 hexagons, 12 pentagons)

Pattern sizes are adjusted for corduroy and do not account for gathered seams.
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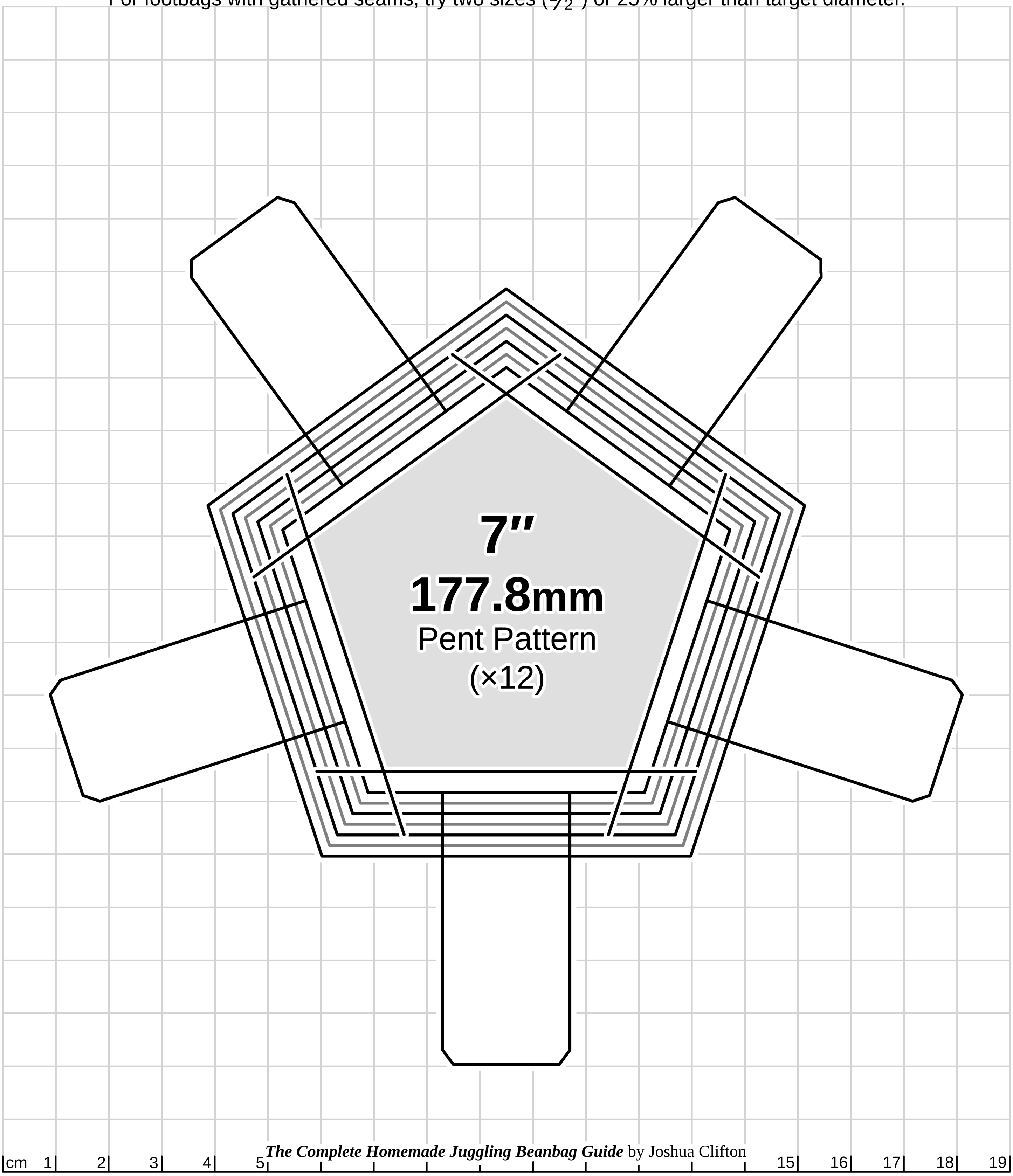
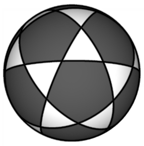
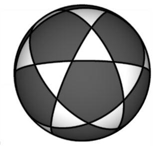


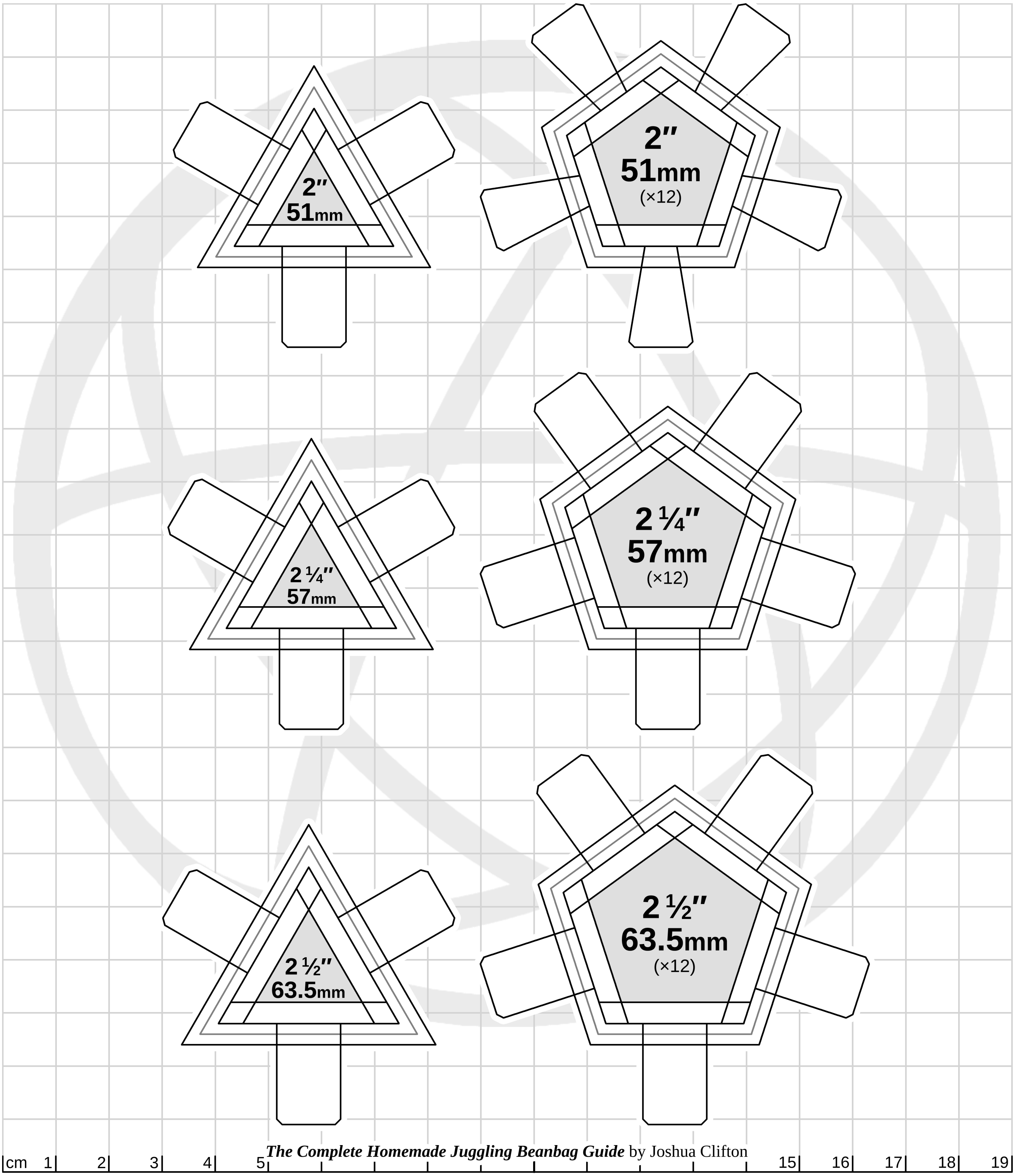


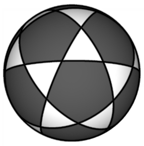
Icosidodecahedron (32 Panels)

Uses 20 triangles, 12 pentagons

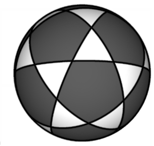
Pattern sizes are adjusted for corduroy and do not account for gathered seams.
For footbags with gathered seams, try two sizes (½″) or 25% larger than target diameter.





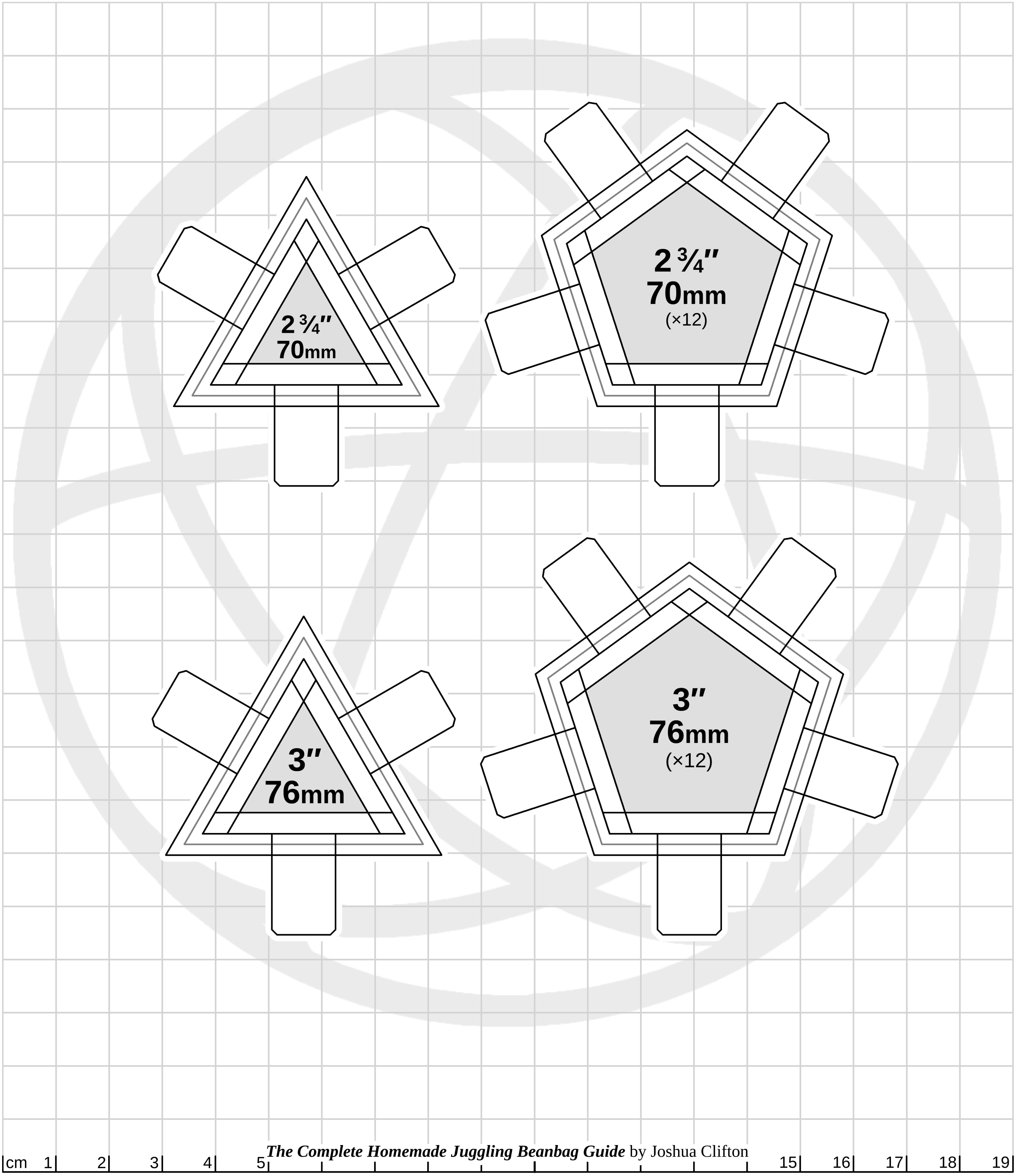


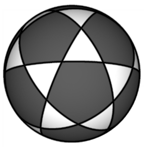
Icosidodecahedron (32 Panels)



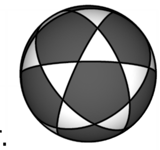
Uses 20 triangles, 12 pentagons

Pattern sizes are adjusted for corduroy and do not account for gathered seams.
For footbags with gathered seams, try two sizes (½″) or 25% larger than target diameter.





Icosidodecahedron (32 Panels)



Uses 20 triangles, 12 pentagons

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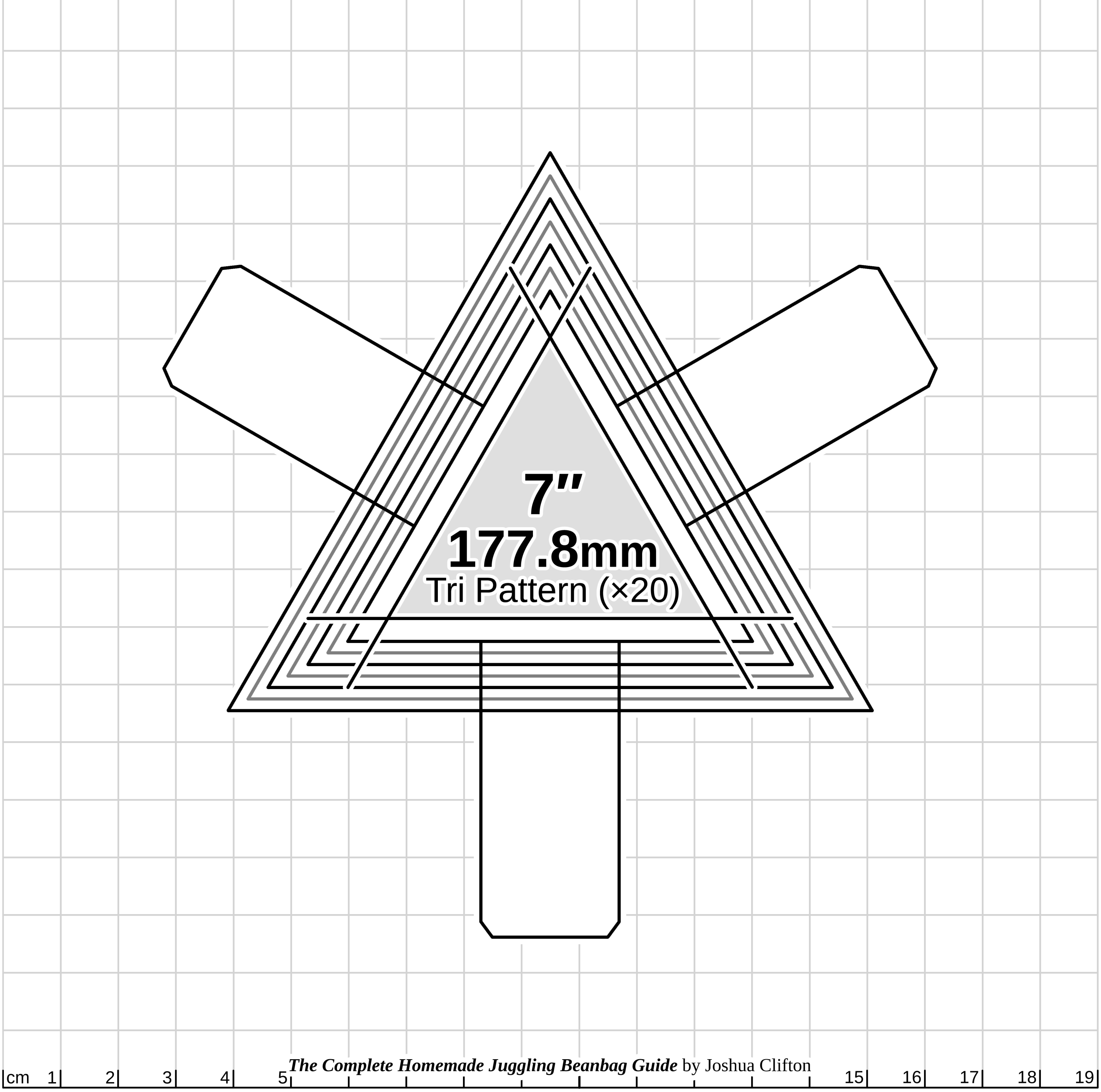

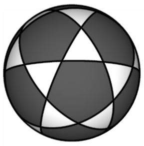

Icosidodecahedron (32 Panels)

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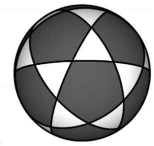
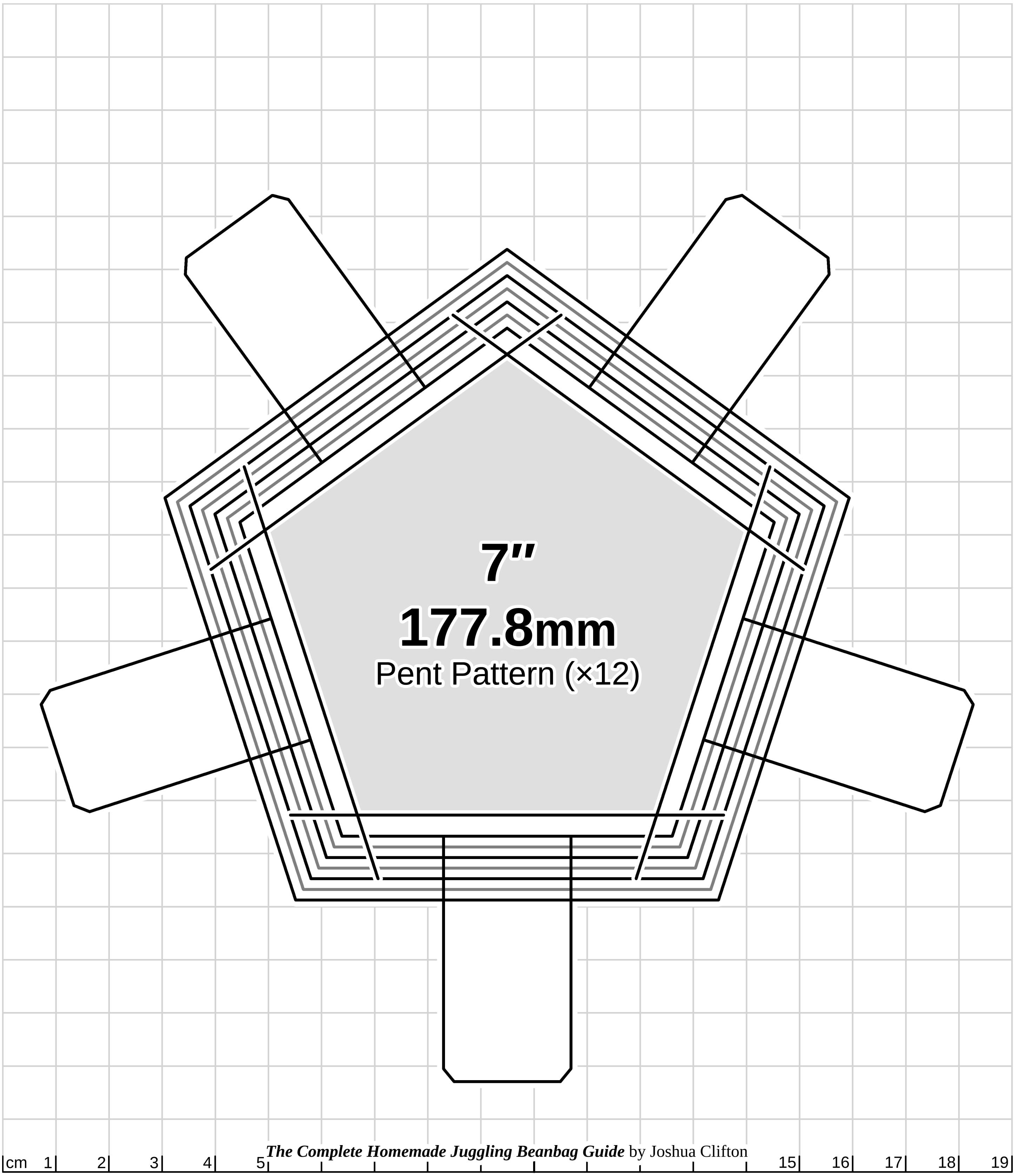


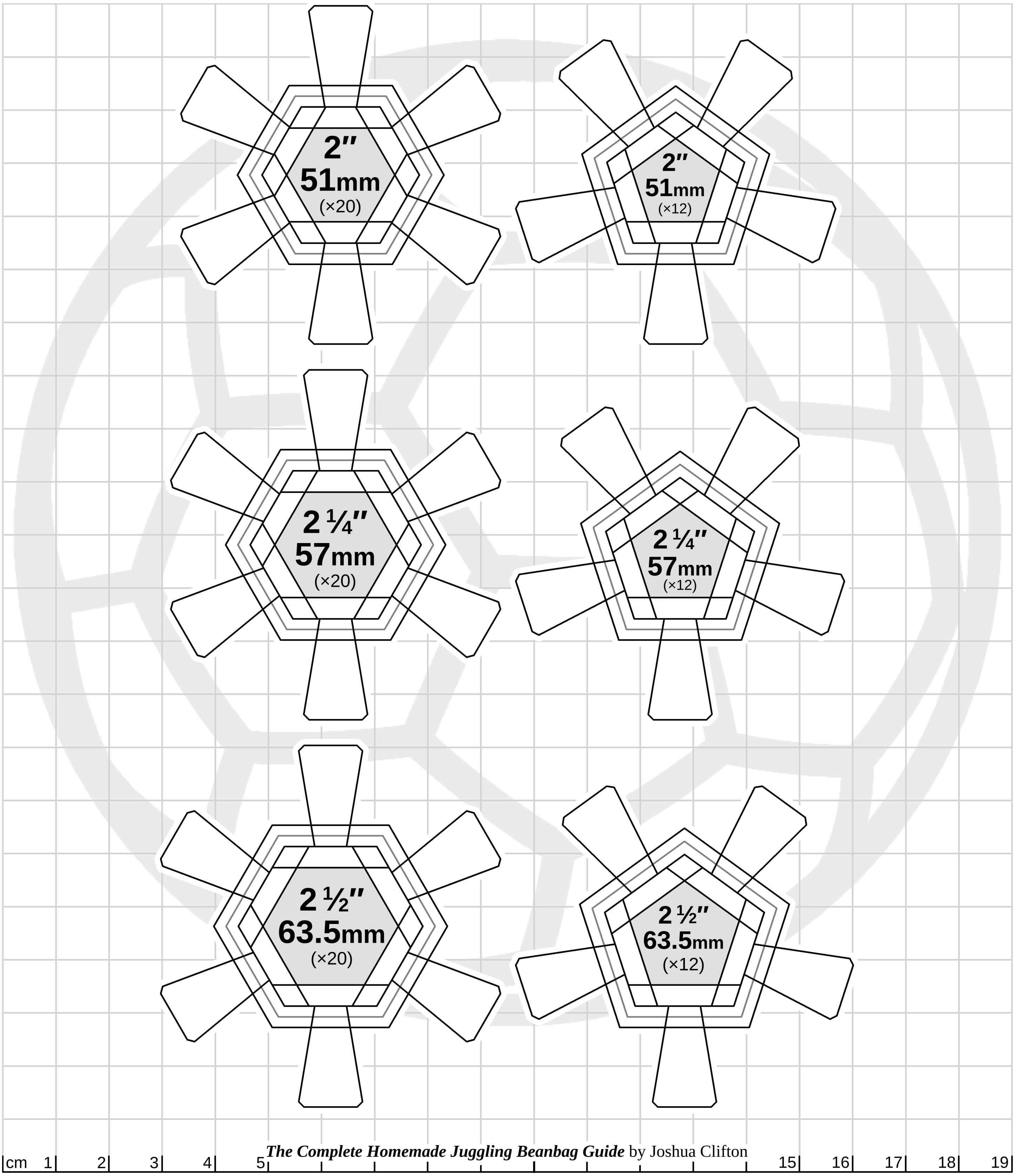


Truncated Icosahedron/Soccer Ball (32 Panels)

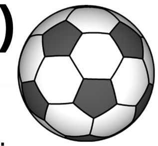
Uses 20 equilateral hexagons, 12 pentagons

Pattern sizes are adjusted for corduroy and do not account for gathered seams.

For footbags with gathered seams, try two sizes (½″) or 25% larger than target diameter.

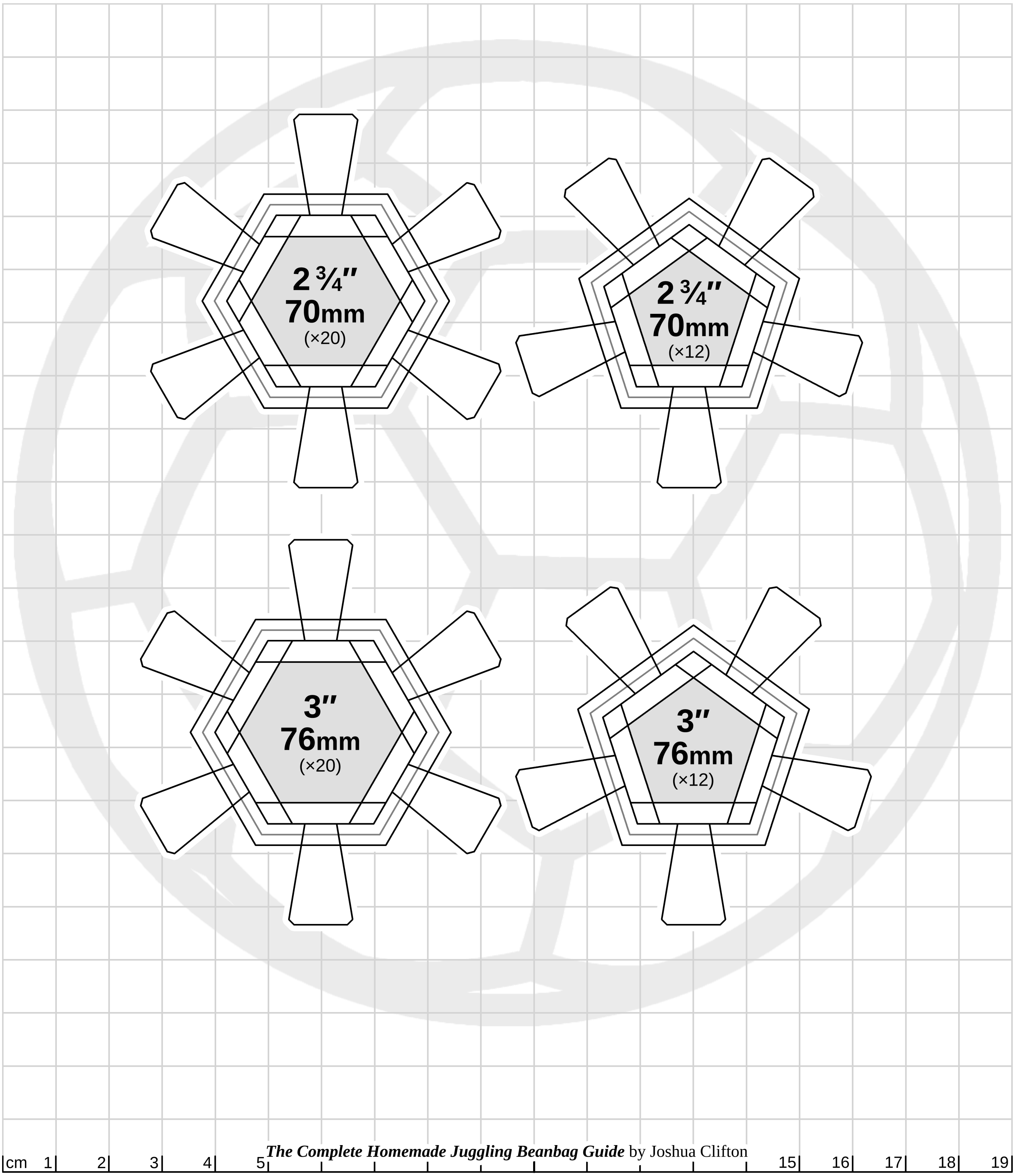


Truncated Icosahedron/Soccer Ball (32 Panels)

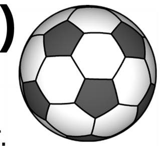


Uses 20 equilateral hexagons, 12 pentagons

Pattern sizes are adjusted for corduroy and do not account for gathered seams.
For footbags with gathered seams, try two sizes (½″) or 25% larger than target diameter.



Truncated Icosahedron/Soccer Ball (32 Panels)



Uses 20 equilateral hexagons, 12 pentagons

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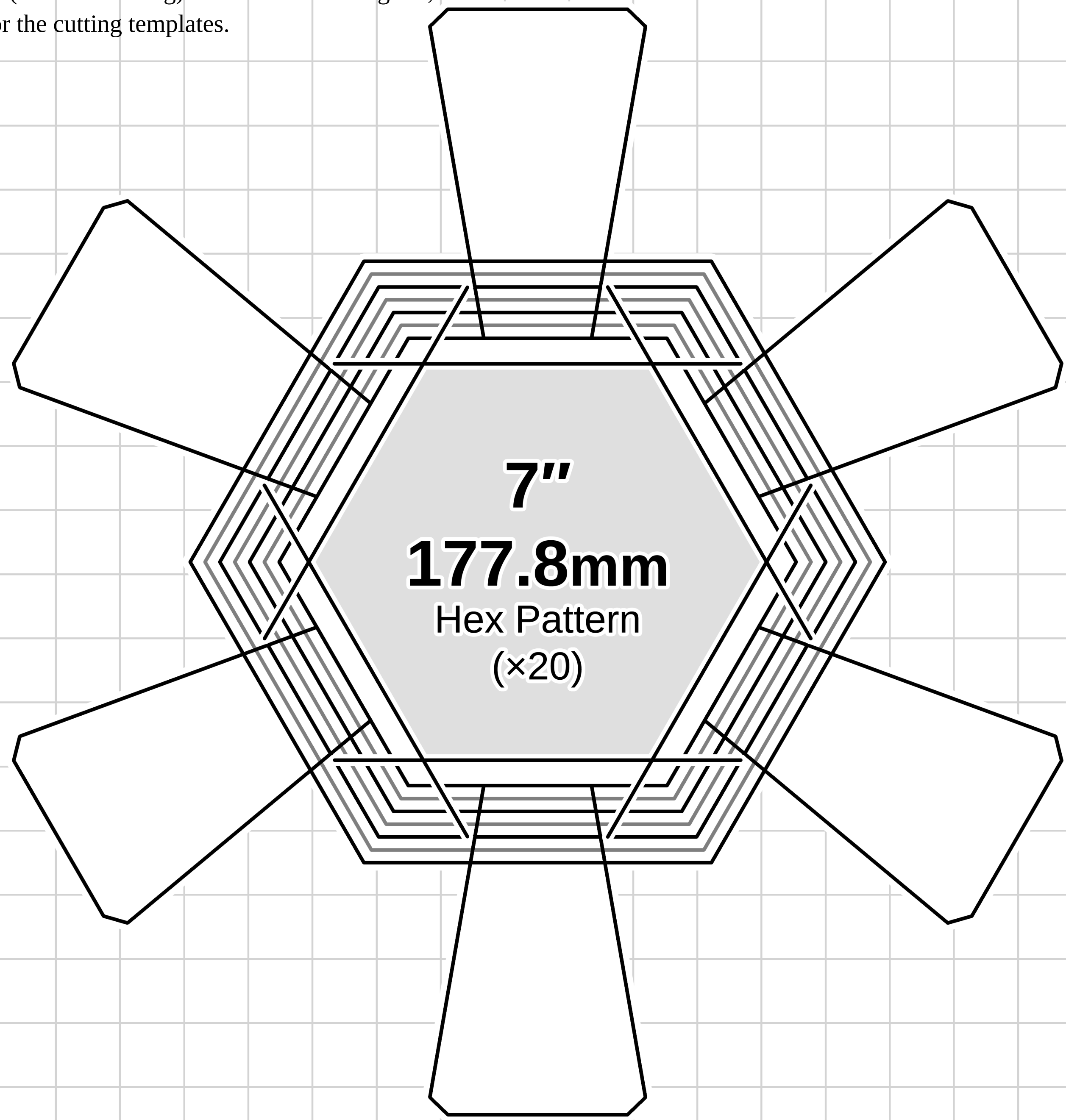

cm 1 2 3 4 5 15 16 17 18 19

Truncated Icosahedron/Soccer Ball (32 Panels)

Uses 20 equilateral hexagons, 12 pentagons

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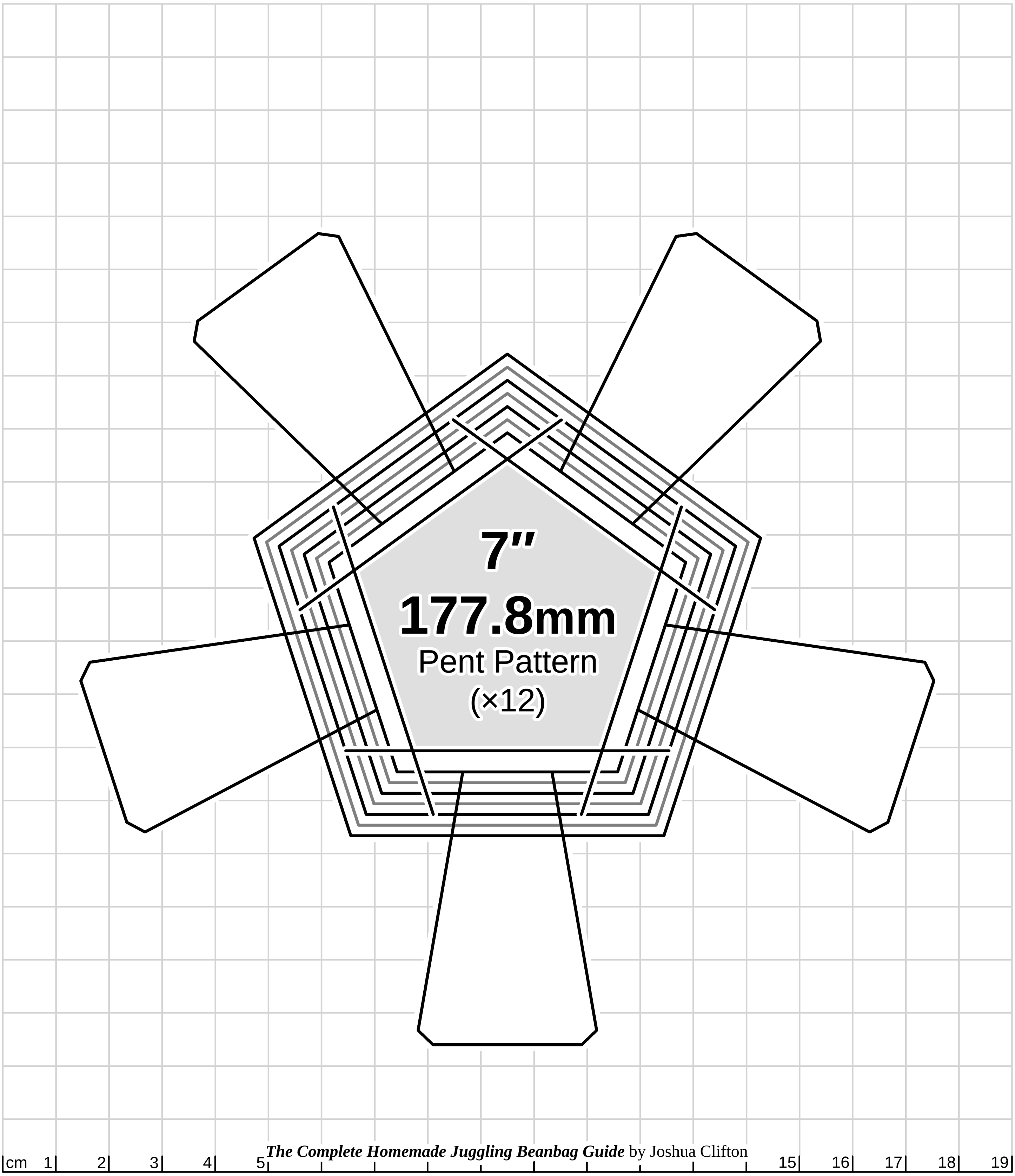

Blank Color Arrangement Diagrams for All Variations

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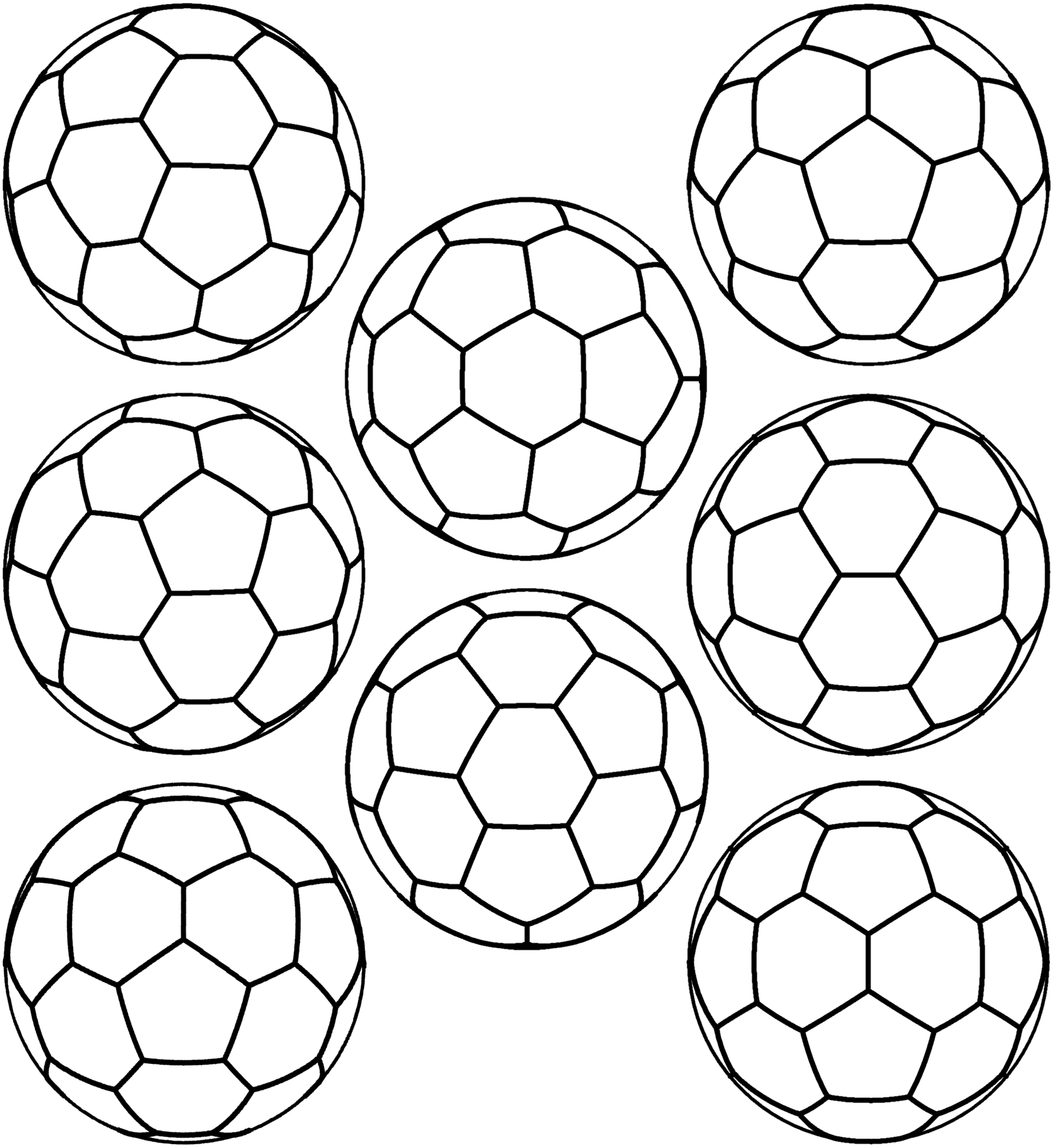
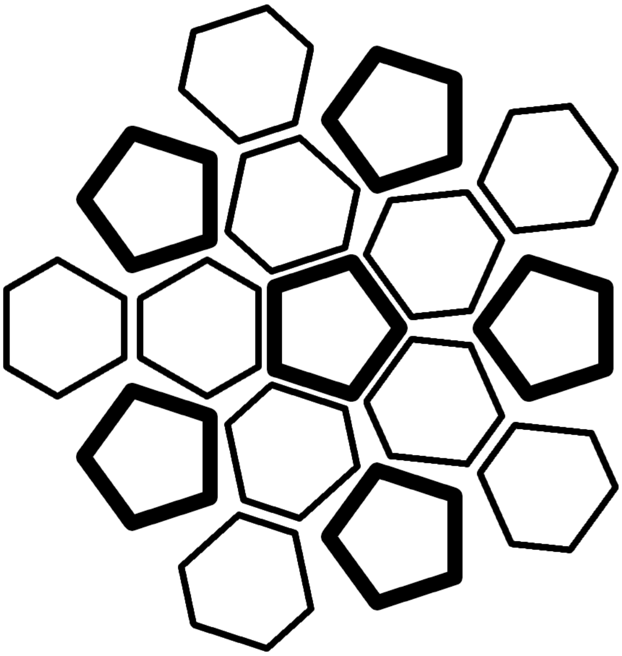
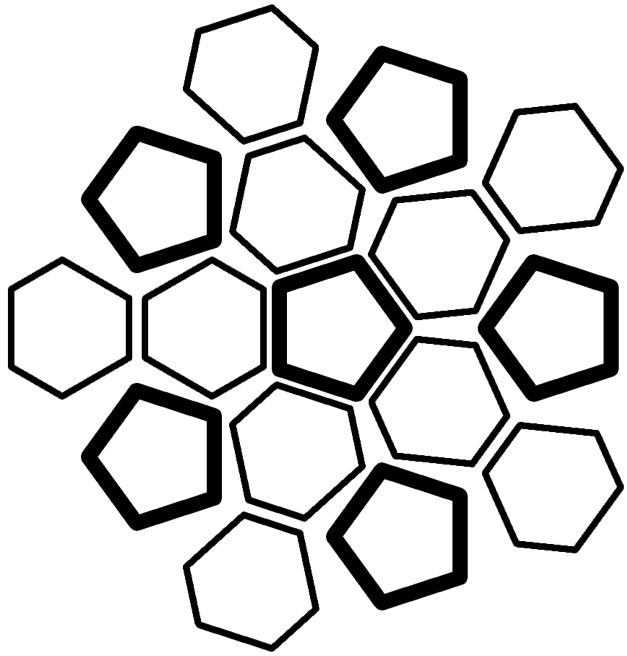
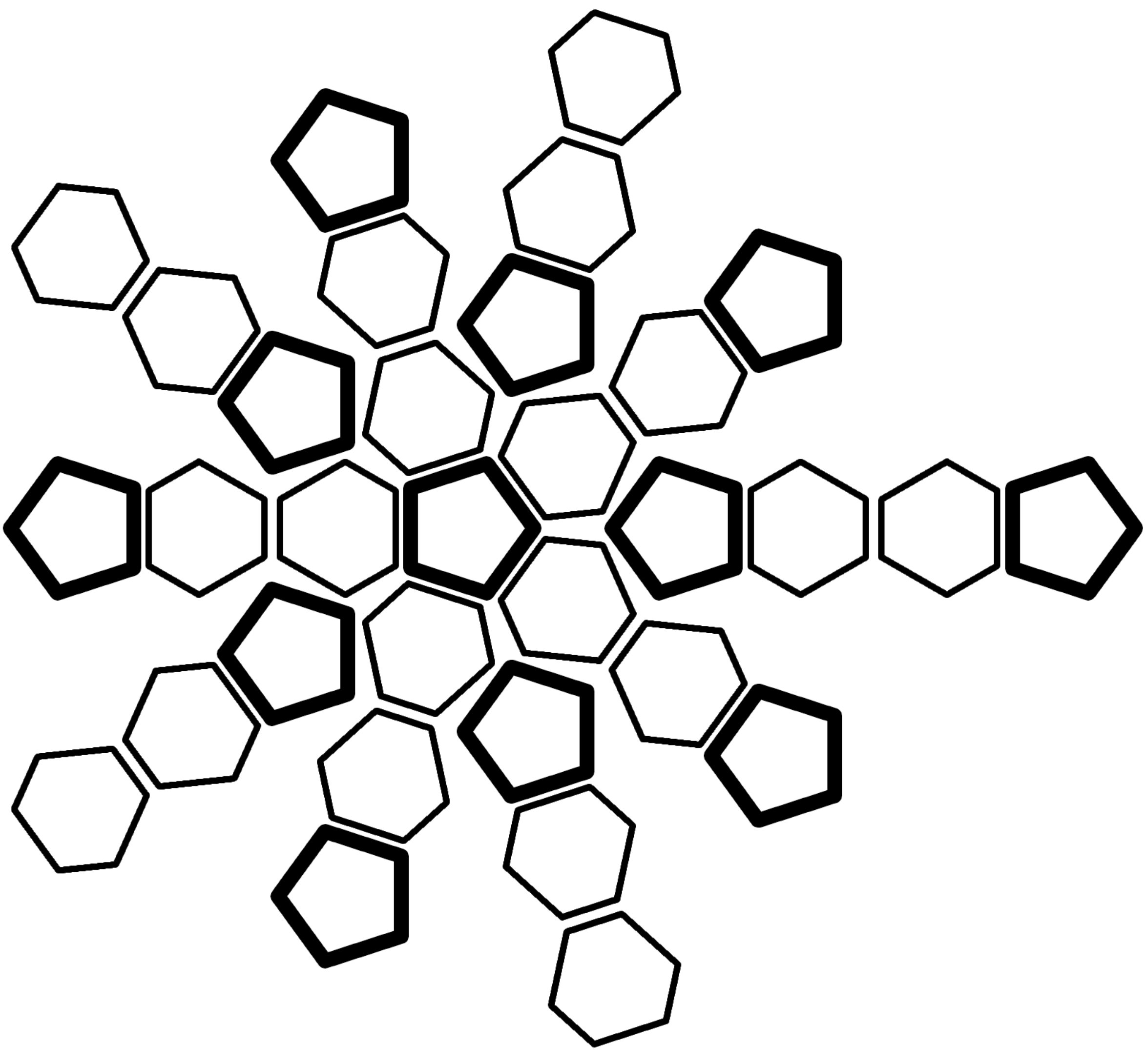
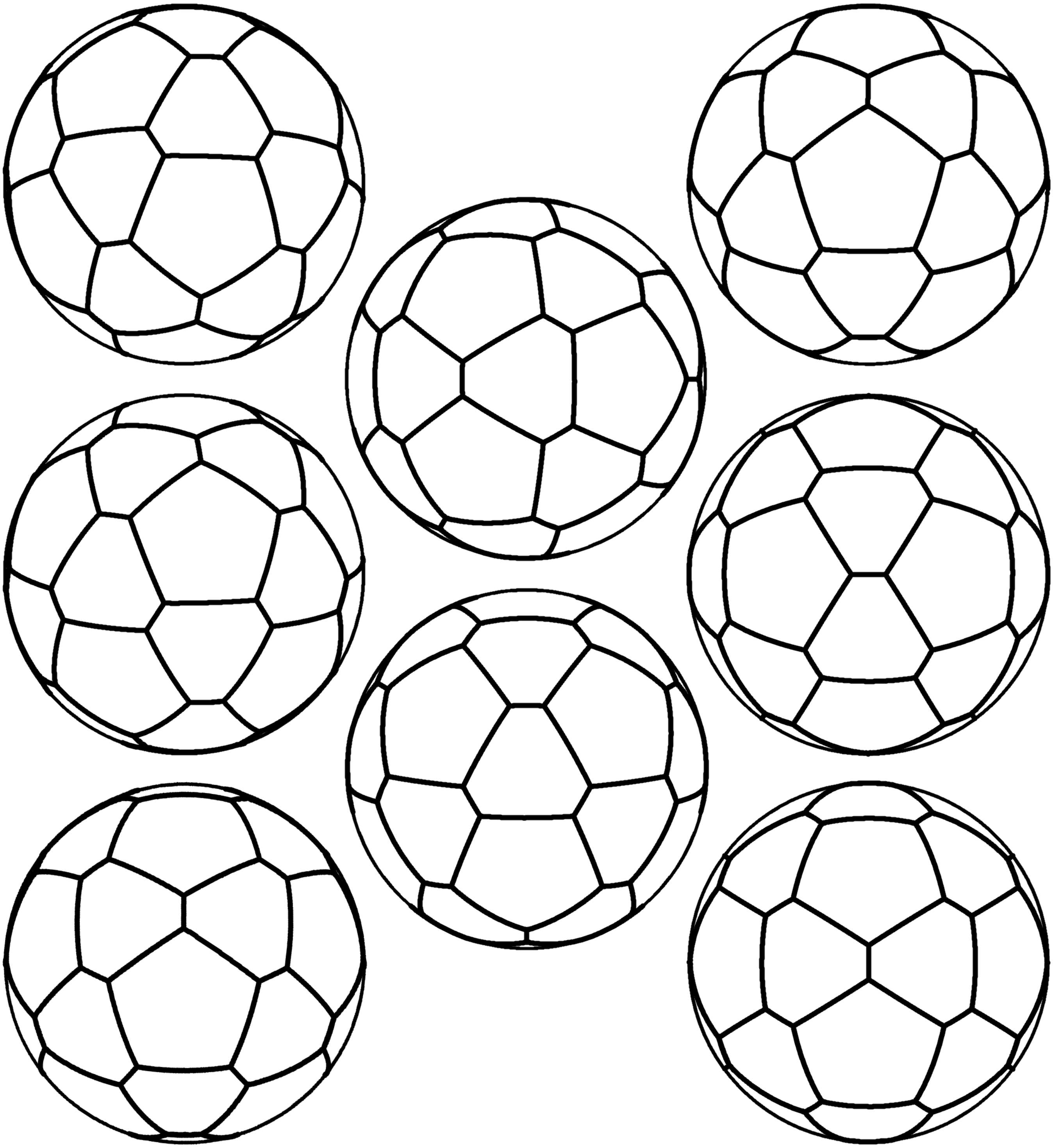
Following are the ball and assembly layout diagrams I used for my color arrangement illustrations **(the main, Equidistant version first, followed by the Footbag Hex and Triangle variations)**. You can use these to experiment with your own arrangements. The Equidistant diagrams are a close enough substitute for the true truncated icosahedron/soccer ball, so I did not include that variation. I also offer PNG format diagrams for download on my website that you can use in an image editor. If they are unavailable, you can capture a screenshot of these pages or export the images and then color them in an image editor. Or you can just print them and color them by hand.

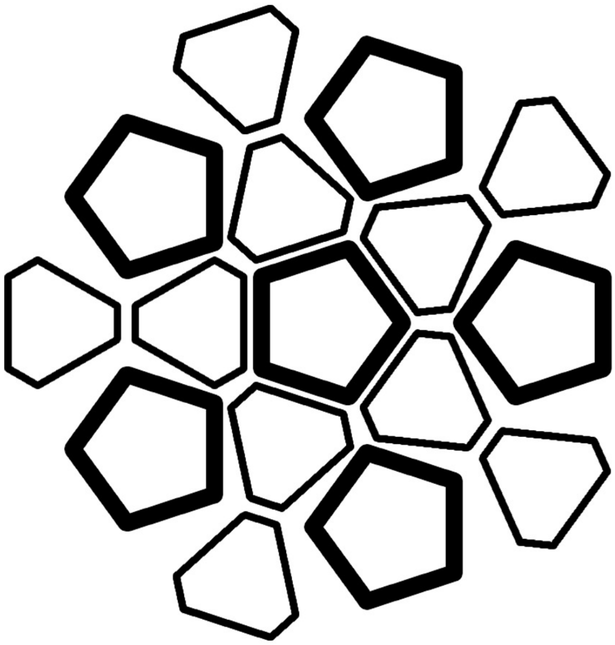


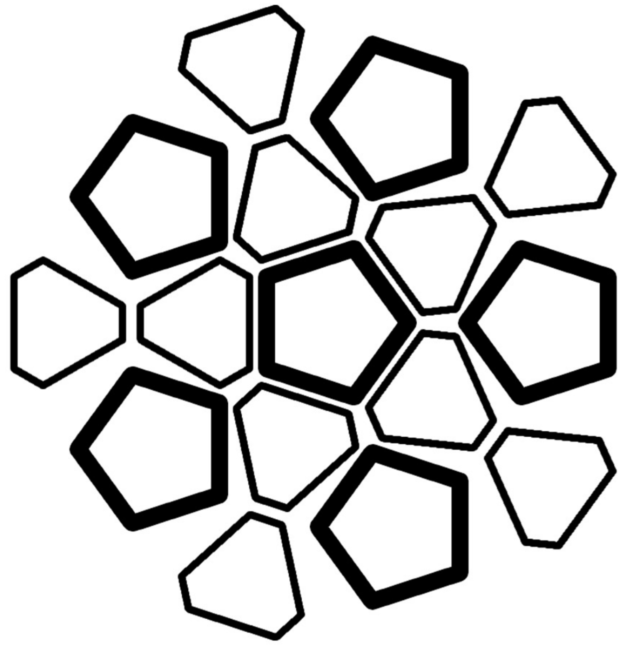


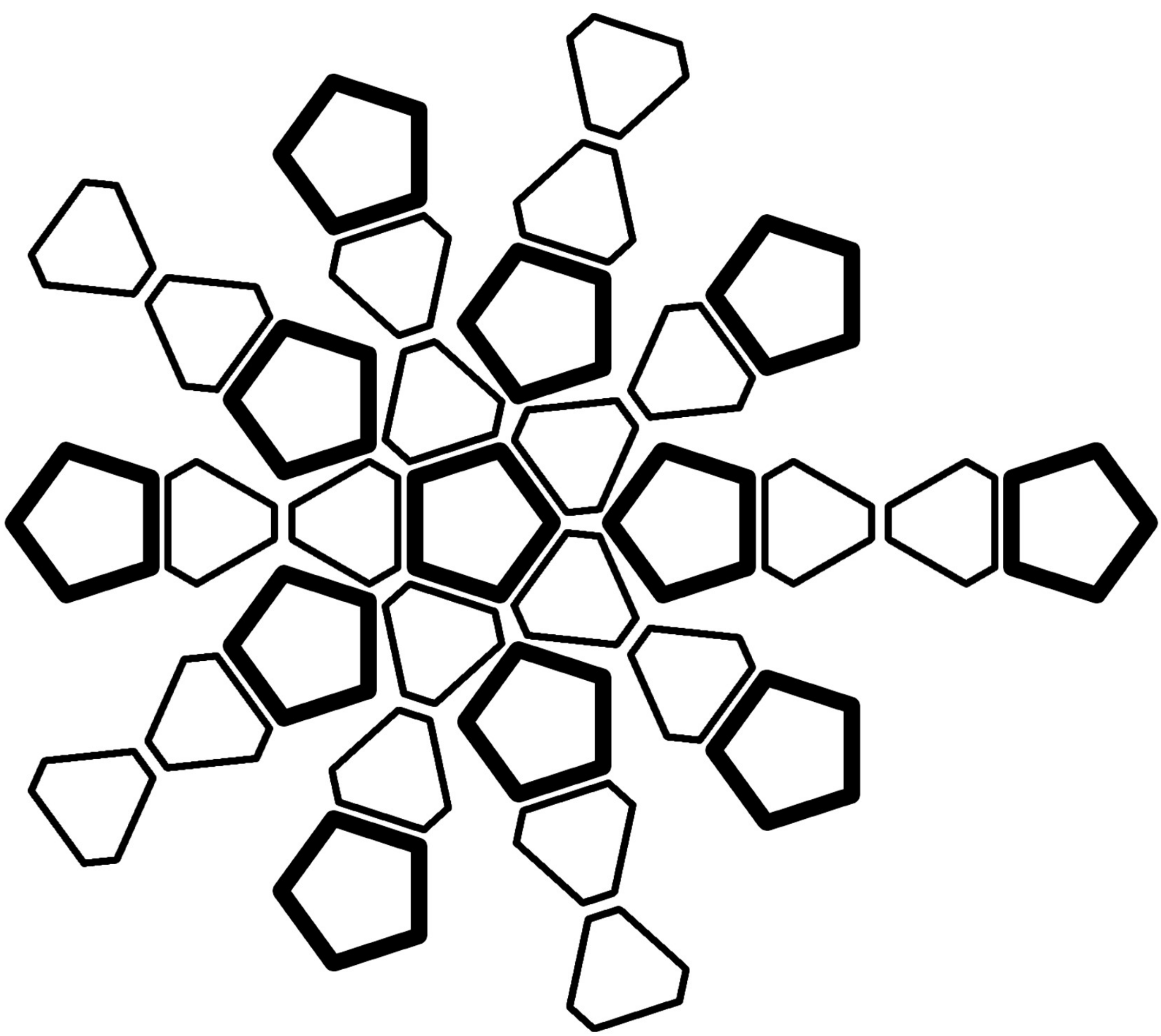


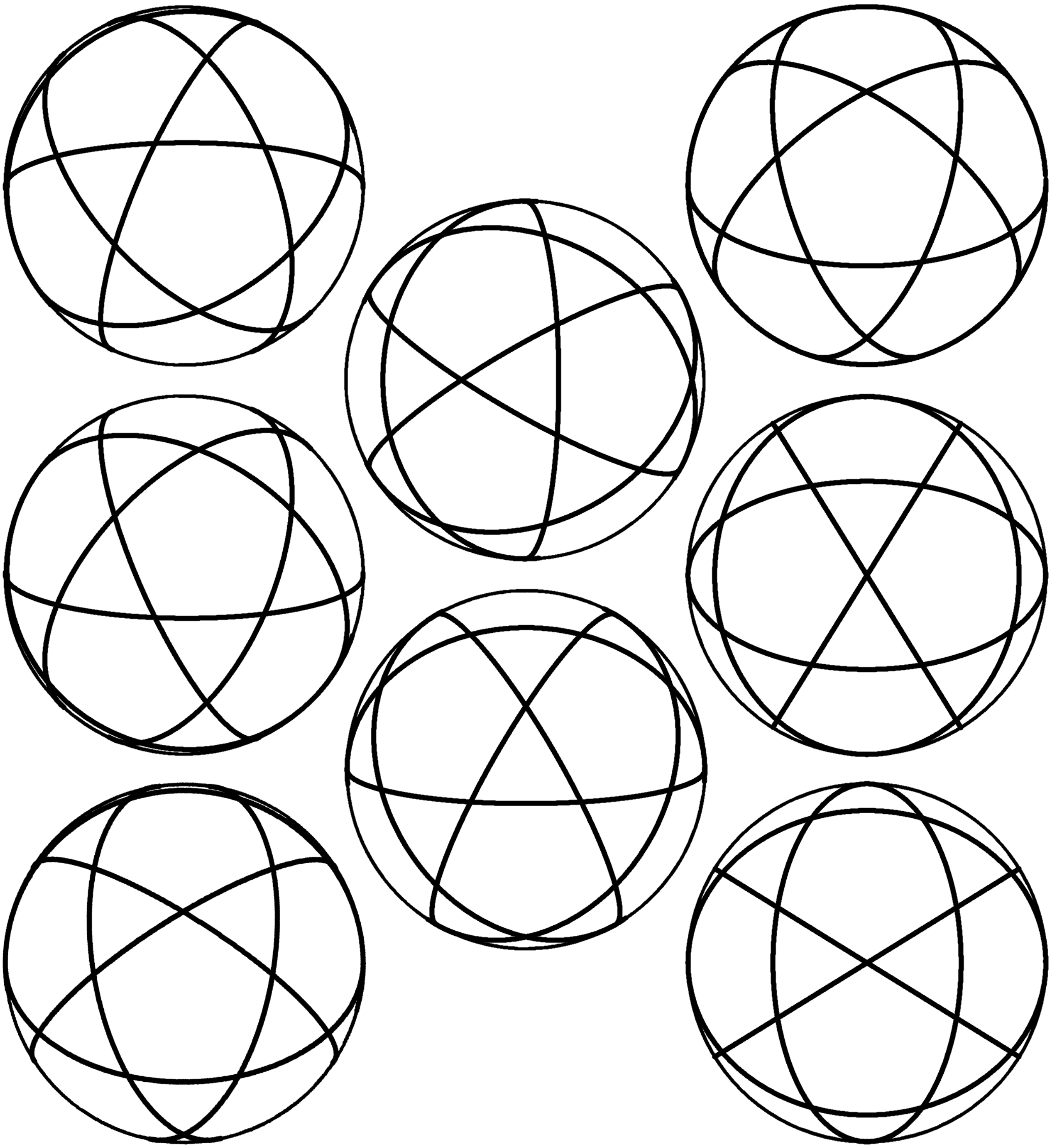


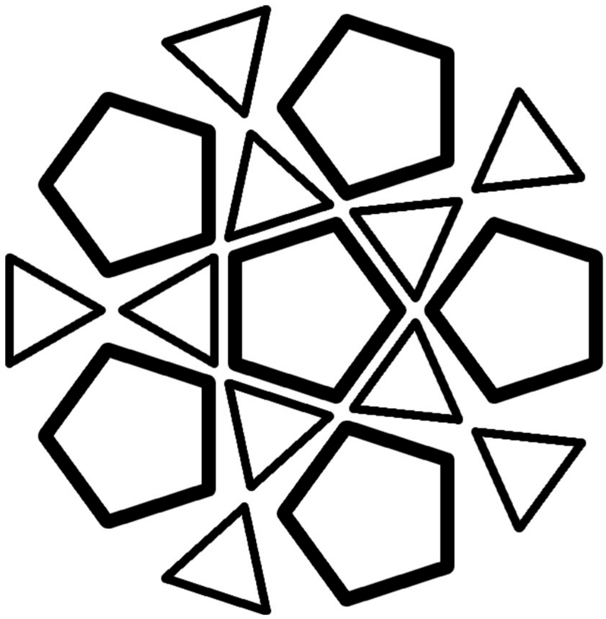


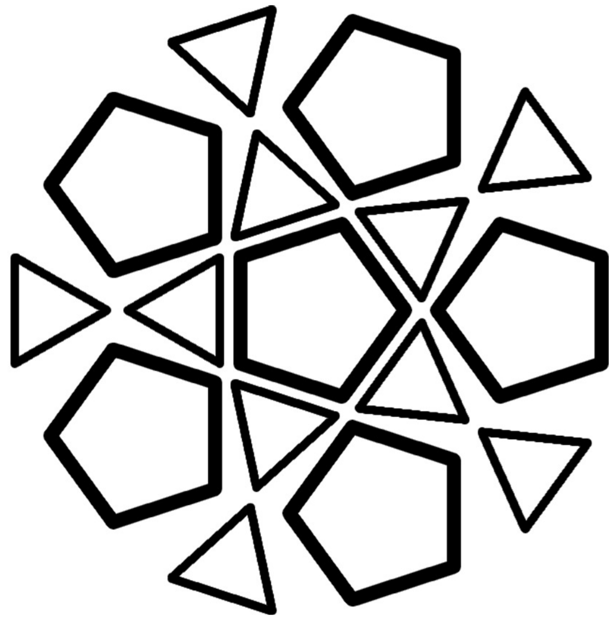


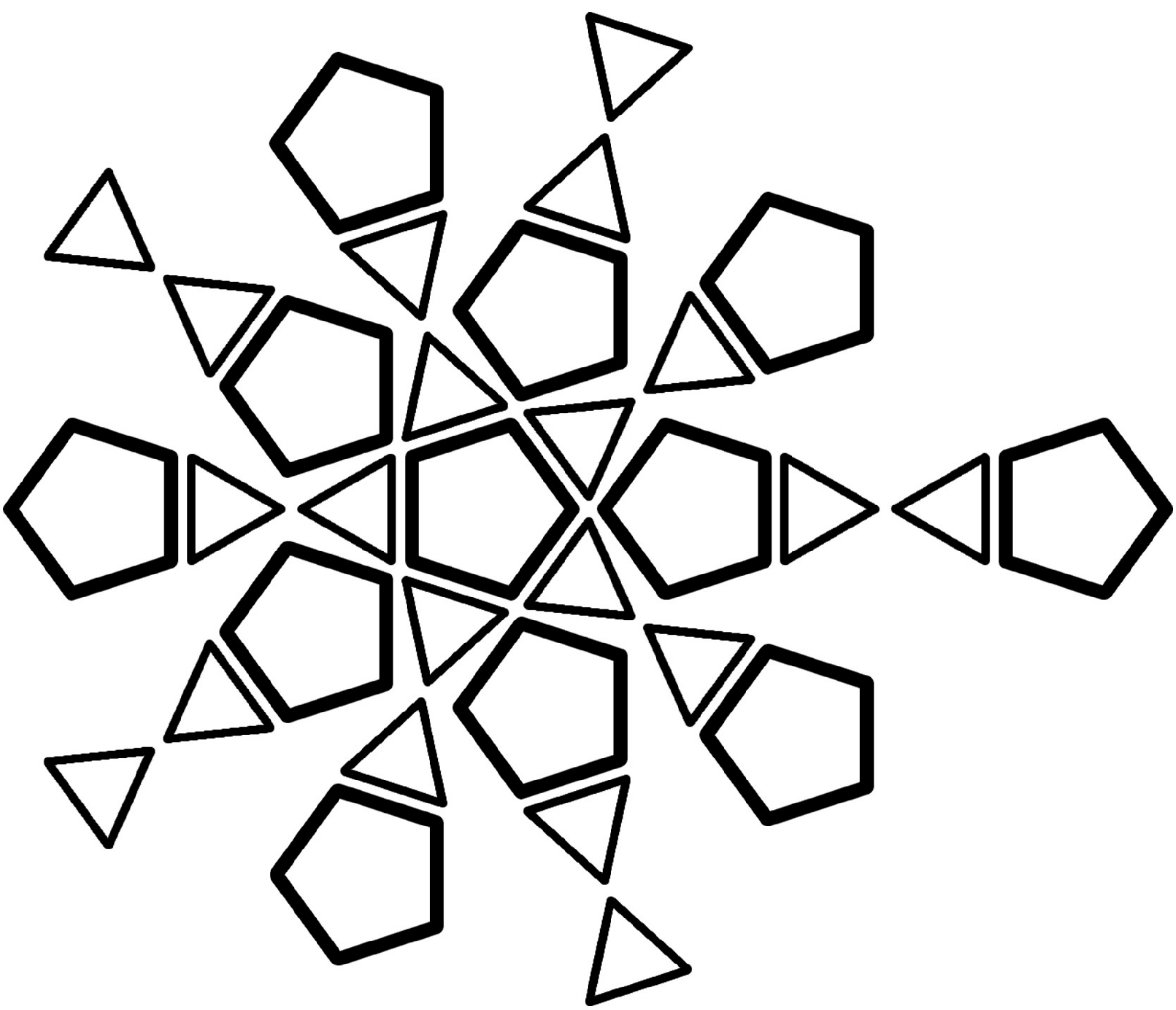












Equidistant Design: Pattern Sizing & Drawing Instructions

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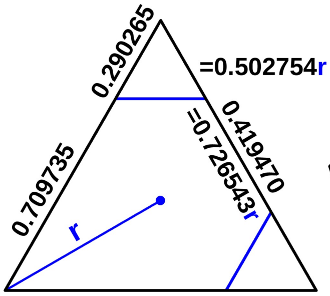

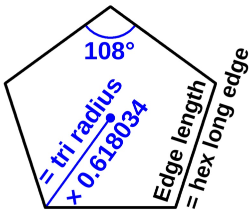

*This section begins with a summary of the design and associated mathematics, then sizing formulas and tables of pre-calculated pattern measurements for drawing custom pattern sizes along with drawing instructions. I provide printable measuring tapes at the end of the **General Information and Techniques** chapter in case you care to measure your beanbags.* ***The patterns for this variation are in the Ready-to-Print Patterns section.***

The main, Equidistant design has a hex panel that is about the same size as the pent panel, which improves the look of color arrangements that do not distinguish between them. **The hex's short edge is 0.691982 of the long edge.**

Color Arrangements

The color arrangement diagrams for the 32-panel designs are in a separate document titled "32-Panel Color Arrangements". Below are examples of the ball illustrations.

Design summary and mathematics

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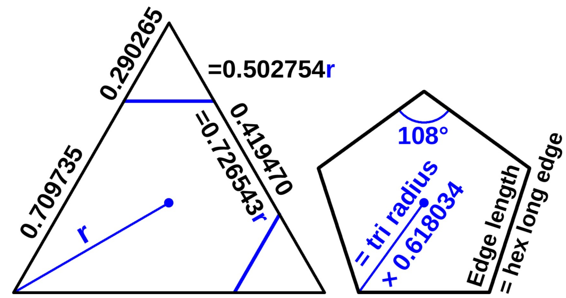
This section shows the mathematics behind the relationship between the pattern dimensions and the ball size. *The numbers in tiny, right-justified typeface are my computer calculator's unrounded values which I display rounded to six places for brevity.*

The key element of the design is the hexagon, which is derived by truncating an equilateral triangle by a specific amount. The pentagon is based simply on the hexagon's long side. The pattern illustrations on the right show the ratios of the triangle to the derived hex, and to the pentagon. For a full discussion of how I created this design, see the "How I Developed This Design" section.



High precision ratios used in my calculations: 0.70973476206080316161762611725617
0.29026523793919683838237388274383   0.50275413978175820381598979201266
0.41946952412160632323525223451234   0.72654252800536088589546675748056
0.61803398874989484820458683436562   Short edge proportion of the long: 0.69198170843764918443329639249149

To define the pattern dimensions in terms of the beanbag size they produce, I need first to define all pattern dimensions and the polyhedron's circumference in common terms, which will be the side length of the starting triangle from which the hex is derived. For the equations I will define the following variables.

h_h = height of the hexagon
h_p = height of the pentagon
r_p = circumradius of the pentagon
r_t = circumradius of the starting triangle
s_t = side length of the starting triangle
s_{hs} = short side length of the hexagon, and the truncation applied to s_t
s_L = hexagon's long side length, and pentagon's side length

Now I will evaluate each of those in terms of the starting triangle's side length, s_t. The values in blue are from the diagrams above.

$$h_h = \textit{triangle height} - \textit{truncated corner height} \approx \frac{\sqrt{3}}{2}s_t - \frac{\sqrt{3}}{2}(0.290265 s_t) \approx 0.614648 s_t$$ 0.61464833389355954485572827474661

$$h_p = \frac{\sqrt{5+2\sqrt{5}}}{2}s_L \approx \left(\frac{\sqrt{5+2\sqrt{5}}}{2}\right)(0.419470 s_t) \approx 0.645497 s_t$$ 0.64549722436790281419654423329701

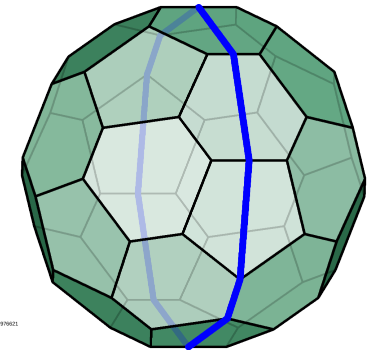
$$r_p = \sqrt{\frac{5+\sqrt{5}}{10}}s_L \quad \text{or} \quad \frac{0.5}{\sin 36°}s_L \approx \frac{0.5}{\sin 36°}(0.419470 s_t) \approx 0.356822 s_t$$ 0.35682208977308993194196984304606

$$\approx \sqrt{3}(0.356822 s_t) \approx 0.618034 r_t$$ 0.61803398874989484820458683436559

$$r_t = \frac{1}{\sqrt{3}}s_t \approx 0.577350 s_t$$ 0.57735026918962576450914878050196

$s_{hs} \approx 0.290265 s_t$ (this is the definition of my equidistant hexagon) 0.29026523793919683838237388274383

$s_L \approx s_t - 2(0.290265 s_t) \approx 0.419470 s_t$ 0.41946952412160632323525223451234



I will define the circumference of the polyhedron as (4 × *height of the pentagon*) + (4 × *height of the hexagon*) + (2 × *hex short edge*). My goal is to define it in terms of the starting triangle from which the hex is derived.

Circumference = $4h_p + 4h_h + 2s_{hs}$

So,
Circumference ≈ $4(0.645497s_t) + 4(0.614648s_t) + 2(0.290265s_t)$
≈ 5.621113s_t

5.6211127089242431129738377976621

I can use that value to express the circumference in terms of the triangle's circumradius:
Circumference ≈ $\sqrt{3}(5.621113\,s_t)$ **≈ 9.736053r_t**

9.7360528069279147701560836250244

Starting triangle and pentagon expressed in terms of ball size

To express the starting triangle in terms of a desired ball size, which is necessary to draw the patterns, I will solve the above expressions for s_t and r_t to express the triangle in terms of the ball Circumference, C, and then the Diameter, D (by multiplying the right sides of the equations by π). The pentagon's side length and circumradius are calculated from the starting triangle using the ratios given previously.

Starting Triangle Side Length, s_t ≈ 0.177901C 0.1779007203346715894453107633685
≈ 0.558892D 0.55889159607173660854024363104935

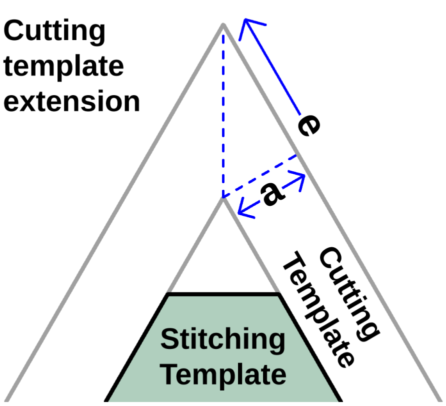
Starting Triangle Circumradius, r_t ≈ 0.102711C 0.10271102877425097229309810066588
≈ 0.322676D 0.32267621343983672042069878548581

Length of Pentagon Side & Hex Long Side, s_L ≈ *Triangle Side Length*(0.419470) 0.41946952412160632323525223451234

Pentagon Circumradius, r_p ≈ *Triangle Circumradius*(0.618034) 0.61803398874989484820458683436562

Cutting pattern adjustments

Designing the cutting patterns requires trigonometry. In the diagrams and equations below, ***e*** is the amount to extend one end of each side to get a seam allowance ***a***. So **2*e*** is the full amount by which to extend each side for the cutting pattern. In the first two diagrams, the triangles outlined in blue dashed lines are 30-60-90 right triangles. Note that I drew those two diagrams for the 14-panel hexagon (I didn't want to draw new diagrams for this design), so the seam allowance is a bit out of proportion to the hexagon's short side. The third diagram is for the pentagon.

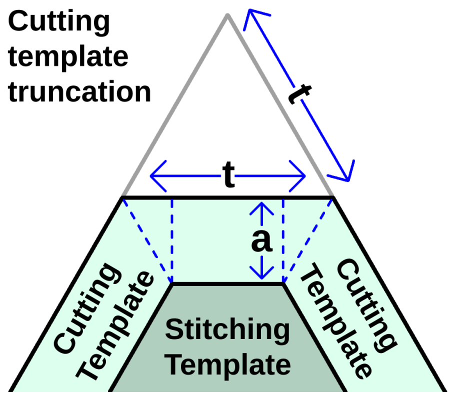


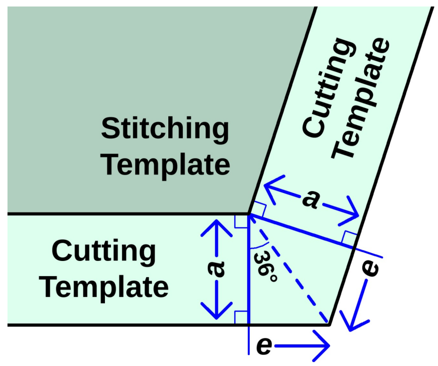
Hex Triangle Adjustment

Side Length Extension, 2*e* = $2(\tan 60°)$ or $2\sqrt{3}a$
≈ 3.464102*a*

3.4641016151377545870548926830117

Circumradius increase = $\frac{1}{\sin 30°}$ or $2\sqrt{3}(\frac{\sqrt{3}}{3})a$ **= 2*a***





Hex Truncation Adjustment

t is the amount of truncation to apply to each end of each side. It equals the short side of the stitching pattern (which is the same as the stitching truncation) plus the tops of the two triangles in blue dashed lines.

Truncation, *t* = *stitching truncation* + 2(tan 30°) or $\frac{2}{\sqrt{3}}a$

≈ *stitching truncation* + 1.154701*a* 1.1547005383792515290182975610039

Pent Adjustment

Side Length increase, 2*e* = $2(\tan 36°)a$ **≈ 1.453085*a*** 1.4530850560107217717909335149612

Circumradius increase = $\frac{1}{\cos 36°}a$ **≈ 1.236068*a*** 1.2360679774997896964091736687312

Adjusting for the influence of fabric attributes on beanbag size

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The bag I made with thick corduroy was 1.54 – 6.08% larger than the mathematical prediction depending on whether I filled it loosely or over-filled it. The moderately filled size was 3.90% larger. I target halfway between the min and max inflations when sizing my patterns, which is **3.8%**. This makes my adjustment factor **1.038**.

I use the adjustment factor to adjust the pattern size to produce a more accurate finished size when using my fabric and stitching techniques. If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may need to figure out your own adjustment factor. For help, see the **General Information and Techniques** chapter under "Adjusting/Scaling a Pattern to Produce an Accurate Ball Size".

The bag I made with my design testing fabric (fairly thin, stiff, tightly-woven, non-stretch) had an inflation of -2.93 – +0.98%, for an average of -0.975%, which is a *deflation*, or negative inflation (adjustment factor = 0.990). The moderately tightly filled size was -2.44%. The bag I made years ago with a thick, stiff, non-stretch denim measured -4.0 – +1.0%, for an average of -1.5% (adjustment factor = 0.985). So if you are using fabrics like these, expect the bag to be quite a bit smaller than with corduroy, and use the Base value in the measurement tables rather than the Adjusted value.

As I understand it, the bag size is affected by fabric attributes as follows. The folding of the fabric at the seams will cause thick, firm fabrics to significantly shrink the bag size unless the fabric has some stretch. Folding thin fabric doesn't consume as much of its size, but my design testing fabric, though fairly thin, has no stretch at all, and so ended up producing about the same size bag as the denim, which stretches a little. Corduroy is a softer, more loosely woven fabric than denim and flexes and compresses more easily, and so not as much of the panels' size is consumed by the folding. My denim and design testing fabric bags have very prominent seams while the corduroy bag has much more subtle seams.

Sizing formulas

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Below are the formulas to calculate the pattern construction elements (*Diameter* and *Circumference* are your target ball size). The value in orange is the adjustment factor. **Don't forget to multiply the final result by 25.4 if you need to convert inches to millimeters.**

Hex Panel

- **Starting Triangle Side Length = *Diameter* × 0.5589 ÷ 1.038**
  = *Circumference* × 0.1779 ÷ 1.038
- **Starting Triangle Circumradius = *Diameter* × 0.3227 ÷ 1.038**
  = *Circumference* × 0.1027 ÷ 1.038
- **Truncation to form the Hexagon = *Side Length* × 0.2903 or *Circumradius* × 0.5028**
- For double-checking: **Hex Height = *Triangle Side*** × **0.6146**
  = *Triangle Circumradius* × **1.0646**

Pent Panel

- **Pentagon Side Length = *Triangle Side Length* × 0.4195**
- **Pentagon Circumradius = *Triangle Circumradius* × 0.6180**
- For double-checking: **Pent Height = *Pent Side*** × **1.5388**
  = *Pent Circumradius* × **1.8090**

Drawing instructions begin on the next page

How to Draw the Equidistant Hexagon

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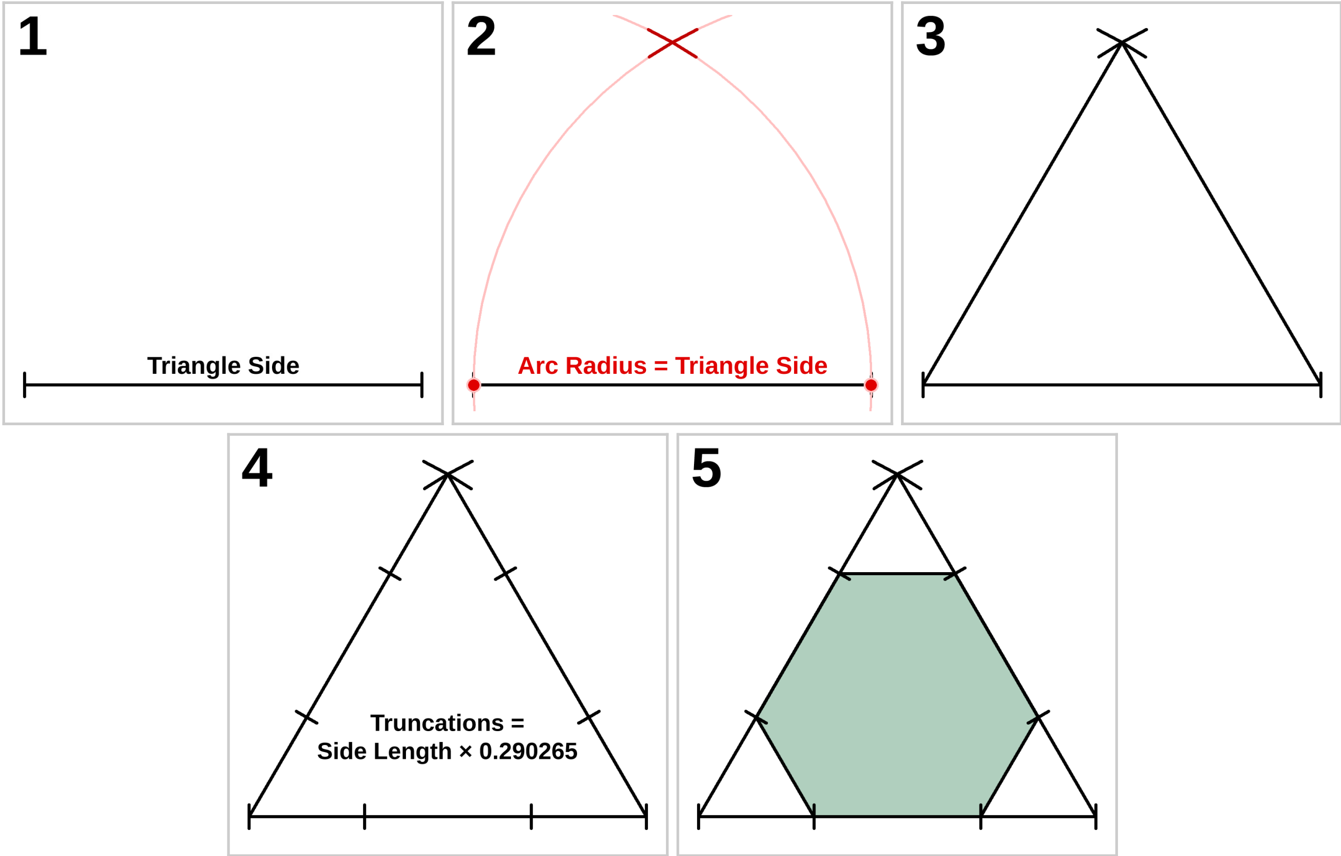
Following the pattern measurement table are manual and SketchUp directions for drawing the hex panel shape. To conserve your template material, I recommend that you draw the pattern on paper and then glue or tape the pattern to your template material before cutting it out.

Hexagon pattern measurement table

The table below has stitching pattern measurements for each ⅛″ diameter increment from 1¾″ – 3″. The values in the **Adjusted** columns account for my 1.038 adjustment factor. The adjusted values decrease the **Base** pattern size so that you will get a more accurate finished size when using corduroy or something similar (a soft, flexible, moderately thick fabric). If you are using a firm denim or a thin, but non-stretch fabric, use the Base value instead. I attempt to explain why in the “Adjusting for the influence of fabric attributes on beanbag size” topic a little earlier in this chapter.

To draw the cutting pattern, increase the Triangle Side Length by the desired allowance × $2\sqrt{3} \approx 3.4641$, or increase the Triangle Circumradius by the allowance × 2. Increase the Truncation by the allowance × $2/\sqrt{3} \approx 1.1547$.

Finished Diameter	**Triangle Side Length** (mm)		**Triangle Circumradius** (mm)		**Truncation** (mm)		**Hex Height** (for double-checking) (mm)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
1¾″ (44.5mm)	24.843	**23.933**	14.343	**13.818**	7.211	**6.947**	15.270	**14.711**
1⅞″ (47.6mm)	26.617	**25.643**	15.367	**14.805**	7.726	**7.443**	16.360	**15.761**
2″ (50.8mm)	28.392	**27.352**	16.392	**15.792**	8.241	**7.939**	17.451	**16.812**
2⅛″ (54.0mm)	30.166	**29.062**	17.416	**16.779**	8.756	**8.436**	18.542	**17.863**
2¼″ (57.2mm)	31.941	**30.771**	18.441	**17.766**	9.271	**8.932**	19.632	**18.914**
2⅜″ (60.3mm)	33.715	**32.481**	19.465	**18.753**	9.786	**9.428**	20.723	**19.964**
2½″ (63.5mm)	35.490	**34.190**	20.490	**19.740**	10.301	**9.924**	21.814	**21.015**
2⅝″ (66.7mm)	37.264	**35.900**	21.514	**20.727**	10.816	**10.420**	22.904	**22.066**
2¾″ (69.9mm)	39.039	**37.609**	22.539	**21.714**	11.332	**10.917**	23.995	**23.117**
2⅞″ (73.0mm)	40.813	**39.319**	23.563	**22.701**	11.847	**11.413**	25.086	**24.167**
3″ (76.2mm)	42.588	**41.028**	24.588	**23.688**	12.362	**11.909**	26.176	**25.218**



Illustrations for the manual directions. The numbers correspond to the step numbers.

Manual directions for the Hexagon

(The terms in bold refer to columns in the pattern measurement table above.)

1. Draw a horizontal line the length of **Triangle Side Length** and mark each end of it.
2. Place a compass on one end of the line, extend it to the other end, and draw a small arc above the center of the line. Draw the same arc from the other end of the line. The resulting X-shaped intersection marks the third corner of the triangle.
3. Draw lines from each end of the first line to the X, forming an equilateral triangle.
4. Measure a distance equal to **Truncation** inward from each end of each side as shown in Illustration 4.
5. Join the truncation marks to form the hexagonal panel shape. Its height, from each long edge to opposite short edge, should equal **Hex Height**. Any error you make will be compounded many times in the juggling bag, so be as precise as you can.
6. To draw a cutting pattern, multiply the desired allowance by 3.4641 and add that to the **Triangle Side Length**, then multiply the allowance by 1.1547 and add that to the **Truncation**. Or, just draw the cutting pattern around the stitching pattern, using its edges as guides.

SketchUp directions for the Hexagon

(The terms in bold refer to columns in the pattern measurement table.)

1. Use the polygon tool (in the Shapes tool drop-down, or in Draw menu -> Shapes) set to 3 sides and draw a triangle with circumradius = **Triangle Circumradius**, which will result in a triangle with sides of length **Triangle Side Length**.
2. Draw lines of length **Truncation** inward from each end of each side (their endpoints will match the locations of the truncation marks in Illustration 4 of the manual instructions).
3. Join the pairs of truncation endpoints across each of the triangle's corners, forming the hexagonal panel shape as shown in Illustration 5. Erase the triangle's corners. The resulting panel shape's height, from each long edge to opposite short edge, should equal **Hex Height**.
4. To draw a cutting pattern, multiply the desired allowance by 2 and add that to the **Triangle Circumradius**, or by 3.4641 and add that to the **Triangle Side Length**, and multiply it by 1.1547 and add that to the **Truncation**. Or, just draw the cutting pattern around the stitching pattern, using its edges as guides.

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How to Draw the Pentagon

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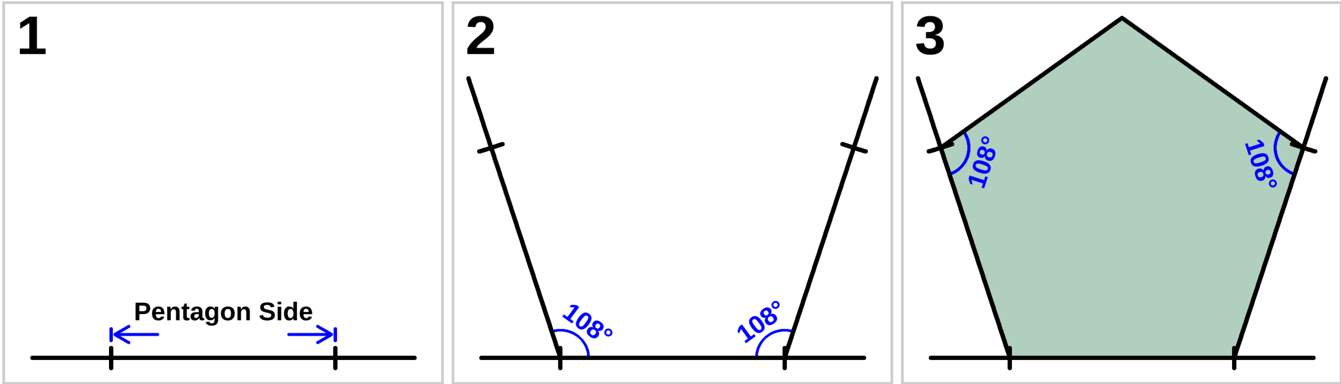
Following the pattern measurement table are manual and SketchUp directions for drawing the pentagon panel shape. To conserve your template material, I recommend that you draw the pattern on paper and then glue or tape the pattern to your template material before cutting it out.

Pentagon pattern measurement table

Below are stitching pattern measurements for each ⅛″ diameter increment from 1 ¾″ – 3″. The values in the **Adjusted** columns account for my 1.038 adjustment factor. The adjusted values decrease the **Base** pattern size so that you will get a more accurate finished size when using corduroy or something similar (a soft, flexible, moderately thick fabric). If you are using a firm denim or a thin, but non-stretch fabric, use the Base value instead. I attempt to explain why in the "Adjusting for the influence of fabric attributes on beanbag size" topic a little earlier in this chapter.

To draw a cutting pattern, increase the Pentagon Side Length by the desired allowance × 2(tan 36°) ≈ 1.4531, or increase the Pentagon Circumradius by the allowance × 1/cos 36° ≈ 1.2361.

Finished Diameter	**Pentagon Side Length** (mm)		**Pentagon Circumradius** (mm)		**Pentagon Height** (mm) (for double-checking)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
1¾″ (44.5mm)	10.421	**10.039**	8.864	**8.540**	16.036	**15.449**
1⅞″ (47.6mm)	11.165	**10.756**	9.498	**9.150**	17.181	**16.552**
2″ (50.8mm)	11.909	**11.473**	10.131	**9.760**	18.327	**17.656**
2⅛″ (54.0mm)	12.654	**12.191**	10.764	**10.370**	19.472	**18.759**
2¼″ (57.2mm)	13.398	**12.908**	11.397	**10.980**	20.618	**19.863**
2⅜″ (60.3mm)	14.142	**13.625**	12.030	**11.590**	21.763	**20.966**
2½″ (63.5mm)	14.887	**14.342**	12.663	**12.200**	22.908	**22.070**
2⅝″ (66.7mm)	15.631	**15.059**	13.297	**12.810**	24.054	**23.173**
2¾″ (69.9mm)	16.375	**15.776**	13.930	**13.420**	25.199	**24.277**
2⅞″ (73.0mm)	17.120	**16.493**	14.563	**14.030**	26.345	**25.380**
3″ (76.2mm)	17.864	**17.210**	15.196	**14.640**	27.490	**26.484**



Illustrations for the manual directions. The numbers correspond to the step numbers.

Manual directions for the Pentagon

(The terms in bold refer to columns in the pattern measurement table above.

1. Draw a horizontal line the length of **Pentagon Side Length** and mark each end of it. Continue the line on either side of the marks to aid in accurately aligning a protractor to it.

2. Place a protractor on the line, center it on each of the endpoint marks in turn, and mark points at 108° angles to the base line. Draw a line from each endpoint through the respective angle mark, and mark each line at the distance of **Pentagon Side Length** from the starting side's endpoints, forming half of the guide pentagon. Continue each line beyond the endpoint to help align the protractor to them.

3. Place the protractor on each new line, centered on the new endpoints, and mark points at 108° as in step 2. Draw the two final sides, completing the pentagon. Make sure the lines meet exactly **Pentagon Side Length** from the previous sides' endpoints. If they do, you drew a perfect pentagon. Any error you make will be compounded many times in the juggling bag, so be as precise as you can. You can also measure the pentagon's height from corner to side and compare it to **Pentagon Height**.

4. To draw a cutting pattern, multiply the desired allowance by 1.4531 and add that to the side length. Or, just draw the cutting pattern around the stitching pattern, using its edges as guides.

SketchUp directions for the Pentagon

(The terms in bold refer to columns in the pattern measurement table above.)

1. Use the polygon tool (in the Shapes tool drop-down, or in Draw menu -> Shapes) set to 5 sides and draw a pentagon with circumradius = **Pentagon Circumradius**, which will result in a pentagon with sides of length **Pentagon Side Length**.

2. To draw a cutting pattern, multiply the desired allowance by 1.2361 and add that to the **Pentagon Circumradius**, or by 1.4531 and add that to the **Pentagon Side Length**.

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14-Panel/Footbag-Style Hex Variation: Pattern Sizing & Drawing Instructions

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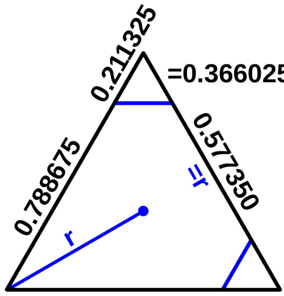

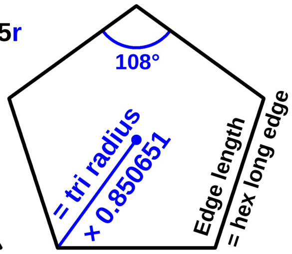

Photos from http://modified.in/footbag/

*This section begins with a summary of the design and associated mathematics, then sizing formulas for drawing custom pattern sizes, pre-calculated pattern measurement tables for a range of standard sizes, and finally, links to the pattern drawing instructions. I provide printable measuring tapes at the end of the **General Information and Techniques** chapter in case you care to measure your beanbags. **The patterns for this variation are in the Ready-to-Print Patterns section.***

If you like the look of a **smaller and more triangular hexagon**, the hexagon I designed for the 14-panel Equidistant Cuboctahedron will work well. This is the typical type of hex used for footbags. **The short edge is 0.366 of the long edge**[3] as opposed to 0.692 in my Equidistant design.

Color Arrangements

The color arrangement diagrams for the 32-panel designs are in a separate document titled "32-Panel Color Arrangements".

This design variation **works better for creature and character faces**. I used it to make the Turtle Ball shown on the left. I wrote an essay about the Turtle Ball further on in this chapter which includes detailed photos and assembly diagrams. In the "32-Panel Color Arrangements" document I provide illustrations for a ladybug, and the photos below show some cartoon and video game characters from https://www.haniabag.com/shop/32-panels/32-panel-footbag-custom/.

3 The patterns I can currently find online use a short edge that is 0.35, 0.4, or 0.5 of the long edge. See https://www.youtube.com/watch?v=xQjZHPBL9F8 and http://modified.in/footbag/viewtopic.php?t=10200. A pattern generator I found (no longer available) defaulted to 0.33. I am not aware of any definitive design for this hexagon.

Design summary and mathematics

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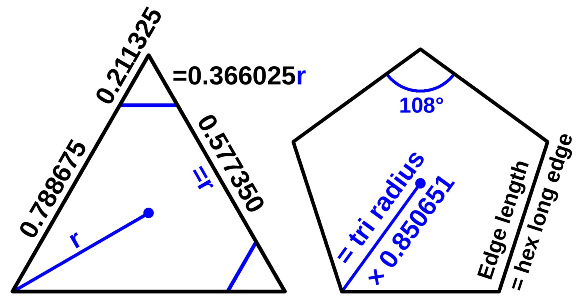
This section shows the mathematics behind the relationship between the pattern dimensions and the ball size. *The numbers in tiny, right-justified typeface are my computer calculator's unrounded values which I display rounded to six places for brevity.*

The key element of the design is the hexagon, which is derived by truncating an equilateral triangle by a specific amount. The pentagon is based simply on the hexagon's long side, which, for this particular hexagon, is equal to the triangle's circumradius. The pattern diagrams on the right show the ratios of the triangle to the hex, and to the pentagon. For a full discussion of how I created this design, see the "How I Developed This Design" section of the 14-Panel Equidistant Cuboctahedron chapter.



High precision ratios used in my calculations: 0.211324865405187117745425609749 0.788675134594812882254574390251
0.577350269189625764509148780502 0.36602540378443864676372317075287 0.85065080835203993218154049706301
Short edge proportion of the long: 0.36602540378443864676372317075287

To define the pattern dimensions in terms of the beanbag size they produce, I need first to define all pattern dimensions and the polyhedron's circumference in common terms, which will be the side length of the starting triangle from which the hex is derived. For the equations I will define the following variables.

h_h = height of the hexagon
h_p = height of the pentagon
r_p = circumradius of the pentagon
r_t = circumradius of the triangle
s_t = side length of the starting triangle (the hex's long side and the pent's side is $0.577350s_t$)
s_{hs} = short side length of the hexagon, which is defined as $0.211325s_t$
s_L = hexagon's long side length, and pentagon's side length

Now I will evaluate each of those in terms of the starting triangle's side length, s_t. The values in blue are from the diagram above.

h_h = *triangle height – truncated corner height* $\approx \frac{\sqrt{3}}{2}s_t - \frac{\sqrt{3}}{2}(0.211325s_t) \approx 0.683013s_t$ 0.68301270189221932338186158537649

$h_p = \frac{\sqrt{5+2\sqrt{5}}}{2}s_L \approx \left(\frac{\sqrt{5+2\sqrt{5}}}{2}\right)(0.577350s_t) \approx 0.888451s_t$ 0.88845070933430607288560060178146

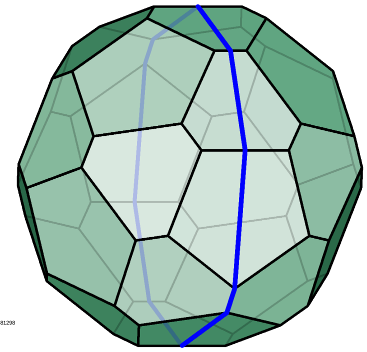
$r_p = \sqrt{\frac{5+\sqrt{5}}{10}}s_L$ or $\frac{0.5}{\sin 36°}s_L \approx \frac{0.5}{\sin 36°}(0.577350s_t) \approx 0.491123s_t$ 0.49112347318842301140783513761764

$\approx \sqrt{3}(0.491123\,s_t) \approx 0.850651r_t$ 0.85065080835203993218154049706308

$r_t = \frac{1}{\sqrt{3}}s_t \approx 0.577350s_t$ 0.57735026918962576450914878050196

$s_{hs} \approx 0.211325s_t$ (this is the definition of my 14-panel design's equidistant hexagon) 0.211324865405187117745425609749

$s_L \approx s_t - 2(0.211325s_t) \approx 0.577350s_t$ 0.577350269189625764509148780502



I will define the circumference of the polyhedron as (4 × *height of the pentagon*) + (4 × *height of the hexagon*) + (2 × *hex short edge*). My goal is to define it in terms of the starting triangle from which the hex is derived.

Circumference = $4h_p + 4h_h + 2s_{hs}$

So,

Circumference ≈ $4(0.888451s_t) + 4(0.683013s_t) + 2(0.211325s_t)$

≈ 6.708503s_t 6.70850337571647582056069996812 98

I can use that value to express the circumference in terms of the triangle's circumradius:

Circumference ≈ $\sqrt{3}(6.708503\,s_t)$ ≈ **11.619469r_t** 11.619468689488261392195473835085

Starting triangle and pentagon expressed in terms of bag size

To express the starting triangle in terms of a desired ball size, which is necessary to draw the patterns, I will solve the above expressions for s_t and r_t to express the triangle in terms of the ball Circumference, C, and then the Diameter, D (by multiplying the right sides of the equations by π). The pentagon's side length and circumradius are calculated from the starting triangle using the ratios given previously.

Starting Triangle Side Length, s_t ≈ 0.149065C 0.1490645445032959915832932000836

≈ 0.468300D 0.4683000779222634818849815246718 4

Starting Triangle Circumradius, r_t ≈ 0.086062C 0.0860624548956068903398815989462

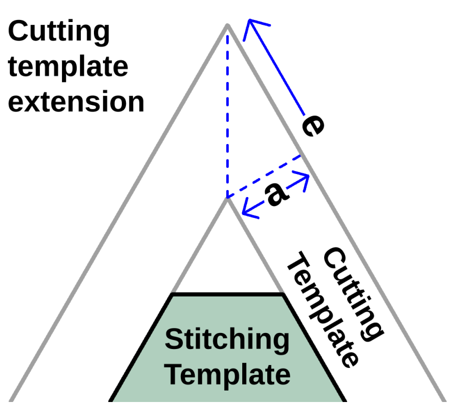
≈ 0.270373D 0.2703731760499415426508738692741 8

Pentagon Side Length, s_L ≈ *Triangle Side Length*(0.577350) 0.5773502691896257645091487805 02

Pentagon Circumradius, r_p ≈ *Triangle Circumradius*(0.850651) 0.8506508083520399321815404970630 1

Cutting pattern adjustments

Designing the cutting patterns requires trigonometry. In the diagrams and equations below, ***e*** is the amount to extend one end of each side to get a seam allowance ***a***. So **2*e*** is the full amount by which to extend each side for the cutting pattern. In the first two diagrams, the triangles outlined in blue dashed lines are 30-60-90 right triangles.

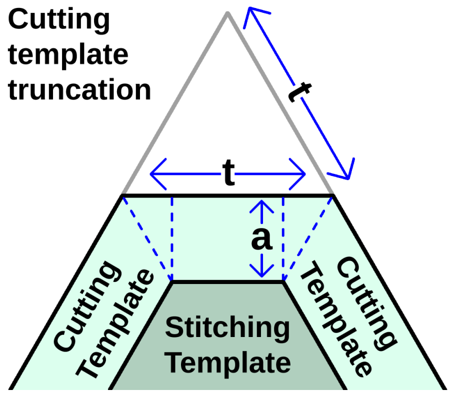


Hex Triangle Adjustment

Side Length Extension, 2*e* = $2(\tan 60°)$ or $2\sqrt{3}a$

≈ 3.464102*a* 3.46410161513775458705489268301 17

Circumradius increase = $\frac{1}{\sin 30°}a$ or $2\sqrt{3}(\frac{\sqrt{3}}{3})a$ **= 2*a***



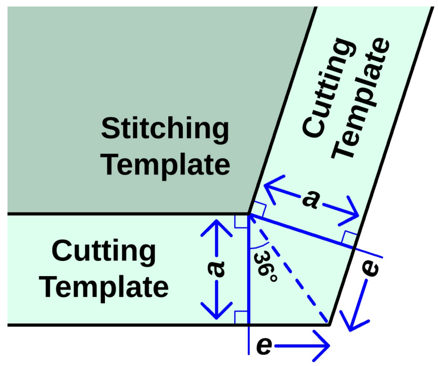
Hex Truncation Adjustment

t is the amount of truncation to apply to each end of each side. It equals the short side of the stitching pattern (which is the same as the stitching truncation) plus the tops of the two triangles in blue dashed lines.

$$\textbf{Truncation, } t = \textit{stitching truncation} + 2(\tan 30°) \text{ or } \frac{2}{\sqrt{3}}a$$

$$\approx \textit{stitching truncation} + 1.154701a$$

1.1547005383792515290182975610039



Pentagon Adjustment

$$\textbf{Side Length Extension, } 2e = 2(\tan 36°)a \approx \mathbf{1.453085}a$$

1.4530850560107217717909335149612

$$\textbf{Circumradius increase} = \frac{1}{\cos 36°}a \approx \mathbf{1.236068}a$$

1.2360679774997896964091736 6

Adjusting for the influence of fabric attributes on beanbag size

The equidistant 32-panel bag I made with thick corduroy was 1.54 – 6.08% larger than the mathematical prediction depending on whether I filled it loosely or over-filled it. I target halfway between the min and max inflations when sizing my patterns, which is **3.8%**. (I'm assuming the inflation will be the same for this variation.) This makes my adjustment factor **1.038**.

I use the adjustment factor to adjust the pattern size to produce a more accurate finished size when using my fabric and stitching techniques. If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may need to figure out your own adjustment factor. For help, see the **General Information and Techniques** chapter under "Adjusting/Scaling a Pattern to Produce an Accurate Ball Size".

The bag I made with my design testing fabric (fairly thin, stiff, tightly-woven, non-stretch) had an inflation of -2.93 – +0.98%, for an average of -0.975%, which is a *deflation* (adjustment factor = 0.990). The moderately tightly filled size was -2.44%. The bag I made years ago with a thick, stiff, non-stretch denim measured -4.0 – +1.0%, for an average of -1.5% (adjustment factor = 0.985). So if you are using fabrics like these, expect the bag to be quite a bit smaller than with corduroy, and use the Base value in the measurement tables rather than the Adjusted value.

As I understand it, the bag size is affected by fabric attributes as follows. The folding of the fabric at the seams will cause thick, firm fabrics to significantly shrink the bag size unless the fabric has some stretch. Folding thin fabric doesn't consume as much of its size, but my design testing fabric, though fairly thin, has no stretch at all, and so ended up producing about the same size bag as the denim, which stretches a little. Corduroy is a softer, more loosely woven fabric than denim and flexes and compresses more easily, and so not as much of the panels' size is consumed by the folding. My denim and design testing fabric bags have very prominent seams while the corduroy bag has much more subtle seams.

Sizing formulas

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Below are the formulas to calculate the pattern construction elements (*Diameter* and *Circumference* are your target ball size). The value in orange is the adjustment factor. **Don't forget to multiply the final result by 25.4 if you need to convert inches to millimeters.**

Hex Panel

- **Starting Triangle Side Length = *Diameter* × 0.4683 ÷ 1.038**
  = *Circumference* × 0.1491 ÷ 1.038
- **Starting Triangle Circumradius = *Diameter* × 0.2704 ÷ 1.038**
  = *Circumference* × 0.0861 ÷ 1.038
- **Truncation to form the Hexagon = *Circumradius* × 0.3660 or *Side Length* × 0.2113**
- For double-checking: **Resulting Hex Long Side = Triangle Circumradius**
- For double-checking: **Hex Height = *Triangle Side Length* × 0.6830**
  = *Triangle Circumradius* × 1.1830

Pent Panel

- **Pentagon Side Length = [same as Starting Triangle Circumradius above]**
- **Pentagon Circumradius = *Triangle Circumradius* × 0.8507**
- For double-checking: **Pent Height = *Pent Side* × 1.5388**
  = *Pent Circumradius* × 1.8090

Hexagon pattern measurement table

Below are stitching pattern measurements for each ⅛″ diameter increment from 1 ¾″ – 3″. The values in the **Adjusted** columns account for my 1.038 adjustment factor. The adjusted values decrease the **Base** pattern size so that you will get a more accurate finished size when using corduroy or something similar (a soft, flexible, moderately thick fabric). If you are using a firm denim or a thin, but non-stretch fabric, use the Base value instead. I attempt to explain why in the "Adjusting for the influence of fabric attributes on beanbag size" topic above.

To draw the cutting pattern, increase the Triangle Side Length by the desired allowance × $2\sqrt{3}$ ≈ 3.4641, or increase the Triangle Circumradius by the allowance × 2. Increase the Truncation by the allowance × $2/\sqrt{3}$ ≈ 1.1547.

Finished Diameter	**Triangle Side Length** (mm)		**Triangle Circumradius** (mm)		**Truncation** (mm)		**Hex Height** (mm) (for double-checking)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
1¾″ (44.5mm)	20.816	**20.054**	12.018	**11.578**	4.399	**4.238**	14.218	**13.697**
1⅞″ (47.6mm)	22.303	**21.486**	12.877	**12.405**	4.713	**4.541**	15.233	**14.675**
2″ (50.8mm)	23.790	**22.919**	13.735	**13.232**	5.027	**4.843**	16.249	**15.654**
2⅛″ (54.0mm)	25.276	**24.351**	14.593	**14.059**	5.342	**5.146**	17.264	**16.632**
2¼″ (57.2mm)	26.763	**25.784**	15.452	**14.886**	5.656	**5.449**	18.280	**17.611**
2⅜″ (60.3mm)	28.250	**27.216**	16.310	**15.713**	5.970	**5.751**	19.295	**18.589**

Finished Diameter	**Triangle Side Length** (mm)		**Triangle Circumradius** (mm)		**Truncation** (mm)		**Hex Height** (mm) (for double-checking)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
2½″ (63.5mm)	29.737	**28.648**	17.169	**16.540**	6.284	**6.054**	20.311	**19.567**
2⅝″ (66.7mm)	31.224	**30.081**	18.027	**17.367**	6.598	**6.357**	21.326	**20.546**
2¾″ (69.9mm)	32.711	**31.513**	18.886	**18.194**	6.913	**6.660**	22.342	**21.524**
2⅞″ (73.0mm)	34.198	**32.946**	19.744	**19.021**	7.227	**6.962**	23.357	**22.502**
3″ (76.2mm)	35.684	**34.378**	20.602	**19.848**	7.541	**7.265**	24.373	**23.481**

Pentagon pattern measurement table

To draw a cutting pattern, increase the Pentagon Side Length by the desired allowance × 2(tan 36°) ≈ 1.4531, or increase the Pentagon Circumradius by the allowance × 1/cos 36° ≈ 1.2361.

Finished Diameter	**Pentagon Side Length** (mm)		**Pentagon Circumradius** (mm)		**Pentagon Height** (mm) (for double-checking)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
1¾″ (44.5mm)	12.018	**11.578**	10.223	**9.849**	18.494	**17.817**
1⅞″ (47.6mm)	12.877	**12.405**	10.953	**10.552**	19.815	**19.090**
2″ (50.8mm)	13.735	**13.232**	11.684	**11.256**	21.136	**20.362**
2⅛″ (54.0mm)	14.593	**14.059**	12.414	**11.959**	22.457	**21.635**
2¼″ (57.2mm)	15.452	**14.886**	13.144	**12.663**	23.778	**22.907**
2⅜″ (60.3mm)	16.310	**15.713**	13.874	**13.366**	25.099	**24.180**
2½″ (63.5mm)	17.169	**16.540**	14.605	**14.070**	26.420	**25.453**
2⅝″ (66.7mm)	18.027	**17.367**	15.335	**14.773**	27.741	**26.725**
2¾″ (69.9mm)	18.886	**18.194**	16.065	**15.477**	29.062	**27.998**
2⅞″ (73.0mm)	19.744	**19.021**	16.795	**16.180**	30.383	**29.271**
3″ (76.2mm)	20.602	**19.848**	17.525	**16.884**	31.704	**30.543**

Drawing instructions

To draw the hexagon, follow the hex instructions in the Equidistant Design section, but use the measurements from the hex table above. Note that the truncation will look different because it is shorter for this hex. I recommend that you copy the measurements you intend to use so you do not need to jump back here.

To draw the pentagon, follow the pentagon instructions in the Equidistant Design section, but use the measurements from the table above. I recommend that you copy the measurements you intend to use so you do not need to jump back here.

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Icosidodecahedron (Triangles and Pents) Variation: Pattern Sizing & Drawing Instructions

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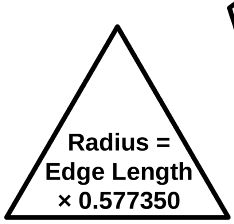
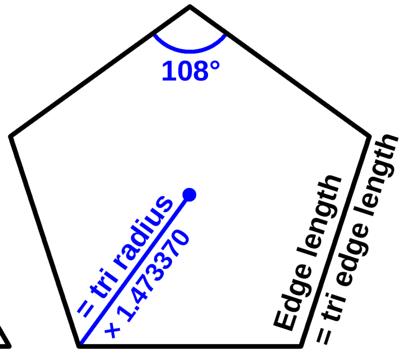
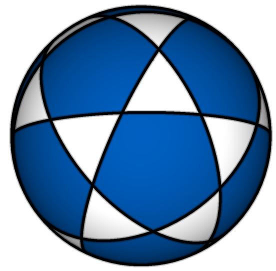






 4

 5

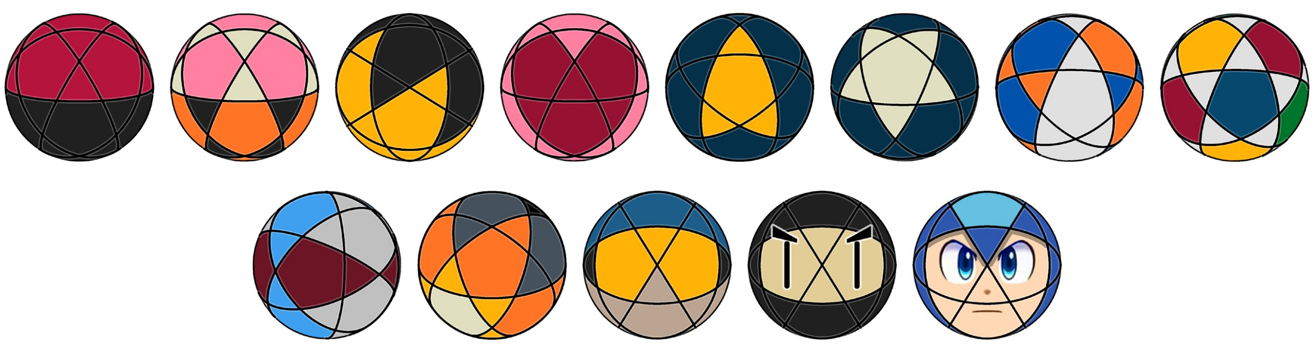
The high precision ratios are
0.57735026918962576450914878050196 1.4733704195652690342235054128528
(0.85065080835203993218154049706301 ÷ tri circumradius ratio)

This section begins with a summary of the design and associated mathematics, then sizing formulas for drawing custom pattern sizes, pre-calculated pattern measurement tables for a range of standard sizes, and finally, links to the pattern drawing instructions. I provide printable measuring tapes at the end of the ***General Information and Techniques*** *chapter in case you care to measure your beanbags.* ***The patterns for this variation are in the Ready-to-Print Patterns section.***

This design uses pentagons and triangles. **It is very versatile and supports many unique arrangements that do not work with hexagons.**

Color Arrangements

The color arrangement diagrams for the 32-panel designs are in a separate document titled "32-Panel Color Arrangements". Below are examples of the ball illustrations.



4 Photo from http://sportgam.com/beanbags-and-foot-bags/hero-black-neon-rainbow-32-panel-hacky-sack-footbag-comes-with-tips-game-instructions/

5 Photo from http://modified.in/footbag/

Design summary and mathematics

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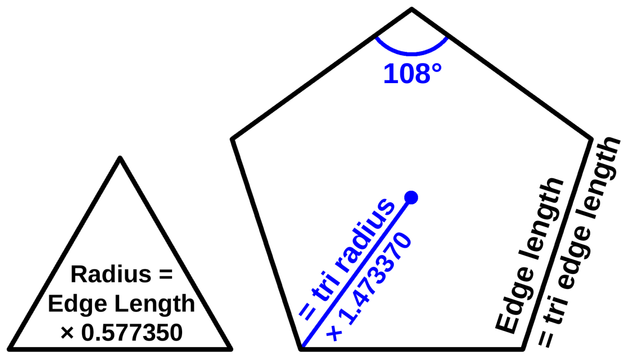
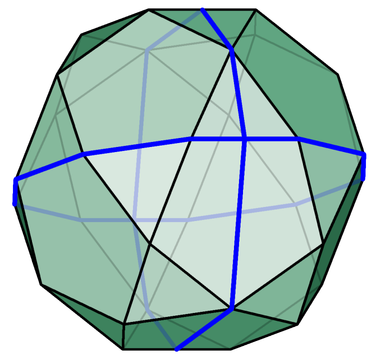
This section shows the mathematics behind the relationship between the pattern dimensions and the ball size. *The numbers in tiny, right-justified typeface are my computer calculator's unrounded values which I display rounded to six places for brevity.*

This design uses a triangle instead of a hexagon. The pattern illustrations on the right show the ratio of the triangle's circumradius to the pentagon's.



The high precision ratios are
0.5773502691896257645091487805019б
1.4733704195652690342235054128528
(0.85065080835203993218154049706301 ÷ tri circumradius ratio)

The circumference of the polyhedron can be calculated as (4 × *pent height*) + (4 × *triangle height*), or as 10 × *edge length*. To be consistent with the other variations, and because it is actually a more accurate predictor of the beanbag size, I will use the first measurement.



To define the pattern dimensions in terms of the beanbag size they produce, I need first to define all pattern dimensions and the polyhedron's circumference in common terms, which will be the side length of the faces. For the equations I will define the following variables.:

h_p = height of the pentagon
h_t = height of the triangle
s = side length of the triangle and pentagon
r_p = circumradius of the pentagon
r_t = circumradius of the triangle

Now I will evaluate each of those in terms of the starting triangle's side length, s_t.

$$h_p = \frac{\sqrt{5+2\sqrt{5}}}{2}s \approx 1.538842s$$

1.5388417685876267012851452880185

$$h_t = \frac{\sqrt{3}}{2}s \approx 0.866025s$$

0.86602540378443864676372317075294

$$r_p = \sqrt{\frac{5+\sqrt{5}}{10}}s \quad \text{or} \quad \frac{0.5}{\sin 36°}s \approx 0.850651s$$

0.85065080835203993218154049706301

$$r_t = \frac{1}{\sqrt{3}}s \approx 0.577350s$$

0.57735026918962576450914878050196

To reiterate, Circumference = $4h_p + 4h_t$

So,
Circumference ≈ $4(1.538842s) + 4(0.866025s)$ ≈ $\mathbf{9.619469}s$

9.6194686894882613921954738350857

I can use that value to express the circumference in terms of the triangle and pentagon's circumradius:
Circumference ≈ $\sqrt{3}\,(9.619469s)$ ≈ $\mathbf{16.661409}r_t$

16.661408512011672874627207221377

Circumference $\approx \dfrac{9.619469}{0.850651}s \approx \mathbf{11.308364}\boldsymbol{r_p}$ 11.308363661140807164017020575883

Triangle and pentagon expressed in terms of bag size

To express the triangle and pentagon in terms of a desired bag size, which is necessary to draw the patterns, I will solve the above expressions for s, r_t, and r_p to express them in terms of the ball Circumference, C, and then the Diameter, D (by multiplying the right sides of the equations by π).

Triangle and Pentagon Side Length, $\boldsymbol{s \approx 0.103956C}$ 0.10395584540887966855087114220256

$\boldsymbol{\approx 0.326587D}$ 0.32658692023425260240131699479191

Triangle Circumradius, $\boldsymbol{r_t \approx 0.060019C}$ 0.06001893533065179828732998558399

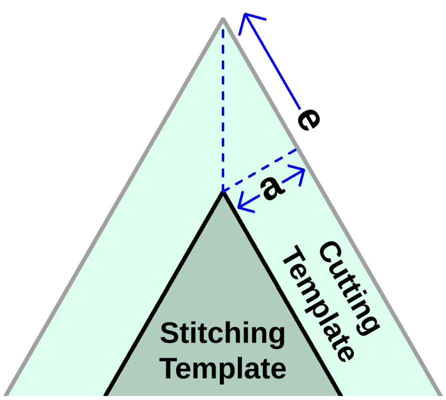
$\boldsymbol{\approx 0.188555D}$ 0.18855504631105657743845895376188

Pentagon Circumradius, $\boldsymbol{r_p \approx 0.088430C}$ 0.08843012392998318919518743277945

$\boldsymbol{\approx 0.277811D}$ 0.27781142769447016275942750393765

Cutting pattern adjustments

Designing the cutting patterns requires trigonometry. In the diagrams and equations below, ***e*** is the amount to extend one end of each side to get a seam allowance ***a***. So **2*e*** is the full amount by which to extend each side for the cutting pattern. In the first diagram, the triangle outlined in blue dashed lines is a 30-60-90 right triangle.

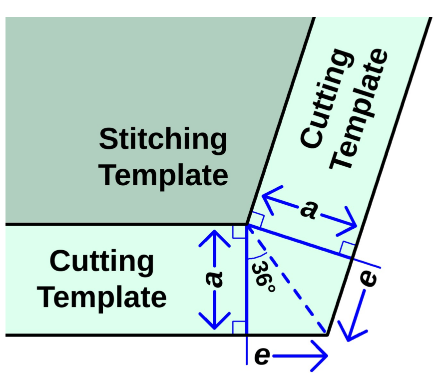


Triangle Adjustment

Side Length Extension, 2*e* = $2(\tan 60°)$ or $2\sqrt{3}a$

$\boldsymbol{\approx 3.464102a}$ 3.4641016151377545870548926830117

Circumradius increase = $\dfrac{1}{\sin 30°}a$ or $2\sqrt{3}\left(\dfrac{\sqrt{3}}{3}\right)a$ $\boldsymbol{= 2a}$



Pentagon Adjustment

Side Length Extension, 2*e* = $2(\tan 36°)a \approx \mathbf{1.453085}\boldsymbol{a}$ 1.4530850560107217717909335149612

Circumradius increase = $\dfrac{1}{\cos 36°}a \approx \mathbf{1.236068}\boldsymbol{a}$ 1.23606797749978969640917366

Adjusting for the influence of fabric attributes on beanbag size

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The equidistant 32-panel bag I made with thick corduroy was 1.54 – 6.08% larger than the mathematical prediction depending on whether I filled it loosely or over-filled it. I target halfway between the min and max inflations when sizing my patterns, which is 3.8%.

However, while the equidistant bag I made with my design testing fabric (fairly thin, stiff, tightly-woven, non-stretch) had an inflation of -2.93 – +0.98%, for an average of -0.975%, which is a *deflation* (adjustment factor = 0.990), the icosidodecahedron bag I finally got around to making (using the same fabric) had an inflation of -4.04 – +0.39%, for an average of -1.82%. So this variation is about 0.8% smaller than the Equidistant version, and I will assume the corduroy bag will exhibit the same difference. So I am using an adjustment factor of **1.03%** for my tables of measurements and printable patterns.

I use the adjustment factor to adjust the pattern size to produce a more accurate finished size when using my fabric and stitching techniques. If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may need to figure out your own adjustment factor. For help, see the **General Information and Techniques** chapter under "Adjusting/Scaling a Pattern to Produce an Accurate Ball Size".

The equidistant design bag I made years ago with a thick, stiff, non-stretch denim measured -4.0 – +1.0%, for an average of -1.5% (adjustment factor = 0.985). So if you are using fabrics like the denim or my design testing fabric, expect the bag to be quite a bit smaller than with corduroy, and use the Base value in the measurement tables rather than the Adjusted value.

As I understand it, the bag size is affected by fabric attributes as follows. The folding of the fabric at the seams will cause thick, firm fabrics to significantly shrink the bag size unless the fabric has some stretch. Folding thin fabric doesn't consume as much of its size, but my design testing fabric, though fairly thin, has no stretch at all, and so ended up producing about the same size bag as the denim, which stretches a little. Corduroy is a softer, more loosely woven fabric than denim and flexes and compresses more easily, and so not as much of the panels' size is consumed by the folding. My denim and design testing fabric bags have very prominent seams while the corduroy bag has much more subtle seams.

Sizing formulas

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Below are the formulas to calculate the pattern construction elements (*Diameter* and *Circumference* are your target ball size). The value in orange is the adjustment factor. **Don't forget to multiply the final result by 25.4 if you need to convert inches to millimeters.**

- **Triangle Side Length = *Diameter* × 0.3266 ÷ 1.03**
  = *Circumference* × 0.1040 ÷ 1.03
- **Triangle Circumradius = *Diameter* × 0.1886 ÷ 1.03**
  = *Circumference* × 0.0600 ÷ 1.03
- **Pentagon Side Length = Triangle Side Length**
- **Pentagon Circumradius = *Triangle Circumradius* × 1.4734**

- For double-checking: **Pent Height = *Pent Side* × 1.5388**
  = *Pent Circumradius* × 1.8090

Triangle pattern measurement table

Below are stitching pattern measurements for each ⅛″ diameter increment from 1 ¾″ – 3″. The values in the **Adjusted** columns account for the 1.03 estimated adjustment factor. The adjusted values decrease the **Base** pattern size so that you will get a more accurate finished size when using corduroy or something similar (a soft, flexible, moderately thick fabric). If you are using a firm denim or a thin, but non-stretch fabric, use the Base value instead. I attempt to explain why in the "Adjusting for the influence of fabric attributes on beanbag size" topic above.

To draw the cutting pattern, increase the Triangle Side Length by the desired allowance × $2\sqrt{3}$ (≈ 3.4641), or increase the Triangle Circumradius by the allowance × 2.

Finished Diameter	**Triangle Side Length** (mm)		**Triangle Circumradius** (mm)		**Triangle Height** (mm) (for double-checking)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
1¾″ (44.5mm)	14.517	**14.094**	8.381	**8.137**	12.572	**12.206**
1⅞″ (47.6mm)	15.554	**15.101**	8.980	**8.718**	13.470	**13.078**
2″ (50.8mm)	16.591	**16.107**	9.579	**9.300**	14.368	**13.949**
2⅛″ (54.0mm)	17.628	**17.114**	10.177	**9.881**	15.266	**14.821**
2¼″ (57.2mm)	18.664	**18.121**	10.776	**10.462**	16.164	**15.693**
2⅜″ (60.3mm)	19.701	**19.128**	11.375	**11.043**	17.062	**16.565**
2½″ (63.5mm)	20.738	**20.134**	11.973	**11.625**	17.960	**17.437**
2⅝″ (66.7mm)	21.775	**21.141**	12.572	**12.206**	18.858	**18.309**
2¾″ (69.9mm)	22.812	**22.148**	13.171	**12.787**	19.756	**19.180**
2⅞″ (73.0mm)	23.849	**23.154**	13.769	**13.368**	20.654	**20.052**
3″ (76.2mm)	24.886	**24.161**	14.368	**13.949**	21.552	**20.924**

Pentagon pattern measurement table

To draw a cutting pattern, increase the Pentagon Side Length by the desired allowance × 2(tan 36°) ≈ 1.4531, or increase the Pentagon Circumradius by the allowance × 1/cos 36° ≈ 1.2361.

Finished Diameter	**Pentagon Side Length** (mm)		**Pentagon Circumradius** (mm)		**Pentagon Height** (mm) (for double-checking)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
1¾″ (44.5mm)	14.517	**14.094**	12.349	**11.989**	22.339	**21.688**
1⅞″ (47.6mm)	15.554	**15.101**	13.231	**12.845**	23.935	**23.238**
2″ (50.8mm)	16.591	**16.107**	14.113	**13.702**	25.530	**24.787**
2⅛″ (54.0mm)	17.628	**17.114**	14.995	**14.558**	27.126	**26.336**

Finished Diameter	**Pentagon Side Length** (mm)		**Pentagon Circumradius** (mm)		**Pentagon Height** (mm) (for double-checking)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
2¼″ (57.2mm)	18.664	**18.121**	15.877	**15.414**	28.722	**27.885**
2⅜″ (60.3mm)	19.701	**19.128**	16.759	**16.271**	30.317	**29.434**
2½″ (63.5mm)	20.738	**20.134**	17.641	**17.127**	31.913	**30.983**
2⅝″ (66.7mm)	21.775	**21.141**	18.523	**17.984**	33.509	**32.533**
2¾″ (69.9mm)	22.812	**22.148**	19.405	**18.840**	35.104	**34.082**
2⅞″ (73.0mm)	23.849	**23.154**	20.287	**19.696**	36.700	**35.631**
3″ (76.2mm)	24.886	**24.161**	21.169	**20.553**	38.295	**37.180**

Drawing instructions

To draw the triangle, follow the hex instructions in the Equidistant Design section up to the point where you complete the triangle, and use the measurements from the triangle table above. I recommend that you copy the measurements you intend to use so you do not need to jump back here.

To draw the pentagon, follow the pentagon instructions in the Equidistant Design section, but use the measurements from the table above. I recommend that you copy the measurements you intend to use so you do not need to jump back here.

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Truncated Icosahedron (Equilateral Hex) Variation: Pattern Sizing & Drawing Instructions

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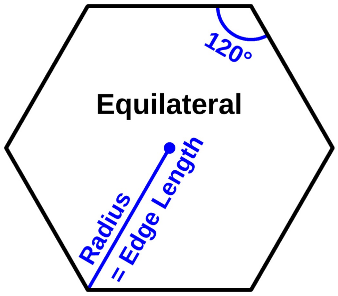
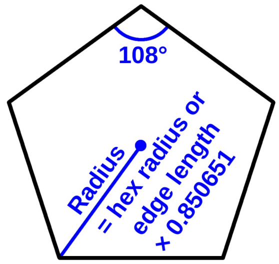






[6]

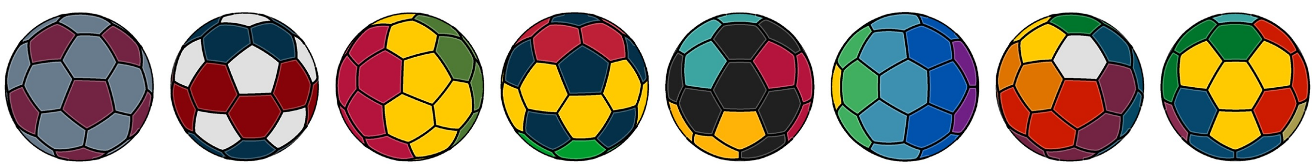
The high precision ratio is 0.85065080835203993218154049706301

*This section has a summary of the design and associated mathematics, then sizing formulas for drawing custom pattern sizes, pre-calculated pattern measurement tables for a range of standard sizes, and finally, links to the pattern drawing instructions. I provide printable measuring tapes at the end of the **General Information and Techniques** chapter in case you care to measure your beanbags.* ***The patterns for this variation are in the Ready-to-Print Patterns section.***

If you want to make a **true soccer ball** design (a truncated icosahedron), or want **hexagons that are larger than the pentagons**, use equilateral hexagons and pentagons. If you draw the panels with SketchUp, which uses a defined circumradius to draw polygons, it is useful to know that the circumradius of a hexagon is equal to its side length.

Color Arrangements

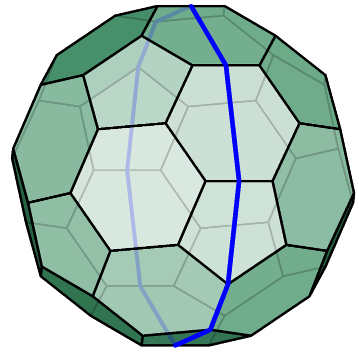
The color arrangement diagrams for the 32-panel designs are in a separate document titled "32-Panel Color Arrangements". Below are examples of the ball illustrations.



6 http://www.adventuretrading.com/footbags/panelled/hero-soccer.html

Design summary and mathematics

This subsection shows the mathematics behind the relationship between the pattern dimensions and the ball size. *The numbers in tiny, right-justified typeface are my computer calculator's unrounded values which I display rounded to six places for brevity.*



This design is composed of equilateral hexagons and pentagons. The pattern illustrations above show the ratio of the pentagon's circumradius to the hexagon's. The hexagon's circumradius is equal to its edge length, and, of course, to the pentagon's edge length. The circumference of the polyhedron is composed of (4 × *pent height*) + (4 × *hex height*) + (2 × *edge length*).

To define the pattern dimensions in terms of the beanbag size they produce, I need first to define all pattern dimensions and the polyhedron's circumference in common terms, which will be the side length of the starting triangle from which the hex is derived. For the equations I will define the following variables:

h_h = height of the hexagon
h_p = height of the pentagon
r_p = circumradius of the pentagon
s = side length (and circumradius of the hexagon)

Now I will evaluate each of those in terms of the starting triangle's side length, s_t.

$$h_p \approx \frac{\sqrt{5+2\sqrt{5}}}{2}s \approx 1.538842s$$ 1.5388417685876267012851452880185

$$h_h = \sqrt{3}s \approx 1.732051s$$ 1.7320508075688772935274463415059

$$r_p = \sqrt{\frac{5+\sqrt{5}}{10}}s \text{ or } \frac{0.5}{\sin 36°}s \approx 0.850651s$$ 0.85065080835203993218154049706301

To reiterate, Circumference = $4h_p + 4h_h + 2s$

So,
Circumference ≈ $4(1.538842s) + 4(1.732051s) + 2s \approx \mathbf{15.083570}\boldsymbol{s}$ 15.083570304626015979250366518098

Circumference ≈ $\frac{15.083570}{0.850651}s \approx \mathbf{17.731800}\boldsymbol{r_p}$ 17.731800353951717148299691301591

Hexagon and pentagon expressed in terms of bag size

To express the hexagon and pentagon in terms of a desired ball size, which is necessary to draw the patterns, I will solve the above expressions for s and r_p to express them in terms of the ball Circumference, C, and then the Diameter, D (by multiplying the right sides of the equations by π).

Side Length and Hex Circumradius, $s \approx 0.066297C$ 0.066297300957539716335544950342730
$\approx 0.208279D$ 0.20827911364103833762853195617641

Pent Circumradius, $r_p \approx 0.056396C$ 0.056395852651089630725833572725 2
$\approx 0.177173D$ 0.17717279638159564890371001498168

Cutting pattern adjustments

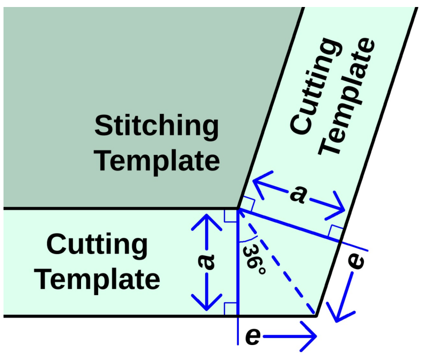
In the diagrams and equations below, ***e*** is the amount to extend one end of each side to get a seam allowance ***a***. So **2*e*** is the full amount by which to extend each side for the cutting pattern.

Hexagon Adjustment

Refer to the pentagon illustration, but for the hexagon the marked angle is 30°.

Side Length Extension, 2*e*, and Circumradius increase = $2(\tan 30°)a$ or $\frac{1}{\cos 30°}a$ or $\frac{2}{\sqrt{3}} \approx \mathbf{1.154701}\boldsymbol{a}$

1.15470053837925152901829756100039



Pentagon Adjustment

Side Length Extension, 2*e* = $2(\tan 36°)a \approx \mathbf{1.453085}\boldsymbol{a}$ 1.4530850560107217717909335149612

Circumradius increase = $\frac{1}{\cos 36°}a \approx \mathbf{1.236068}\boldsymbol{a}$

1.23606797749978969640917366

Adjusting for the influence of fabric attributes on beanbag size

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The equidistant 32-panel bag I made with thick corduroy was 1.54 – 6.08% larger than the mathematical prediction depending on whether I filled it loosely or over-filled it. I target halfway between the min and max inflations when sizing my patterns, which is **3.8%**. (I'm assuming the inflation will be the same for this variation.) This makes my adjustment factor **1.038**.

I use the adjustment factor to adjust the pattern size to produce a more accurate finished size when using my fabric and stitching techniques. If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may need to figure out your own adjustment factor. For help, see the **General Information and Techniques** chapter under "Adjusting/Scaling a Pattern to Produce an Accurate Ball Size".

The bag I made with my design testing fabric (fairly thin, stiff, tightly-woven, non-stretch) had an inflation of -2.93 – +0.98%, for an average of -0.975%, which is a *deflation* (adjustment factor = 0.990). The moderately tightly filled size was -2.44%. The bag I made years ago with a thick, stiff, non-stretch denim measured -4.0 – +1.0%, for an average of -1.5% (adjustment factor = 0.985). So if you are using fabrics like these, expect the bag to be quite a bit smaller than with corduroy, and use the Base value in the measurement tables rather than the Adjusted value.

As I understand it, the bag size is affected by fabric attributes as follows. The folding of the fabric at the seams will cause thick, firm fabrics to significantly shrink the bag size unless the fabric has some stretch. Folding thin fabric doesn't consume as much of its size, but my design testing fabric, though fairly thin, has no stretch at all, and so ended up producing about the same size bag as the denim, which stretches a little. Corduroy is a softer, more loosely woven fabric than denim and flexes and compresses more easily, and so not as much of the panels' size is consumed by the folding. My denim and design testing fabric bags have very prominent seams while the corduroy bag has much more subtle seams.

Sizing formulas

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Below are the formulas to calculate the pattern construction elements (*Diameter* and *Circumference* are your target ball size). The value in orange is the adjustment factor. **Don't forget to multiply the final result by 25.4 if you need to convert inches to millimeters.**

- **Hexagon Side Length and Circumradius = *Diameter* × 0.2083 ÷ 1.038**
  = *Circumference* × 0.0663 ÷ 1.038
- For double-checking: **Hex Height (side to side) = *Side Length* (or *Circumradius*) × 1.7321**
- For double-checking: **Hex Width (corner to corner) = *Side Length* (or *Circumradius*) × 2**
- **Pentagon Side Length = Hexagon Side Length**
- **Pentagon Circumradius = *Hex Circumradius* × 0.8507**
- For double-checking: **Pent Height = *Pent Side* × 1.5388**
  = *Pent Circumradius* × 1.8090

Hexagon pattern measurement table

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Below are stitching pattern measurements for each ⅛″ diameter increment from 1 ¾″ – 3″. The values in the **Adjusted** columns account for my 1.038 adjustment factor. The adjusted values decrease the **Base** pattern size so that you will get a more accurate finished size when using corduroy or something similar (a soft, flexible, moderately thick fabric). If you are using a firm denim or a thin, but non-stretch fabric, use the Base value instead. I attempt to explain why in the "Adjusting for the influence of fabric attributes on beanbag size" topic above.

To draw the cutting pattern, increase the Circumradius and Side Length by the desired allowance × $2(\tan 30°)$ or $2/\sqrt{3} \approx 1.1547$.

Finished Diameter	**Hex Side Length and Circumradius** (mm)		**Hex Height** (mm) (for double-checking)	
	Base	**Adjusted**	Base	**Adjusted**
1¾″ (44.5mm)	9.258	**8.919**	16.035	**15.448**
1⅞″ (47.6mm)	9.919	**9.556**	17.181	**16.552**
2″ (50.8mm)	10.581	**10.193**	18.326	**17.655**
2⅛″ (54.0mm)	11.242	**10.830**	19.471	**18.759**
2¼″ (57.2mm)	11.903	**11.467**	20.617	**19.862**
2⅜″ (60.3mm)	12.564	**12.104**	21.762	**20.966**
2½″ (63.5mm)	13.226	**12.742**	22.908	**22.069**
2⅝″ (66.7mm)	13.887	**13.379**	24.053	**23.172**
2¾″ (69.9mm)	14.548	**14.016**	25.198	**24.276**

Finished Diameter	Hex Side Length and Circumradius (mm)		Hex Height (mm) (for double-checking)	
	Base	Adjusted	Base	Adjusted
2⅞″ (73.0mm)	15.210	**14.653**	26.344	**25.379**
3″ (76.2mm)	15.871	**15.290**	27.489	**26.483**

Pentagon pattern measurement table

To draw a cutting pattern, increase the Pentagon Side Length by the desired allowance × 2(tan 36°) ≈ 1.4531, or increase the Pentagon Circumradius by the allowance × 1/cos 36° ≈ 1.2361.

Finished Diameter	Pentagon Side Length (mm)		Pentagon Circumradius (mm)		Pentagon Height (mm) (for double-checking)	
	Base	Adjusted	Base	Adjusted	Base	Adjusted
1¾″ (44.5mm)	9.258	**8.919**	7.875	**7.587**	14.247	**13.725**
1⅞″ (47.6mm)	9.919	**9.556**	8.438	**8.129**	15.264	**14.705**
2″ (50.8mm)	10.581	**10.193**	9.000	**8.671**	16.282	**15.686**
2⅛″ (54.0mm)	11.242	**10.830**	9.563	**9.213**	17.299	**16.666**
2¼″ (57.2mm)	11.903	**11.467**	10.125	**9.755**	18.317	**17.646**
2⅜″ (60.3mm)	12.564	**12.104**	10.688	**10.297**	19.335	**18.627**
2½″ (63.5mm)	13.226	**12.742**	11.250	**10.839**	20.352	**19.607**
2⅝″ (66.7mm)	13.887	**13.379**	11.813	**11.381**	21.370	**20.588**
2¾″ (69.9mm)	14.548	**14.016**	12.376	**11.922**	22.388	**21.568**
2⅞″ (73.0mm)	15.210	**14.653**	12.938	**12.464**	23.405	**22.548**
3″ (76.2mm)	15.871	**15.290**	13.501	**13.006**	24.423	**23.529**

Drawing instructions on the next page.

Drawing instructions

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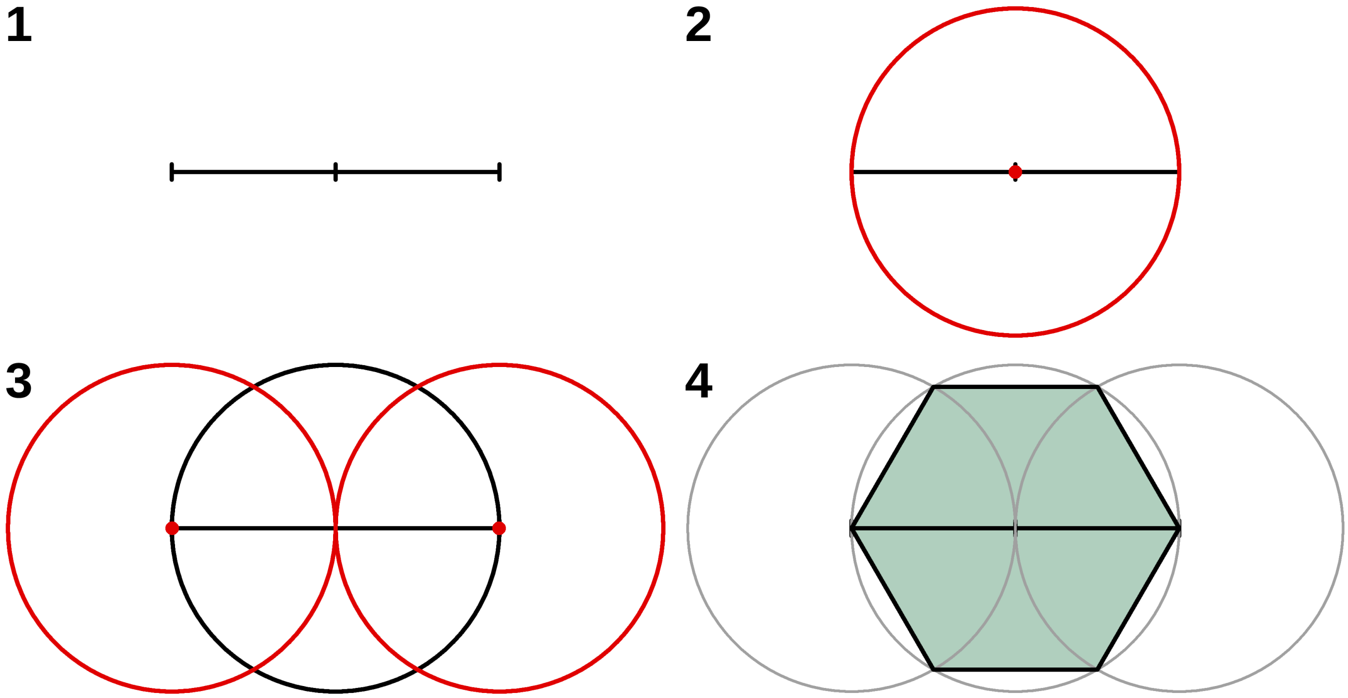
To draw a regular hexagon in SketchUp, simply use the polygon tool set to 6 sides (the default) and draw a hexagon with circumradius = **Hex Side Length and Circumradius**. To draw it by hand, you can either measure out each edge and the **120° angles**, or you can use the compass method below.



1. Mark a center point for the hexagon and draw a line the length of **Hex Side Length and Circumradius** out from each side of it (the line will be double the length of the hex side). Mark the ends of the line.

2. Place a compass in the center, extend it to one of the ends, and draw a circle that encompasses the line.

3. Leaving the compass radius unchanged, place the compass on each of the endpoints of the line in turn and draw a circle.

4. The four intersections of the three circles and the endpoints of the original line form the six corners of the hexagon. Just draw lines connecting them. To ensure you drew everything correctly, measure the hex's height from side to side and compare that to **Hex Height**. Its width, corner to corner, should equal twice the side length/circumradius (the length of the initial line).

To draw the pentagon, follow the pentagon instructions in the Equidistant Design section, but use the measurements from the table above. I recommend that you copy the measurements you intend to use so you do not need to jump back here.

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How I Developed This Design

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Photo from http://www.flyingclipper.com/home/fly/page_1024_134/tossaball_phat_tyre_32_panel_juggle_ball.html.
CG Equidistant Truncated Icosahedron drawn by me in SketchUp.

Conceptualization and overview of related polyhedra

In April, 2013 I discovered the Tossaball Phat Tyre 32-panel juggling bag made by Flying Clipper (photo above). I loved the look of it. This type of panel structure is commonly used for footbags (Hacky Sacks), but Flying Clipper is the only manufacturer I have seen using it for a juggling bag.

The geometric shape of this structure is somewhere between the icosidodecahedron, which has 12 pentagons and 20 triangles, and the truncated icosahedron, which is the polyhedron classic soccer balls are based on and has 12 pentagons and 20 equilateral hexagons (all the polyhedra I mention are illustrated on the following page). The truncated triangles or semiregular hexagons used here look a lot like the ones in my 14-panel Equidistant Cuboctahedron, but I think these are more truncated (closer to a regular hexagon, less triangular). Flying Clipper holds a patent on the footbag version of this design (Patent # 386230 dated Nov. 11, 1997).

For months I wondered if an equidistant version of the icosidodecahedron would produce those truncated triangles. In case you haven't read the development section of the 14-panel Equidistant Cuboctahedron chapter, I mean by "equidistant" that all faces are the same distance from the center or, to put it another way, opposing faces of both shapes are the same distance from each other. By my measurements the distance between opposing triangular faces of the icosidodecahedron is normally 9.82% greater than between the pentagons. The distance between the pentagonal faces of the truncated icosahedron is 2.65% greater than between the hexes, so there is not much difference there.

On July 29, 2013 I finally felt ambitious enough to do the calculations and crack the Equidistant Icosidodecahedron. A big step forward was when I realized that the dodecahedron can be used as the basis for building the icosidodecahedron (I should have realized this from the name) just as a cube (or octahedron) can be used to build a cuboctahedron. An icosidodecahedron is really a fully truncated dodecahedron, or, as it is officially called, a "rectified" dodecahedron (meaning that each vertex is chopped off down to the middle of the edges). I did not know before this how to build an icosidodecahedron.

I later realized that the same is true of an icosahedron, and starting with an icosahedron is a much easier way to construct an icosidodecahedron and greatly reduces the number of calculations needed to design the hex panel shape. The method I learned for constructing an icosahedron is also easier than for the dodecahedron. Because I ended up truncating an icosahedron to create this design, I changed the name of my design years later from "Equidistant Icosidodecahedron" to "Equidistant Truncated Icosahedron".

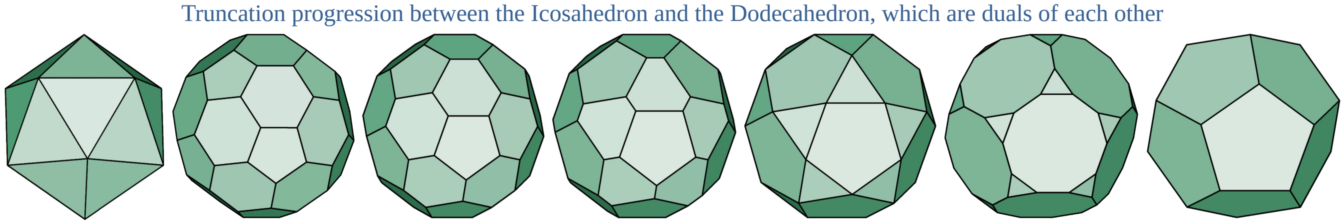
It turns out that the equidistant version has hexes that are more nearly equilateral (less triangular) than Flying Clipper's appear to be (compare the photo and the CG equidistant version above). The short side is 69.2% the length of the long. I tried building a version that uses the hex from the Equidistant Cuboctahedron whose long side equals the circumradius of the guide triangle. That hex has a short side that is 36.6% the length of the long. I think Flying Clipper's hexes are somewhere in between.

The benefit of the equidistant version, aside from its equidistance which is not of great importance in terms of roundness with so many panels, is that the two panel shapes are nearly the same size which will improve the look of color arrangements that do not distinguish between them.

The illustrations below show how all these polyhedra are related. As the icosahedron's vertices are progressively truncated, it becomes the truncated icosahedron (equilateral hexagons and pentagons), my polyhedron, the 14-panel hex version of my polyhedron, and finally the icosidodecahedron at the point when the hexagons' short edges are reduced to their vanishing point, forming triangles. If the pentagonal faces are then further truncated, enlarging them and shrinking the triangles, the pentagons become decagons (10 sides), forming the truncated dodecahedron, and then, at the point where the triangular faces disappear and the decagonal faces become pentagons again, the shape becomes the dodecahedron.

If the dodecahedron is fully truncated, through the truncated dodecahedron stage, the result is the icosidodecahedron. If the icosidodecahedron's triangular faces are then further truncated, shrinking the pentagonal faces and converting the triangular faces into hexagons, the reverse progression of polyhedra is formed until the pentagonal faces shrink to nothing and become the vertices of the icosahedron. That is progression direction from which I formed my polyhedron. The interrelations of polyhedra is part of why I find the study of them fascinating.

Truncation progression between the Icosahedron and the Dodecahedron, which are duals of each other



Icosahedron	**Truncated Icosahedron** (equilateral hexagons and pentagons)	My **Equidistant Truncated Icosahedron**	A further truncated version forming my 14-panel hex shape	**Icosidodeca-hedron**	**Truncated Dodecahedron** (equilateral decagons and triangles)	**Dodecahedron**

Solving the Equidistant Design

To solve the Equidistant Icosidodecahedron/Truncated Icosahedron, I used a method similar to the one I used for the Equidistant Cuboctahedron. First, some measurements of icosahedrons from Wikipedia[7]. (My computer calculator's full-precision values are in tiny, right-justified typeface after each rounded value.)

If the edge length of a regular icosahedron is a, the radius of a circumscribed sphere (one that touches the icosahedron at the vertices) is

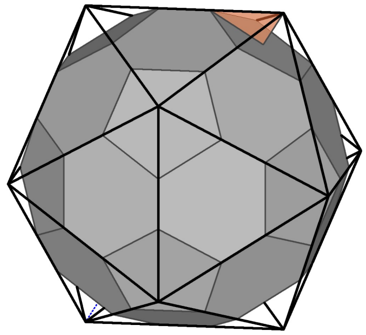
$$r_c = \frac{a}{4}\sqrt{10+2\sqrt{5}} = a\sin\left(\frac{2\pi}{5}\right) \approx 0.951057a$$

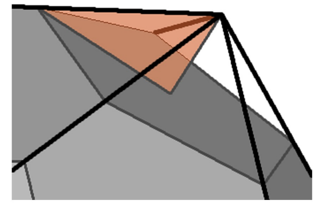
0.95105651629515357211643933337938

7 http://en.wikipedia.org/wiki/Icosahedron

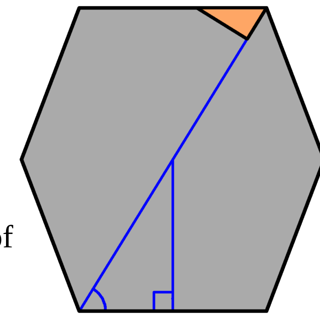
and the radius of an inscribed sphere (touches each of the icosahedron's faces) is

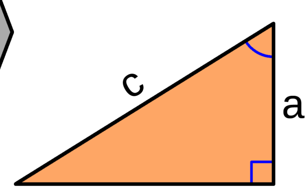
$$r_i = \frac{\sqrt{3}}{12}(3+\sqrt{5})a \approx 0.755761a$$ 0.75576131407617073048013370202501





I began by forming a hypothetical triangle (illustrated in orange) to define the position of the equidistant pentagonal face in relation to the vertex of the icosahedron. The short leg of the triangle runs from the vertex to the center of the proposed pentagonal face (it would continue to the diagonal opposite vertex). The long leg is the circumradius of the pentagonal face.





The grey hexagon on the right is a 2D view of the above diagram. (It is a cross section of a unit icosahedron. The top and bottom edges are the edges of the icosahedron and the diagonal edges are the heights of the faces). The diagonal line that runs through it is the diameter from corner to corner of the icosahedron, which has a length of $2r_c$.

The marked angles are equal to $\cos^{-1}\left(\frac{1}{2r_c}\right) \approx 58.282526°$ 58.282525588538994675786096860227

This is derived by using the triangle in the grey cross section diagram (which has the same angles as the orange triangle) and knowing that the adjacent side has a length of 0.5 and the hypotenuse is the circumscribed radius r_c.

Side *a* has a length, by definition, equal to $r_c - r_i \approx 0.195295$ 0.19529520221898284163630563135437

The reasoning for this is that the inscribed radius gives me the pentagonal faces' target distance from the center (the distance at which the triangular and derived hexagonal faces sit), and the circumscribed radius gives me the distance from center to vertex. The difference between them is the distance from the vertex at which the pent faces must be positioned to be equally distant from the center as the triangular/hexagonal faces.

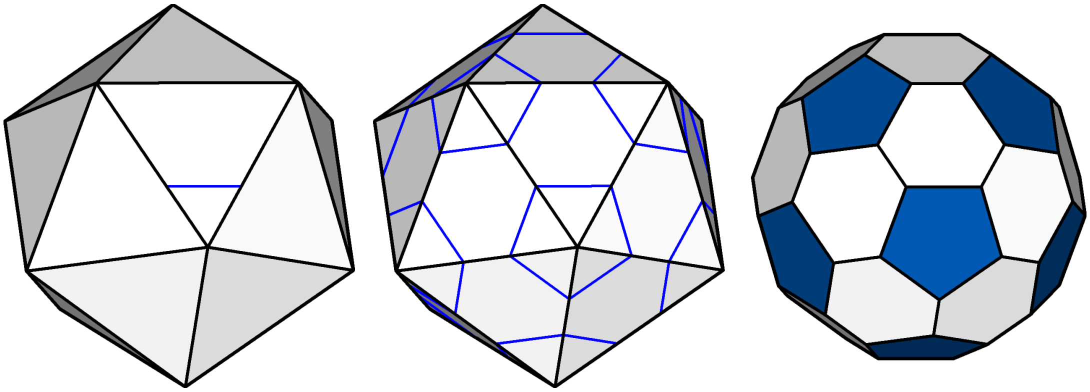
Side *b* has a length equal to

$$\tan\left(\cos^{-1}\left(\frac{1}{2r_c}\right)\right)(r_c - r_i) \approx 0.315994$$ 0.31599427503009812257945227421943

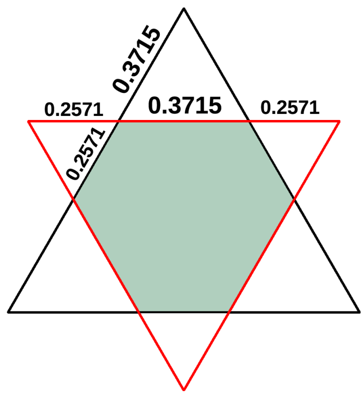
The hypotenuse, *c*, has a length equal to

$$\frac{r_c - r_i}{\cos\left(\cos^{-1}\left(\frac{1}{2r_c}\right)\right)} = \frac{r_c - r_i}{\frac{1}{2r_c}} = 2r_c(r_c - r_i) \approx 0.371474$$ 0.37147354934308673396507945588261

The length of the hypotenuse allows me to build the Equidistant Truncated Icosahedron from an icosahedron as shown below. The initial blue line in the first step intersects the edges of the triangular face at a point that is 0.371474 or 37.1474% distant from the shared corner. I draw those around every vertex and then truncate the vertices into pentagonal faces.



The 0.371474 truncation allows me to draw the panel shape (shown in green below) from an equilateral triangle, but it is the long truncation meaning that the hexagon will be oriented with its long sides facing the triangle's corners as shown by the black triangle. I would prefer (though it doesn't matter greatly) to truncate my triangles using the short truncation as depicted by the red triangle.



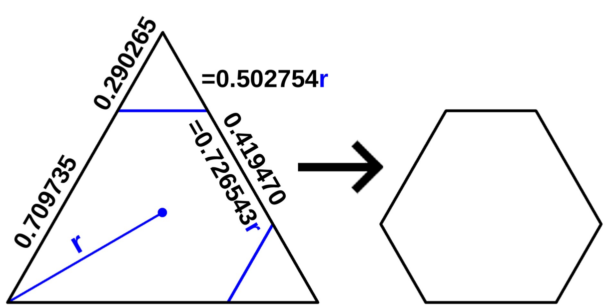
High precision values: 0.37147354934308673396507945588261,
0.25705290131382653206984108823478,
0.88557935197073979810476163235217

The side of the red triangle measures (0.257053 × 2) + 0.371474 = 0.885579. That value is the ratio of the length of the red triangle's side to that of the black triangle, so if I divide the hex's short side, 0.257053, by 0.885579, I will get the percentage of the short truncation on the red triangle's side.

The truncation, then, to construct the hex panel shape from an equilateral triangle is

$$\frac{0.257053}{0.885579} = 0.290265$$

0.29026523793919683838237388274383



How to draw the Equidistant Truncated Icosahedron and variations

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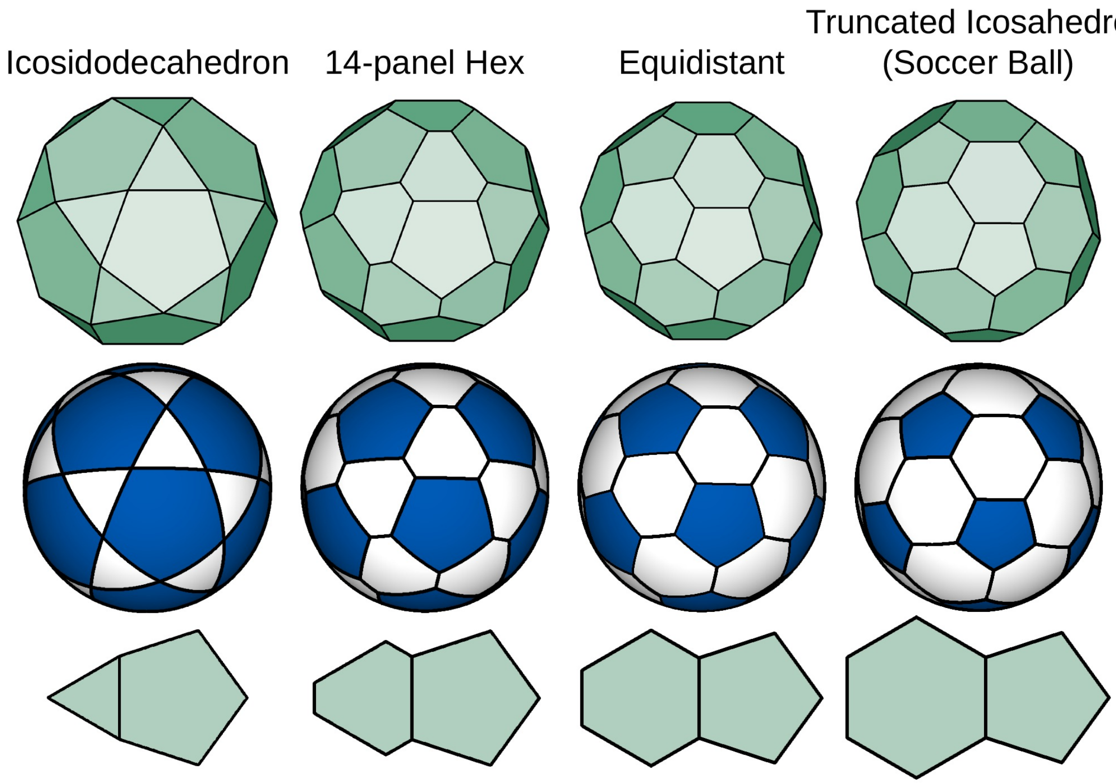
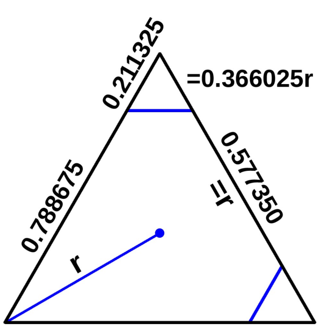


To truncate an icosahedron to produce hex faces matching my semiregular hexes, you must reverse the conversion I did above which redefines the hex according to a triangle with the corners at the short sides. This is so that you can truncate according to the hex's long side proportion, but in relation to the icosahedron's edge length. Here is the formula that expresses the truncation as a percentage of the icosahedron's edge length. Let L be the hex's long side proportion (in this case 0.419470) and S be the short side proportion (0.290265).

$$\text{Icosahedron Truncation Percentage} = \frac{L}{2L+S}$$

For my hex above this results in the 0.371474 or 37.1474% truncation I used to create the equidistant polyhedron. If you wanted to create the polyhedron based on the hex from my 14-panel equidistant design (right) the formula would be as follows:



$$\frac{0.577350}{2(0.577350)+0.211325} \approx 0.422650 = 42.2650\%$$

This makes the truncation on the icosahedron 42.2650% distant from its vertices.

To create a true Archimedean Truncated Icosahedron (or soccer ball) which has equilateral hexagons and pentagons, the truncation is one third of the icosahedron's edge length. To create an Icosidodecahedron, which has equilateral triangles and pentagons, the truncation is half the edge length. Basically, you just make one truncation across the centers of each edge.

To convert the polyhedra into spheres, follow the directions in *Appendix II*.

Turtle Ball

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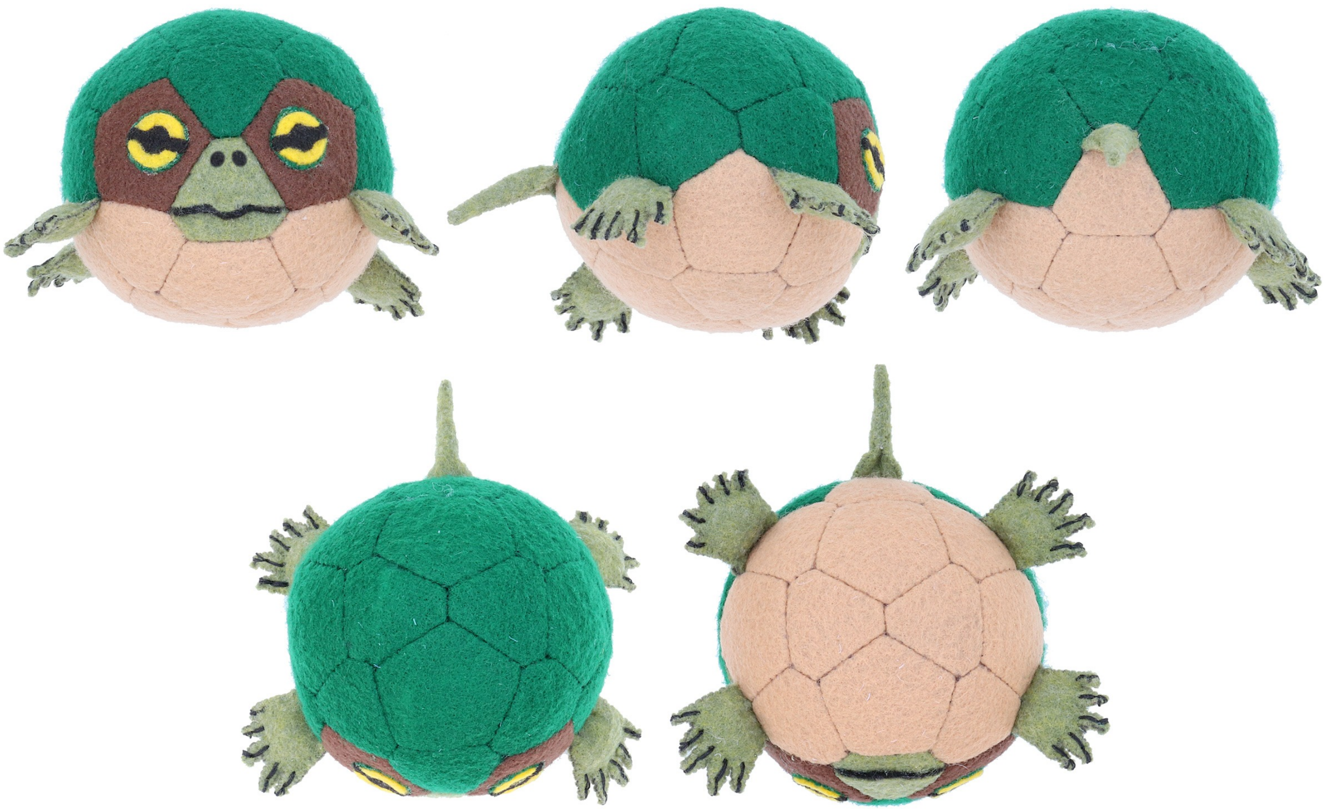
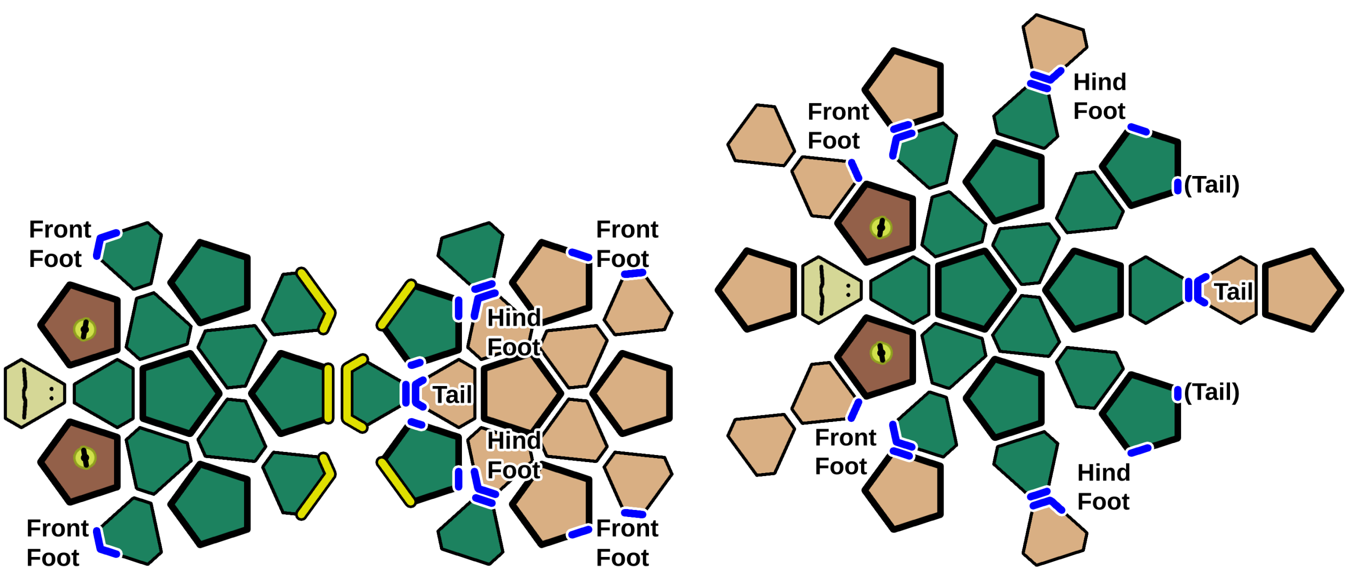


Assembly summary



Assembly diagrams. Dual-hemisphere method on the left, pent-centric, ring by ring method on the right.

I used the first assembly diagram type above – the dual-hemisphere method described in my "Assembly" section. I used my 14-panel hex variation for this ball and the 3″ pattern size, resulting in a ball that is 3.28″ in diameter (I stitched it a bit generously, and stretched it out tightly with fiber fill).

The blue highlighted portions of the seams show where I sewed the feet and tail into them. The yellow highlights are where I suggest leaving the seams open to turn the bag out, and to fill the bag. I needed only the upper three seams open to turn the bag out, though (along with some loose stitches beyond them), and then I actually closed the bag fully before adding the two main fillers and then loosened the last several stitches enough to push the funnel in between them (that is easier than stitching the ball closed with filler trying to push out).

If you include the feet and tail in your ball, I strongly advise marking those seams before assembly to remind you to add those accessories. I forgot to add the hind feet, forgetting that they were not at the edges of the hemispheres like the front feet, and had to reopen some completed seams.

Story and ball description

Inspired by my Turtle Ball color arrangement idea, which was based on the Big-Eyed Ladybug, I decided to make a felt Turtle Ball for my sister's fiancé, Chris, who loves turtles. But I decided to embellish it with feet and a tail (Chris is not a juggler, so this is just a toy or shelf curio). I also gave it a special ability: my Turtle Ball is self-righting! It will not get stuck on its back, but will roll over onto its belly. I accomplished this by filling the belly portion with crushed walnut shell, which is a fairly heavy filler, and the upper portion with tightly packed fiber fill. And just to be sure the bottom was heavy enough, I filled the four panels in the center of the belly with steel BBs. The resulting ball rights itself perfectly – just like a Weeble Wobble! It also sits perfectly level.

This was the most difficult and complicated sewing project I have ever done. I finished it on December 18, 2020, just in time for a trip to visit some family for a pre-Christmas party. I wanted to present it to Chris at the party so I could show it off to everyone. Though I had started this project around two months before (I don't remember when, but it was well before publishing the Second Edition guide), I had a very hard time finding the motivation to work on it and had to rush to get it done in time. My depression has been very bad and that made the difficulty of the project intimidating. After getting a start on it I ran out of steam and took a break from it for over a month.

I had to simplify the top shell from my original idea, which was to add narrow strips of a mottled, light brown felt into the seams to make outlines around each shell panel. I sewed a few panels that way and they look great, but it was so difficult to do right that I couldn't stay motivated to make the entire shell that way.

Figuring out how to get three layered fillers into the ball and positioning and balancing the BBs correctly at the bottom was a challenge. I originally wanted to glue or sew four small, fabric pillows full of BBs to the four central belly panels. This would prevent them from moving while I added the other fillers, and would prevent them from intermixing with the crushed walnut shell when the ball was thrown or squeezed.

This turned out to be too tedious and difficult, so I just held the loose BBs in place temporarily with hard drive magnets while I closed the ball and filled it, and then hoped that the tightly packed stuffing would prevent them from redistributing. I added the crushed walnut shell over the top of them with a funnel, and then, keeping the ball level so the crushed walnut shell stayed level, I used the funnel and the eraser end of a pencil to push the fiber fill into the upper portion of the ball.

I am proud of the face. I used no paint. Every feature is composed of hand-cut pieces of felt that I glued together. The eyes have a green circular piece for the base with a pupil and a pair of yellow, half-ring-shaped iris sections glued to them.

After I glued all four pieces of each eye together, I glued the completed eyes to the brown panels (before assembly). Then I whip-stitched all the way around each eye with transparent thread to ensure it would never peel off. I chose the type of turtle eye that looks like frogs' eyes because I like frogs, and the shape and color of their eyes.

The beak panel has three pieces: the base panel for the 32-panel structure, a shorter black panel that extends on the upper three sides only to just inside the stitching pattern (so it does not add thickness to the seam), and a slightly shorter top piece that extends all the way out to the edges of the base panel on those three sides. I glued the three layers together before assembly.

Only while making the assembly diagrams did I realize that I had forgotten to add nostrils to my turtle's beak. Rats. So I cut out two tiny ovals of black felt and glued them on. Unfortunately, I had already taken all the photos and didn't want to set them up again. So instead I Photoshopped nostrils onto the photos by cutting out circular sections of the pupils and then positioning, shaping, and feathering them to match my felt nostrils. It's not perfect, but it works well enough for a casual inspection.

My artistic sister (the one to whom Chris is engaged) helped me draw the pattern for the feet. Even such a simple sketch was difficult for me to get right, and I was unmotivated to keep trying. (I also consulted with her to select the ball's colors.)

Each foot is composed of two matching layers that I whip-stitched together around the palms and around each toe with transparent thread. I then used a black upholstery thread to make the claws, with the thread traversing from one toe to the next within the palm, and knotted inside the wrist so that there are no knots on the outside.

I filled the feet with crushed walnut shell so they had some rigidity and bulk. The wrists have enough seam allowance to be sewn into the seams. They extend across one full short seam and about a third of the adjacent long seam.



The tail took a few tries to get right. The pattern shape is like a Christmas tree with a flat top. The sides fold inward to the middle and the whole thing is folded in half, except near the base where it opens back out. The seam is whip-stitched together with transparent thread to form a slightly tapered shaft. The stitching divides into two near the base to allow the base to unfold and widen. What would be the trunk of the Christmas tree extends beyond the base of the tail and is the portion that gets stitched into the ball's seam, along with just a bit of the tail's base so there isn't an edge showing under the tail.

The base of the tail is a little wider than the short seam it attaches to and was supposed to extend slightly into the adjacent seams, which would give it some depth and rigidity by bending it around two downward angles. But I forgot to add the tail to the seam until I had already finished the adjacent seams to the right of the tail. So the tail is a bit misaligned with the seam, extending only into the left adjacent seam as shown in the third photo above.

I am very pleased with my Self-Righting Turtle Ball, and Chris loves it!



(Below is what it looked like without the nostrils. None of us noticed the lack of them, but they really improve it, and now it looks weird without them.)



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