The Complete Homemade Juggling Beanbag Guide

12-Panel Spherical Rhombic Dodecahedron Chapter

Small file size version (150dpi images)

By Joshua Clifton
Contact me on Reddit under Joshuacliftojm
or at juggle@joshuaclifton.com

Published 4/20/2023 Last edited 9/27/2024

This is part of a multi-document guide. Hyperlinks with the icon[1] open other guide documents[2], if they are saved to the same folder (**CTRL+Click** opens them in a new tab).

Visit my website to download those, and check back occasionally for revisions and corrections:

www.joshuaclifton.com/juggle

Compare the Last Edited date above on the right with the one on the web page to see if I have submitted changes.

Please contact me with your thoughts! Feedback on this project would be helpful and encouraging. You may also request custom patterns or other help with your project.

If this guide is useful to you, please **consider donating at my website** linked on the left. I am not monetizing the guide, and I am in need of income.

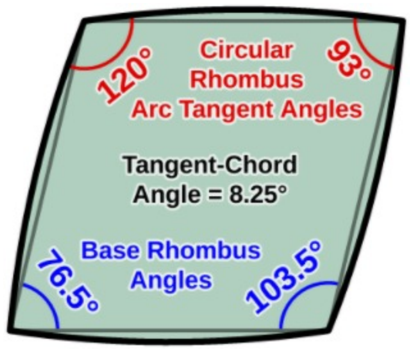
My website also provides blank **color arrangement diagrams** for experimenting with new arrangements in an image editor.

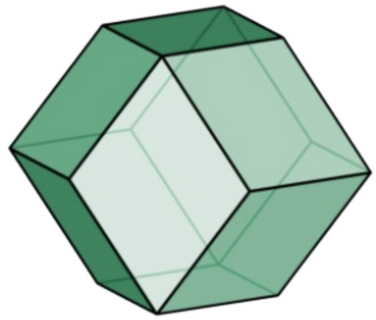
---

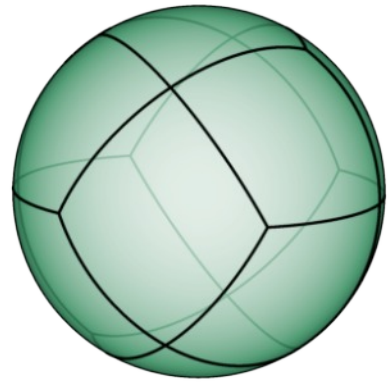
1 Icon from https://freesvg.org/vector-illustration-of-external-link-icon

2 **If the linked PDF does not open at the specified location**, keep it open, switch to the previous PDF's tab, and click the link again. **Cross-document links may not work in mobile PDF readers.** In that case you must open the document yourself and find the linked topic.

12-PANEL SPHERICAL RHOMBIC DODECAHEDRON INSTRUCTIONS







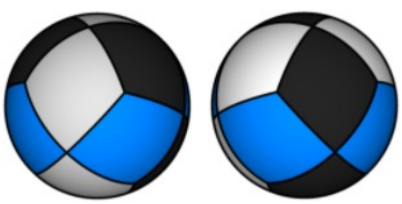

Design testing fabric ball.


“Diamond Ring with Dual Caps” arrangement

Chapter Index

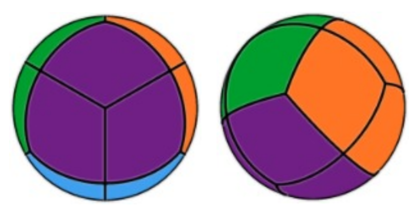
Design Notes

This design provides an alternative to the regular dodecahedron, having the same number of panels but very different color arrangement possibilities, and an even longer list of arrangements. It is also **easier and simpler to assemble**. Depending on the fabric used it may not be quite as uniform a sphere as the regular dodecahedron due to the elongated shape of the panels and the two different vertex types, but the inferiority would be significant only to perfectionists.



Its structure can be thought of as three rings, each composed of four diamonds joined at their acute corners, that encircle the ball perpendicularly to each other. This is illustrated on the left by each of the colors. Many of the color arrangements are based on this fact. Because of this structure, **the stretch direction of woven fabrics will be balanced simply by orienting the grain along the same diagonal on all panels**.

The spherical form's edges correspond to a pair of spherical tetrahedra in their dual positions, and so by using four colors, each on a triangular patch of three panels, **it can be made to look like a tetrahedron**. **It can also be assembled like a tetrahedron** after forming the four triangular patches.



Though the rhombic dodecahedron's faces are all the same, it has two different vertices. It is a Catalan solid, not a Platonic or Archimedean solid. It is the dual polyhedron of the 14-face cuboctahedron, and its 3-way vertices correspond to the cuboctahedron's triangular faces (and also to the vertices of the cube) and its 4-way vertices correspond to the squares (and to the vertices of the octahedron).

Supplies

- **For the templates**
  - Cardboard or Template Plastic, Scissors, Glue Stick or Double-Sided Adhesive Tape (to attach the pattern to the template material). **For drawing the pattern by hand:** Paper, Compass, metric Ruler, fine-point Pencil.
- **For the beanbag**
  - Fabric, Needle and durable Thread, Scissors, Fabric Marker or soft Pencil, beanbag Filler, Funnel.
- **For your information**
  - Unless you are experienced with this sort of thing, I recommend that you browse through the **General Information and Techniques** chapter (in the ***01 – Homemade Juggling Beanbag Guide – Index & Supplementary Chapters*** document) before starting. You may find some tips there that will improve your experience and your beanbags.

Printing and Drawing the Pattern

Back to Chapter Index

Later in this chapter I provide ready-to-print patterns. (When printing them, be sure to tell the Print Dialog to print only the page(s) you want so you don't print the entire document.) After those are sizing formulas, pre-calculated pattern measurements, and instructions for drawing the pattern yourself. Click the hyperlinks or look to the Chapter Index to locate those sections.

Color Arrangements

Back to Chapter Index

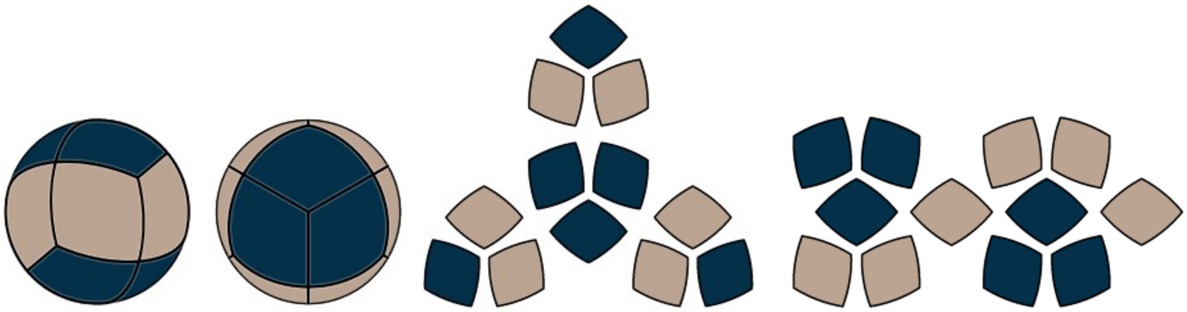
Following are color arrangement ideas, **grouped by the number of colors they use**. Each arrangement includes **two assembly layouts: a tetrahedron method** (form four triangular 3-panel patches and then assemble those as you would a tetrahedron), **and my optimized method** (form two 5-panel patches and attach them with a pair of additional panels). I actually prefer the tetrahedron method as it is a little simpler and easier, but I include the other method in case anyone prefers that (it requires fewer restitched seams and fewer threads).

To help me figure out the arrangements and create my diagrams, I stuck colored thumbtacks into an all-white beanbag I made using my design-testing fabric. I recommend this as a way to design new arrangements or to use as a reference to aid you in correctly assembling the bags.

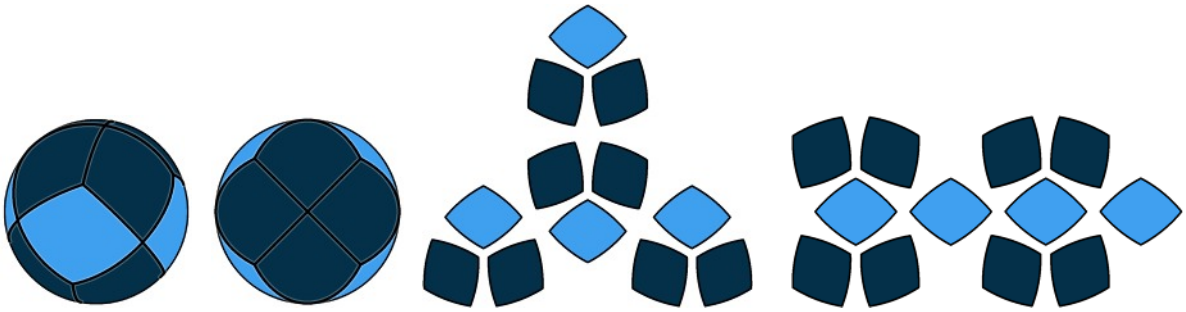
I also provide printable blank color arrangement diagrams for the ball views and the assembly layouts after the printable patterns. You can use those to experiment with color arrangements without having to make a beanbag. Look at the Chapter Index to locate them.

2 colors

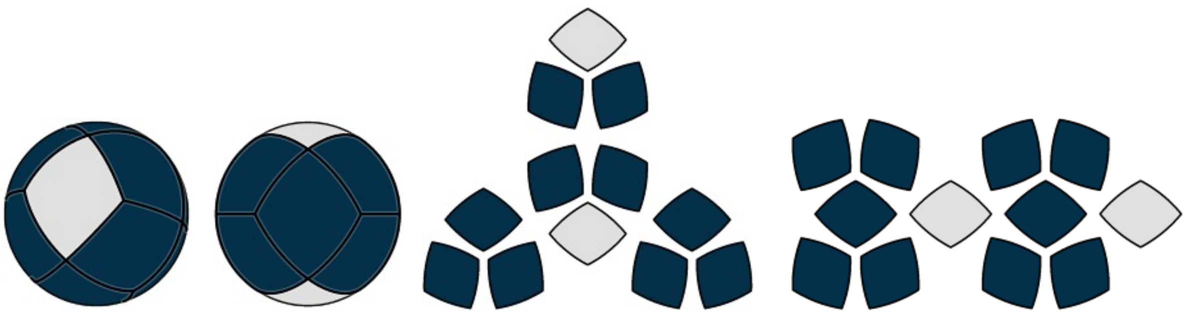
Back to Chapter Index



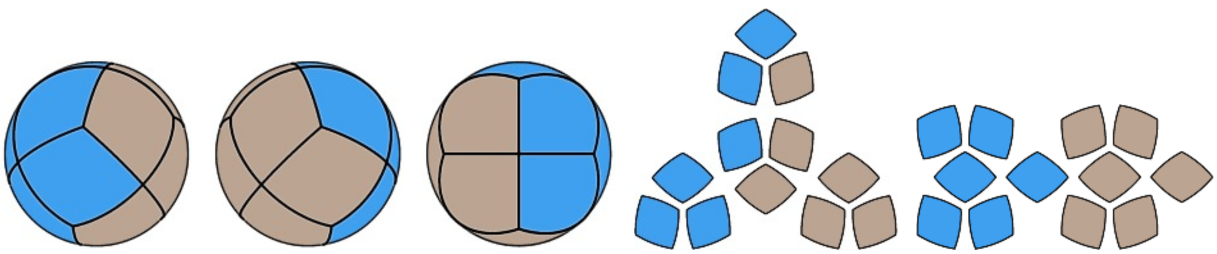
#1: Belt. A wavy ring of 6 panels around the middle between two triangular caps of a second color.



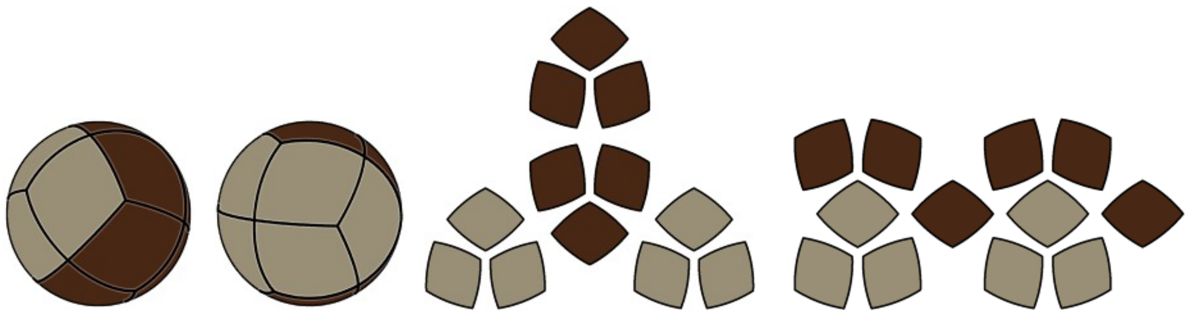
#2: Diamond Ring. Four diamonds joined at their acute corners that encircle the ball, against a contrasting background.



#3: Gems. Two diamonds opposite each other that contrast with the rest of the panels.



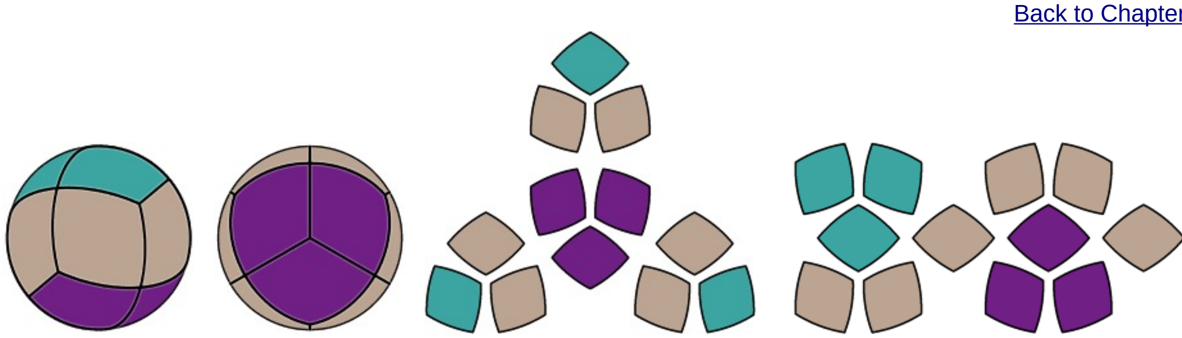
#4: Interlocking Plates. Each color on a patch of 6 panels with a half-diamond notch on one side and a protruding diamond on the opposite side that fits into the other plate's notch. The plates are shaped as shown in the right-most assembly diagram: a group of five with a sixth attached to its side.



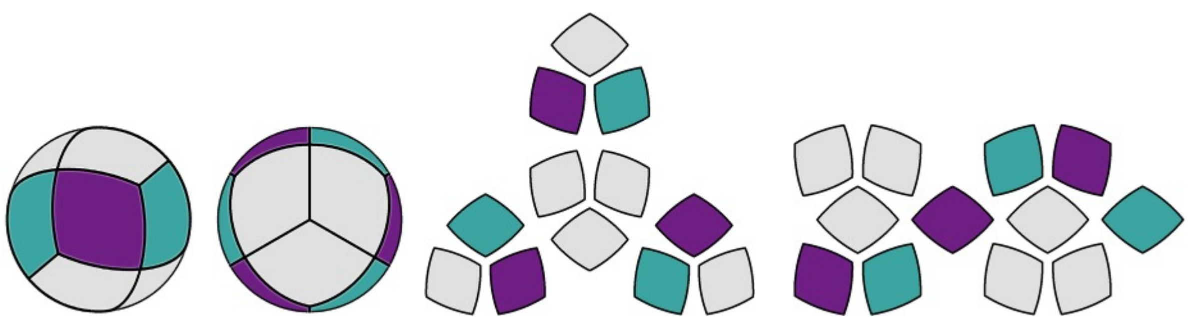
#5: Baseball. Two 6-panel patches, each a double triangle and each of a different color, that wrap around two-thirds of the ball in perpendicular directions similarly to the panels of a baseball.

3 colors

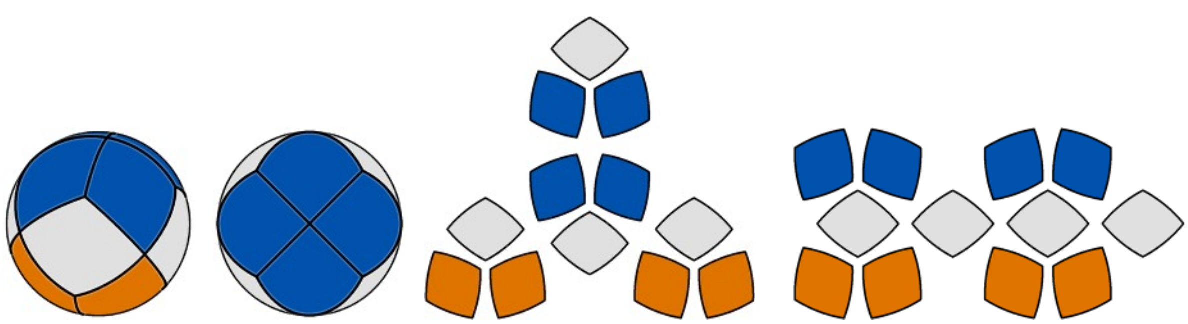
[Back to Chapter Index]



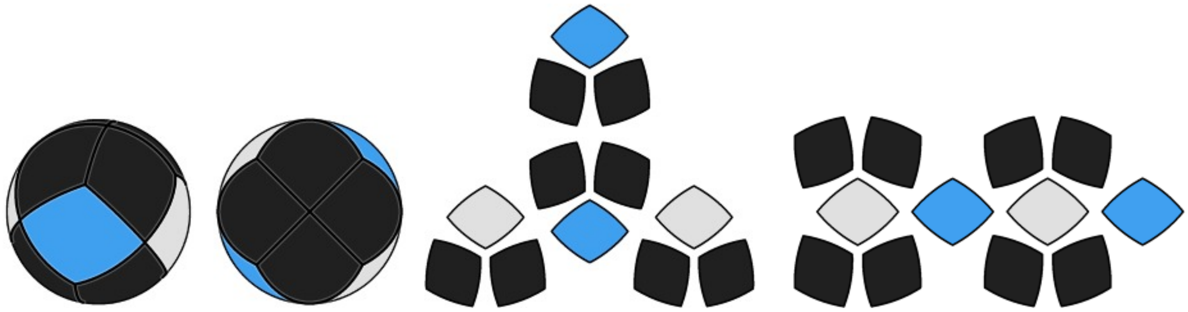
#6: Belt with Dual Caps. Same as the 2-color Belt arrangement but each triangular cap above and below the belt is a unique color.



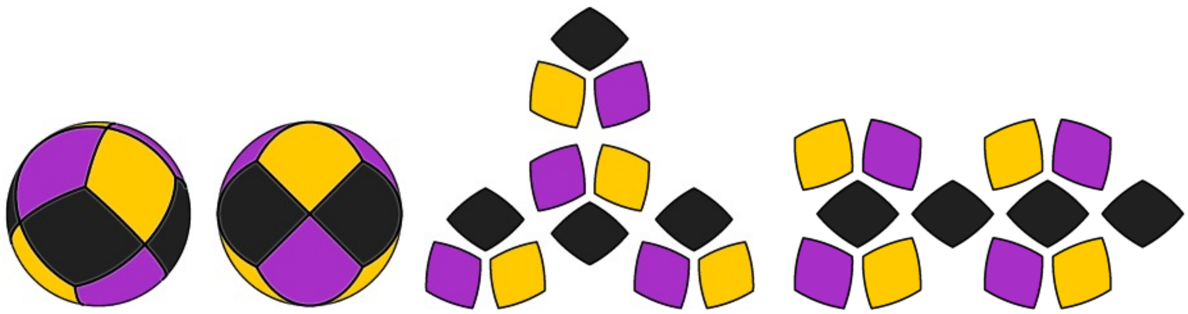
#7: Bi-Color Belt. Same as the 2-color Belt arrangement, but the belt consists of two alternating colors (3 panels of each color).



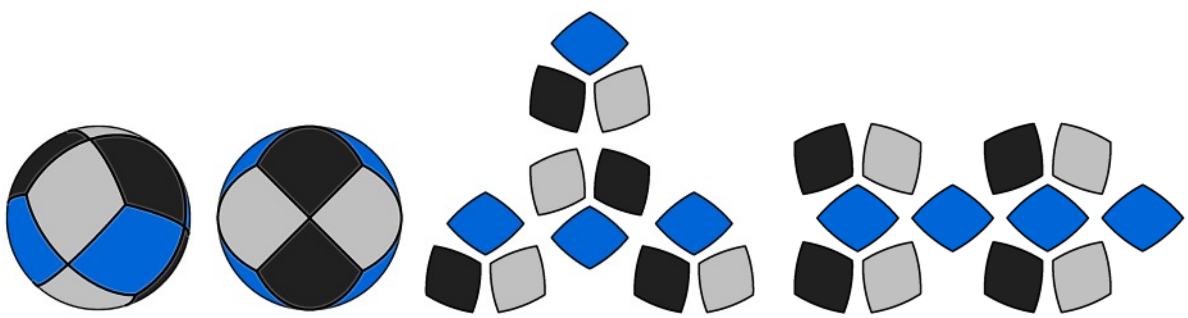
#8: Diamond Ring with Dual Caps. Same as the 2-color Diamond Ring arrangement, but each 4-panel cap above and below the ring is a unique color.



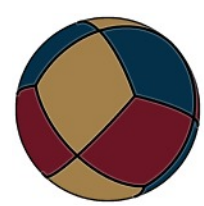
#9: Bi-Color Diamond Ring. Same as the 2-color Diamond Ring arrangement, but the ring consists of two alternating colors.



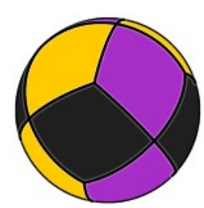
#10: Jester Ball. Similar to the Diamond Ring arrangement (four contrasting diamonds joined at their acute corners that encircle the ball, which are black in this case), but each 4-panel cap above and below the ring is now a checkered pattern of two colors that also alternate with those of the opposing cap. (On the right is an alternate color scheme that I also liked.)

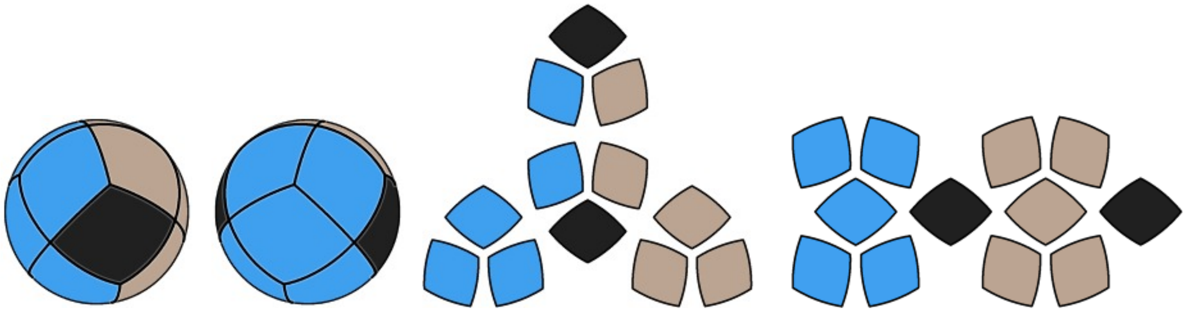


#11: Checker Ball/Orthogonal Diamond Rings. Three “diamond rings” (four diamonds joined at their acute corners that encircle the ball) that are oriented in each of the three orthogonal directions. This forms a checker pattern of two colors around each 4-way vertex. On the right is my first draft of this arrangement, which I also like.



This arrangement is related to the Jester Ball. If the colors on one cap of the Jester Ball are rotated/swapped so that they align with the other cap instead of alternating with it, they form two complete rings around the ball. This is illustrated on the right.



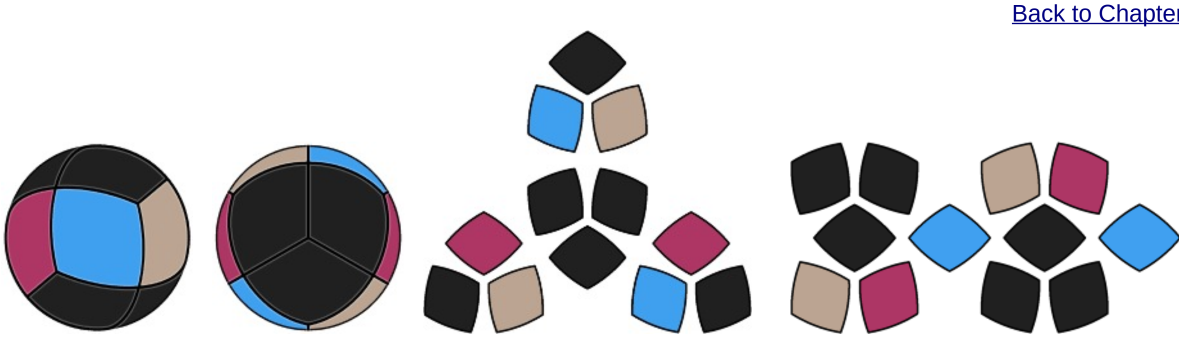


#12: Gems with Dual Plates. This arrangement can be thought of either as the first Gems arrangement (in the "2 colors" category) with each remaining hemisphere a unique color, or as the Interlocking Plates arrangement but with the two diamonds outside the main 5-groups assigned a third color.

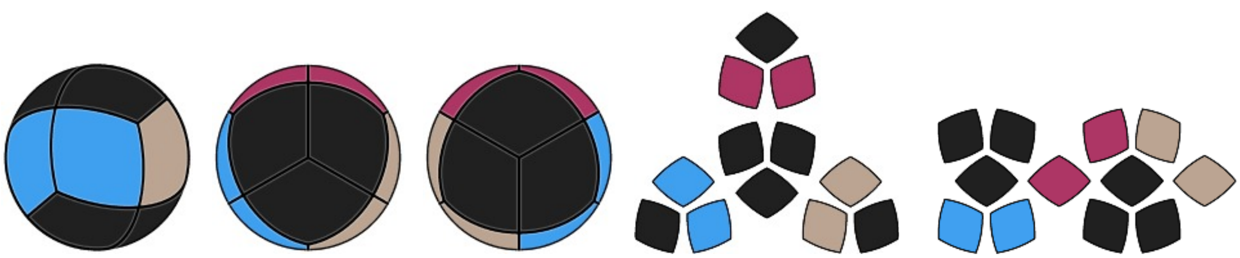
#13: Dual Gems. Same as the 2-color Gems arrangement, but each gem is a unique color.

4 colors

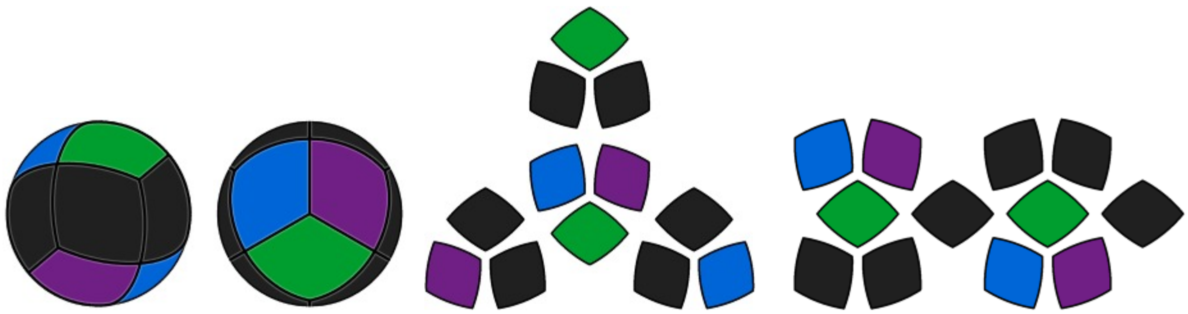
Back to Chapter Index



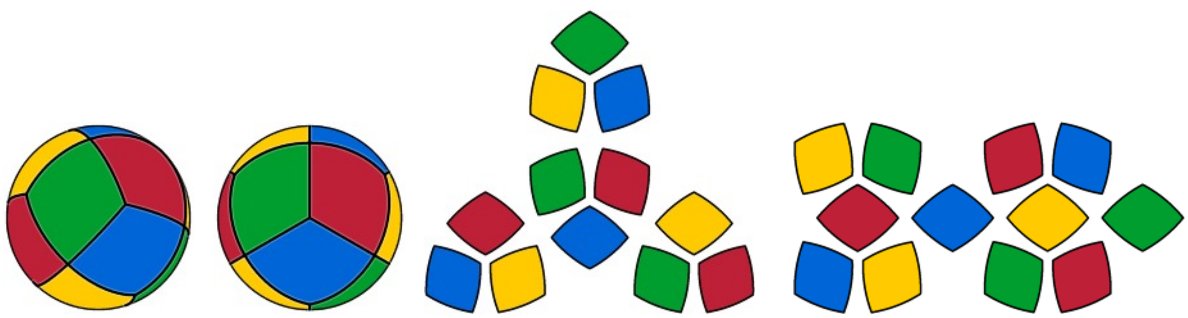
#14a: Tri-Color Belt Type A. Same as the 2-color Belt arrangement, but the belt consists of a repeating sequence of three colors. Each color is opposite its match.



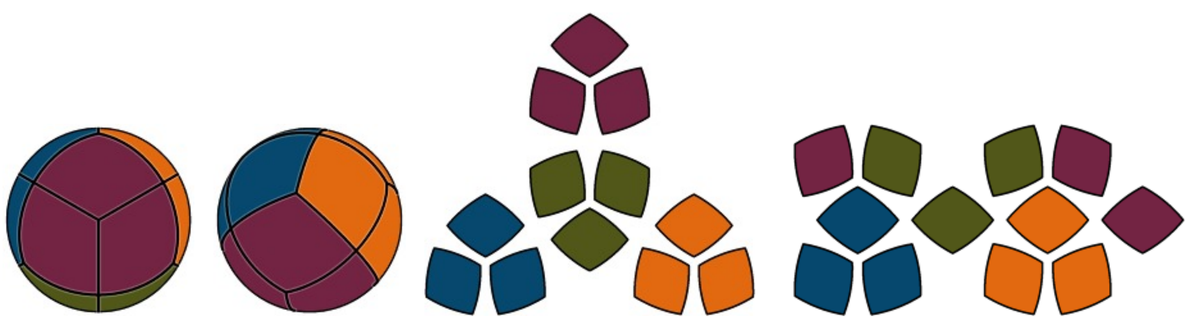
#14b: Tri-Color Belt Type B. In this variation of the above arrangement each of the three belt colors is on a pair of adjacent panels. Viewed from one side the pairs will align with the polar triangle's sides, and from the other side each pair will surround a corner of the polar triangle.



#15: Belt with Trio Caps. Same as the 2-color Belt arrangement, but each triangular cap above and below the belt is a trio of colors. Each color is opposite its match, and so the sequence on one cap will be the reverse of the other in terms of clockwise/counter-clockwise direction.



#16: Four-Color Patchwork Ball. Four colors arranged so that no panel touches a matching one. Each color is arranged in a ring around a triangle-shaped patch as shown by the yellow panels in the second illustration.

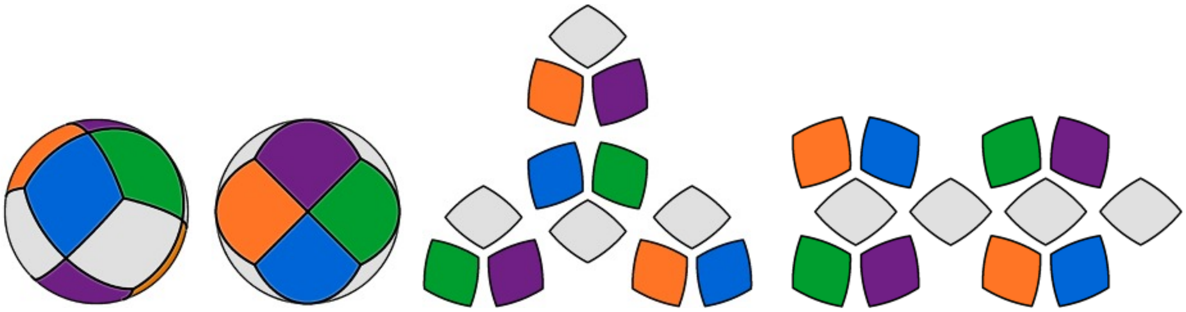


#17: Tetrahedron. Each color on a triangular patch of three panels, which forms the appearance of a spherical tetrahedron.

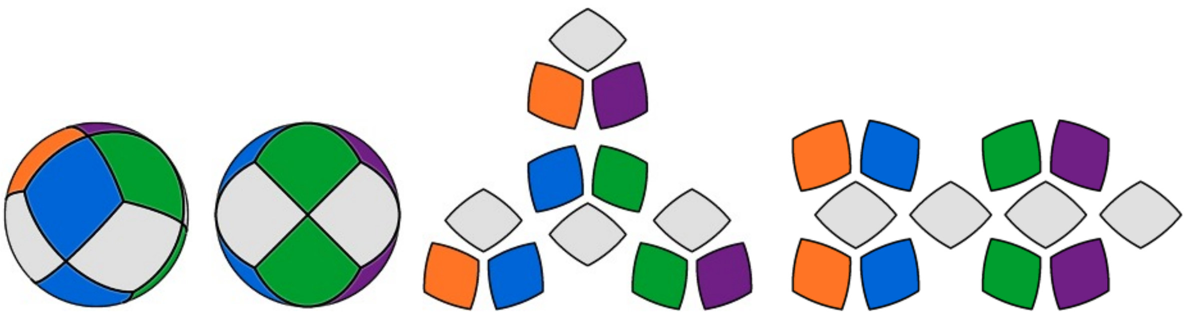
5 colors

Back to Chapter Index

#18: Quad-Color Diamond Ring. Same as the 2-color Diamond Ring arrangement, but the ring consists of four unique colors.



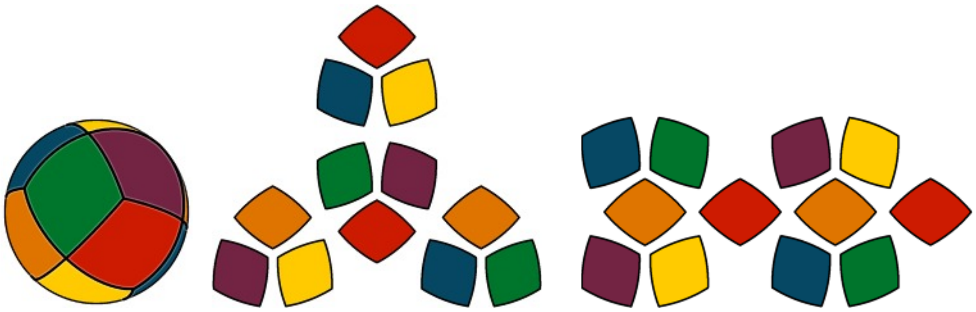
#19: Pinwheels. Related to the Diamond Ring arrangement (four contrasting diamonds joined at their acute corners that encircle the ball), but the ring is a neutral color (white in this example) and each 4-panel cap above and below the ring now consists of four bold colors. Each color is opposite its match, which means that the sequence on one cap will be the reverse of the other in terms of clockwise/counter-clockwise direction.



#20: Four Checkered Stripes. Similar to the Pinwheels arrangement above, but the four bold colors on each 4-panel cap are aligned with those of the opposite cap, forming four longitudinal stripes across the neutral diamond ring in the middle.

6 colors

Back to Chapter Index



#21: Six-Color Patchwork Ball. Each of the six colors is on a pair of opposite panels.

Cutting Out the Templates

Back to Chapter Index

To make an exterior type template, simply use scissors to cut along the pattern. If you want to make a stencil (interior) type, you will need to use an X-Acto knife. If you lack the skill to cut curves with a knife, you can convert the curves into three or more straight cuts that approximate the curve.

If you use a thick marker to trace the patterns, remember to **stitch on the side of the pattern lines where the edges of the template were** (inside the lines for exterior templates, outside the lines for stencil or combo), so you don't change the size of the ball. If the marker soaks through the fabric you're using, however, you will need to stitch inside the patterns to hide the lines within the seams. In that case, when using stencil or combo templates, cut out the templates' interiors slightly outside the lines, shifting the edges outward by the width of the marker lines. Then the edges of the patterns they produce will be correctly positioned for stitching inside them. For combo templates, shift the outer edges by the same amount to maintain the same seam allowance.

I recommend keeping the inner part that you cut out of stencil or combo templates for use in drawing the front stitching patterns. Step 2 of the Assembly instructions explains why.

Making the Panels

Back to Chapter Index

Depending on the type of template you're using (exterior or interior) and whether it is translucent or not, you must be careful which pattern, cutting or stitching, you trace first so that the **second template doesn't hide the lines of the first** and prevent you from aligning the two. **Do not necessarily use them in the sequence below**.

1. You will need 12 panels, and **you will be tracing the patterns onto the back of the fabric (the side that will be inside the bag)**. If you use a cutting template, first trace that. **I recommend orienting the pattern so that the grain or cords of woven fabric runs along a diagonal, and the same diagonal on all panels.** I orient them along the long diagonal.
2. Use the smaller, stitching template to trace the stitching pattern within each cutting pattern, being sure to center it well (centering it allows you to align the patterns more easily as you sew, but is not otherwise important).

3. Cut out the panels.

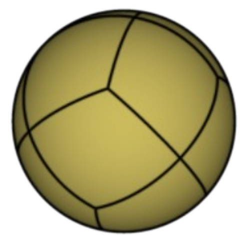
Assembly

Back to Chapter Index

I devised two methods of assembling this structure. The first and simpler method uses six threads and duplicate stitching on four seams. It involves forming four triangular triplets of panels and then assembling those like a tetrahedron. The second method is more optimized, requiring only four threads and two restitched seams, but is a little more complicated and difficult. It involves forming two patches of five panels and then joining them with a pair of additional panels between them. The illustrations show both methods, and the written instructions apply to both.

I figured out both methods before actually making a beanbag, and then, after making several beanbags and trying both methods, **I found I preferred the tetrahedron method**. It feels less tedious, and the tri-panel patches are easier to make and join to each other than the penta-panel patches. But the other method works well, and some may prefer it, so I included both.

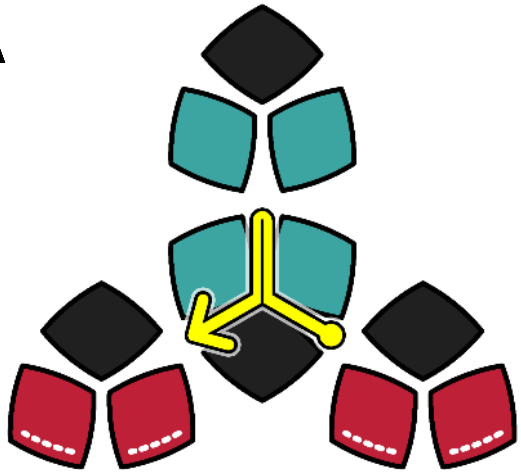
Helpful Hints: Remember that when placing panels together to sew them, or when any pair of panels are folded together along their joined seam, they should match up corner to corner, edge to edge. Each type of corner, acute and obtuse, will join only to others of its type. The sharper corners point into 4-way vertices and the blunter corners point into 3-way vertices. And be careful not to join five corners together.



The assembly illustrations begin on the next page and the written instructions begin on the page following that.

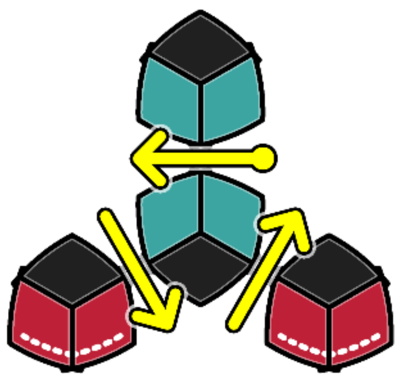
Tetrahedron Method

A



Stage 1: Arrange the panels, draw the front stitching patterns (the dashed lines), and sew each group of three panels into a triangular patch.

B



Stage 2: Sew around the central patch (either direction), attaching each of the other patches to its edges.

C

Continue stitching around the two remaining sides of the top-most patch.

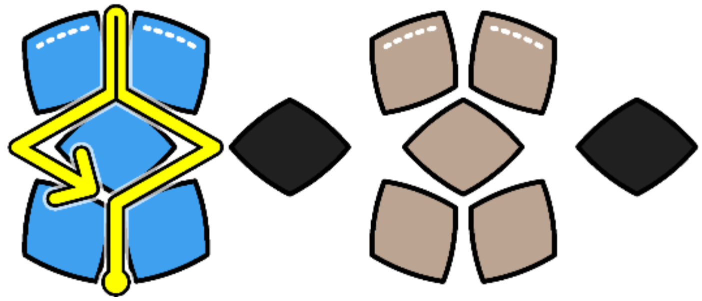
D

Note that in C & D the ball is still inside out and so the front stitching lines (the dashed lines) would not actually be visible. I show them just for positional reference.

Sew a few starter stitches along the final pair of seams with the front stitching patterns.

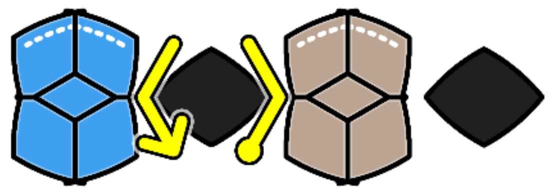
Optimized Method

A



Stage 1: Arrange the panels, draw the front stitching patterns (the dashed lines), and sew each group of five panels together following the arrow.

B



Stage 2: Sew around the single panel between the patches (either direction), connecting it to the two patches. Start and end at the corner opposite the front patterns.

C

Continue along the pair of seams between the two patches.

D

Add the last panel and continue to sew around that one.

E

Note that in D & E the ball is still inside out and so the front stitching lines (the dashed lines) would not actually be visible. I show them just for positional reference.

Sew a few starter stitches along the final pair of seams with the front stitching patterns.

1. ***Stage 1, Illustration A:*** **Lay the panels out as shown for your preferred assembly method** (I prefer to place them front face up) and **arrange them according to your color pattern**.
2. Use the stitching template to **draw stitching lines on the *fronts*** of the four panel edges shown with dashed lines in the diagrams. My stitching pathway leaves these edges partially unsewn so the bag can be turned out between them. They will then be **sewn from the outside following the front stitching lines**. (If you use a thin or flexible fabric and don't need such a large opening, just skip marking the inner or outer pair of panels.) Be sure to align the template as well as possible with the stitching patterns on the backs.

   If you want to **hide the stitching lines within the seams**, sketch them a millimeter or two nearer to the panel edges and stitch slightly inside them (toward the middle of the panels). **If you use a Stencil or Combo type template,** use the inner portion that you cut out of the template to draw these patterns, since the main template will cover the area near the edge.

   I have found it helpful to **add marks along the front stitching lines for each stitch** so that I can more easily keep the exterior stitches even with each other and not get a skewed seam. I space the stitch marks ⅛″ (3mm) apart. If you **make these marks on your template first**, you can more easily transfer them onto these and future panels.
3. **Sew each group of three or five panels together** as shown by the yellow arrow in Illustration A of your chosen assembly method. Place two panels **front faces together** and sew along the edge from the acute corner to the obtuse corner. Then add the next panel, making sure that its obtuse corner joins with the obtuse corners of the first two panels, and continue into that seam. For the tetrahedron method, double back to the center and then sew the third seam. For the optimized method, continue around that third panel, attaching the fourth panel, then add the fifth and sew it to the fourth. Then double back to the middle panel and continue around it.

   If you are using the backstitch, you can make the duplicate stitches up to twice as long without causing the fabric to ripple as long as you're careful how tightly you pull them (if you pucker the fabric, wiggle it straight again). Tie off the thread and trim it after each patch. Place each finished patch in the original position among the others.

   When all the patches are finished, consider ironing their seam allowances flat; see the **General Information and Techniques** chapter under "Better Seams by Ironing".
4. ***Stage 2, Illustrations B – E:*** Now **sew the patches together**, following the arrows in the remaining illustrations of your chosen assembly method. **Place the patches front faces together to sew them.**

   For the tetrahedron method, sew around the central patch (in either direction), attaching each of the others to its edges. Start and end on one of the top corners so that you can then continue the thread up and down the top patch's remaining edges as shown in Illustration C. This leaves one open pair of seams, which have the front stitching lines drawn on them.

   For the optimized method, attach the central lone panel to the lower edge (opposite the front patterns) of either patch and sew toward the middle of that patch. Continue around the panel, attaching the four edges of the two patches to it. Continue the thread into the seams between the two patches as shown in Illustration C (make sure these are the seams that do *not* have the front stitching lines on them). Then continue around the second lone panel as shown in Illustration D. This leaves one open pair of seams, which have the front stitching lines drawn on them.

When you reach the 4-way seam intersections in the middle of each patch's edge and have to **cross the perpendicular seams**, I recommend referring to the "Stitching Techniques" section of the **General Information and Techniques** chapter under "Crossing seam intersections...". The retreating stitches I recommend at the intersections help to tightly close the 4-way vertices, which can open up a bit if they are not cinched shut well. **To really close the intersections well**, follow the second method in the "Closing seam intersections tightly" topic in the same section. **In short, stitch each panel corner to the one diagonally opposite it (the thread will form an X across the intersection) and cinch them together.**

5. Start a new thread and **sew a few starter stitches** at one end of the final seams to make it easier to continue from the outside. If you don't need the entire opening to turn the bag out, continue to sew as much as you don't need. To **reduce the number of stitches you need to make from the outside**, you can make extra stitches and then loosen them to allow the panels to spread enough to turn the bag out. Then you can pull them tight again from the outside. If you want to do this, or if you want to be able to loosen the last several stitches enough to push a funnel between them, **your final thread will need several inches of extra length**.
6. At this point, **consider ironing the remaining seam allowances**.
7. **Turn the bag right side out through the opening**. A good method for this is to use the back end of a pen or other slender tool to push the fabric through the opening from the opposite side and then either invert the bag around the tool, or remove the tool and work the bag through with your fingers. **Be gentle so as not to pop any stitches**.
8. **Pull out the last stitch so that the thread is on the outside** where you can get to it. Continue sewing the opening closed following the front stitching lines. For help, see the "Stitching Techniques" section of the **General Information and Techniques** chapter under "Backstitch from the exterior Approaches". Fill the bag at some point during this final sewing with a funnel. I find it helpful to **put some filler in first to prevent the bag from collapsing** while I sew, and to hold the seam allowances in place and give me something to push the needle against.

   You can sew the entire opening closed before fully filling the bag, which prevents the filler from spilling back out while you sew. Just loosen the last several stitches enough to push the funnel between them, or at least to push some filler in with your fingers. Then re-tighten the stitches (see "Tips on finishing the bag").

Back to Chapter Index

Ready-to-Print Patterns

Back to Chapter Index

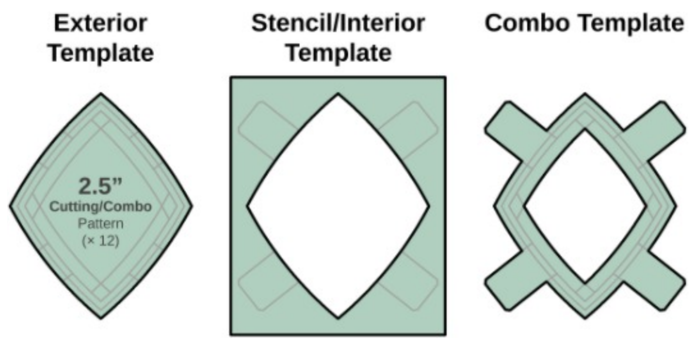
The pattern pages are 8.27″×11″ (210mm×279mm) to fit both "Letter" and "A4" sizes. ***Make sure the print is not being scaled to fit the printer margins*** *(select Default/None scaling/Actual size/Ignore printer margins). To verify correct sizing,* ***compare the centimeter grid to a ruler*** *and adjust the next print if necessary. (Note that PDF viewers and printers can both contribute to slight size inaccuracy.)*

On the following pages are patterns for beanbag diameters from 2″ – 3″ in ¼″ increments, and a 7″ pattern for scaling to larger sizes. The patterns are reduced by 2.73% from the mathematical calculation to account for the inflation in size I observed in my corduroy bag. **If you use a dense/stiff or completely non-stretch fabric, I recommend enlarging the pattern to about 103% to get the intended ball size.**

To make the template, I recommend cutting out the portion of the paper with the pattern you want and gluing or taping it to your template material, and then cutting along the pattern.

The cutting patterns have 4mm, 6mm, and 8mm allowances so you can choose the amount that works best for your fabric and preference (the lighter, 6mm pattern is a hair under ¼″), and they include **tabs for the optional combo type template** (stitching pattern on the inside, cutting pattern on the outside, with the tabs to help you hold it down). Two tabs may be sufficient with a rigid enough template.

The examples on the right show the **three ways you can cut out the Cutting/Combo templates** (using the 8mm allowance). Remember that the cutting patterns have slightly different curve radius to panel size proportions from the stitching patterns (they are parallel, not proportional), so you should not use them as stitching patterns.



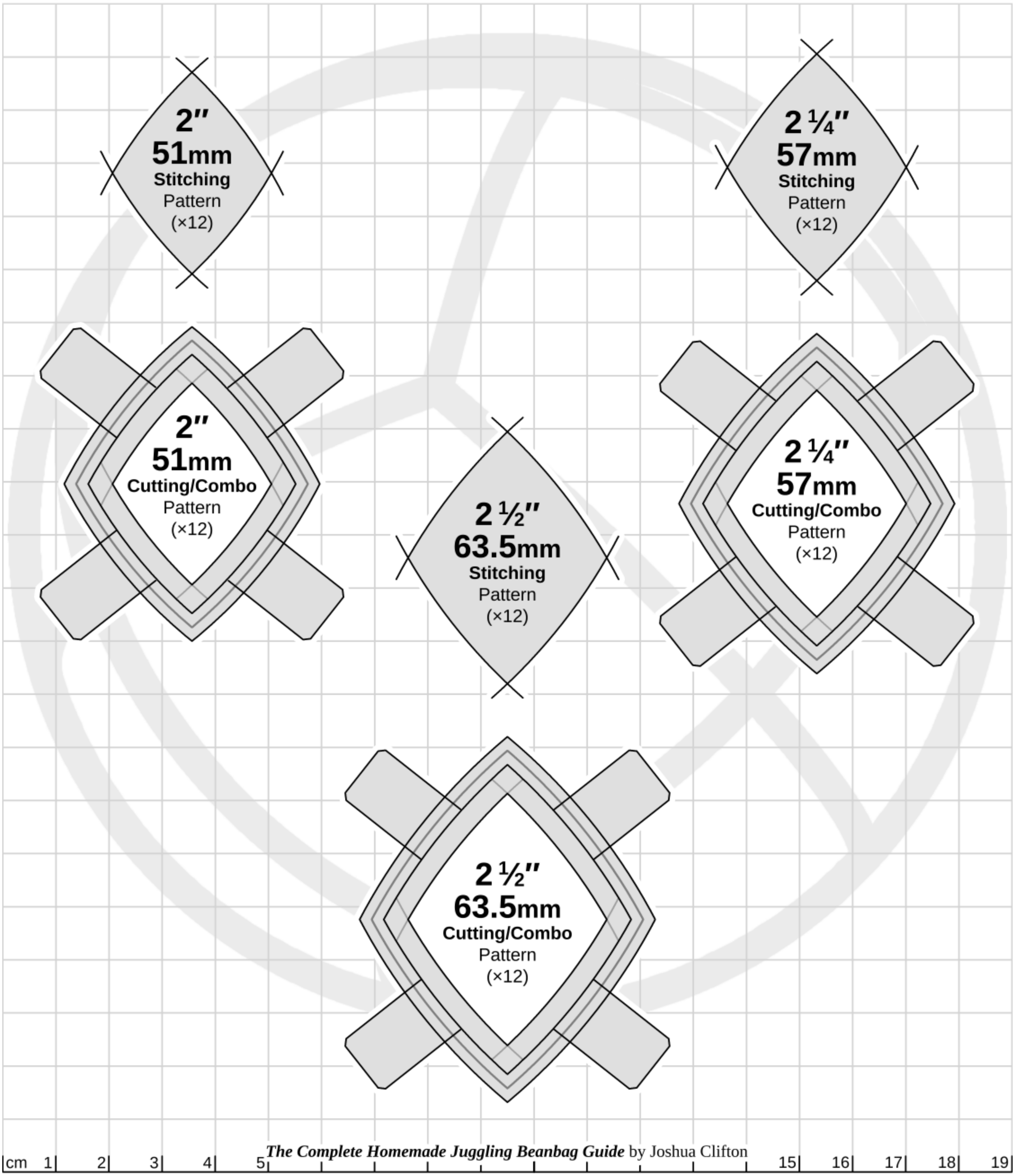
To produce other pattern sizes or correct an incorrectly sized beanbag, adjust the size scaling in the print dialog. For example, to reduce my 2.5″ pattern to the 2.3″ size recommended by the Juggling Store for small hands and numbers juggling, divide 2.3 by 2.5, multiply the result by 100, and that is your scale (92% in this case). If your beanbag ends up not being the expected size, see the **General Information and Techniques** chapter under "Adjusting/Scaling a Pattern to Produce an Accurate Ball Size" for help with correcting it.

The table below provides the scaling for the ⅛″ increments between my ¼″ sizes. The centimeter grid can be used to verify correct scaling.

Target Diameter	Print this pattern size	At this scale
1¾″ (44.5mm)	2″	87.5%
1⅞″ (47.6mm)	2″	93.8%
2⅛″ (54.0mm)	2¼″	94.4%
2⅜″ (60.3mm)	2½″	95%
2⅝″ (66.7mm)	2¾″	95.4%
2⅞″ (73.0mm)	3″	95.8%

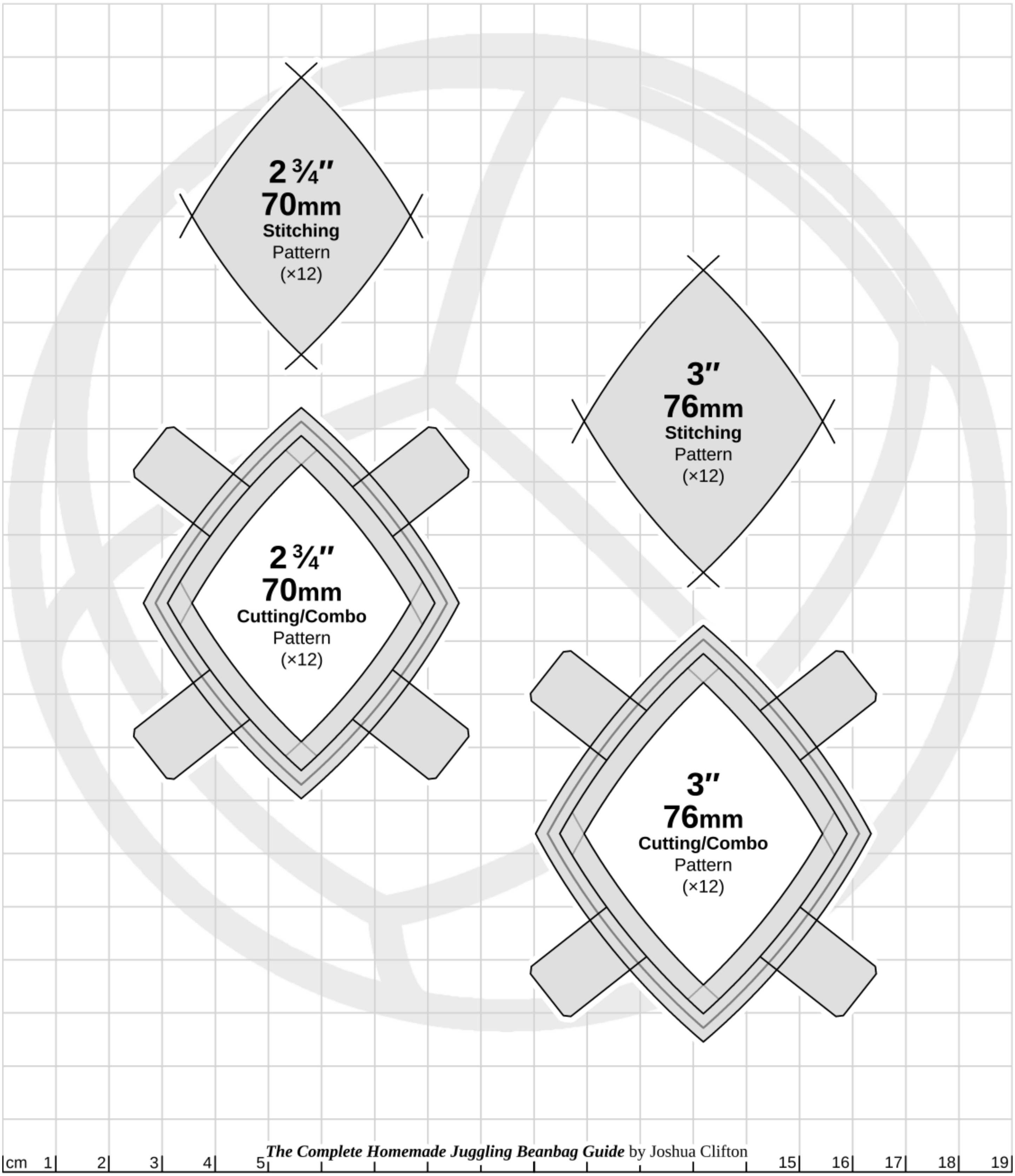
Rhombic Dodecahedron (12 panels)

(Pattern sizes are adjusted for corduroy and do not account for gathered seams)



Rhombic Dodecahedron (12 panels)

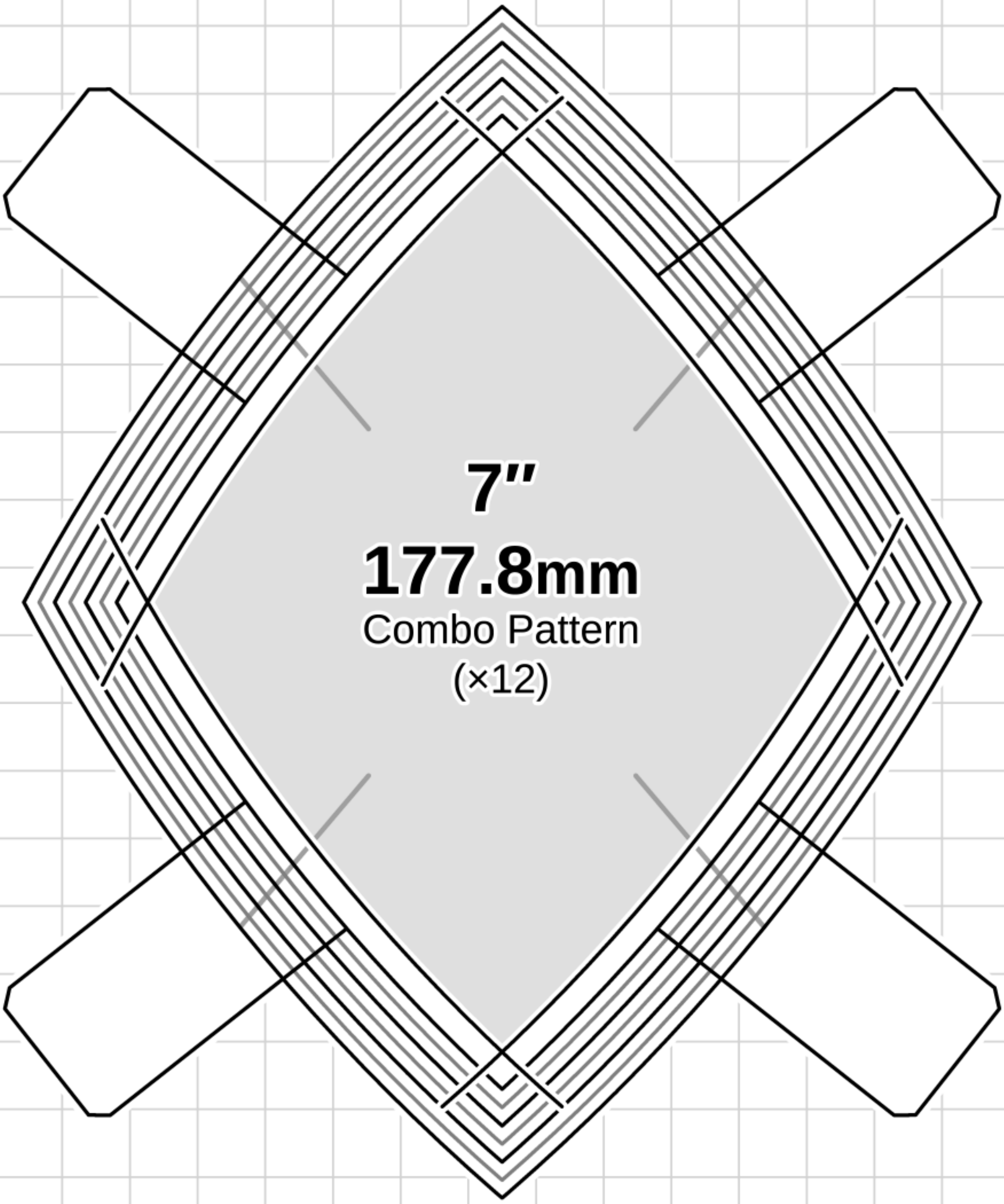
(Pattern sizes are adjusted for corduroy and do not account for gathered seams)

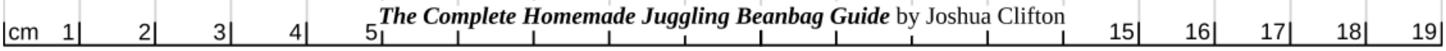


Rhombic Dodecahedron (12 panels)

(Pattern sizes are adjusted for corduroy and do not account for gathered seams)

Extra large and versatile pattern for scaling to larger sizes in the Print Dialog. Print twice if you want both a stitching template and a cutting template (or cut out a combo template). The inner pattern (filled with gray) is the stitching pattern. Each dark pattern outside of that marks a 4mm seam allowance interval (at 100% scaling). Use those or the lighter, half-intervals between them to cut out the amount of allowance you want for the cutting template.

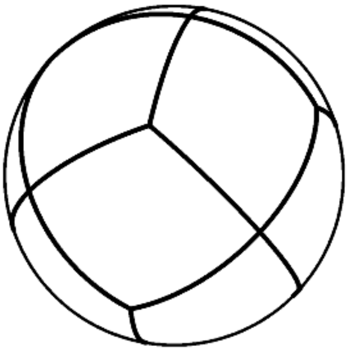




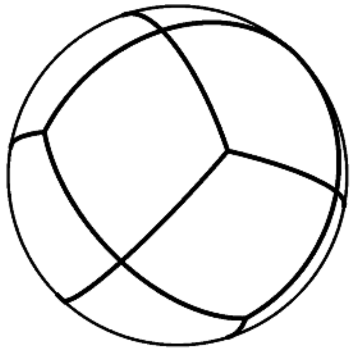
Blank Color Arrangement Diagrams

Back to Chapter Index

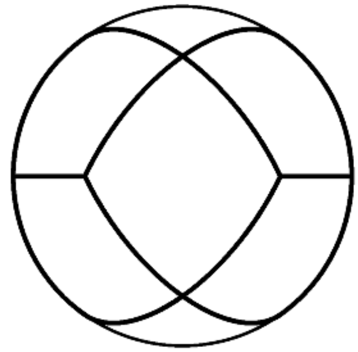
These are the ball diagrams and the assembly layout diagrams I used for my color arrangement illustrations. You can use these to experiment with your own arrangements. I also offer PNG format diagrams for download on my website that you can use in an image editor. If they are unavailable, you can capture a screenshot of these pages or export the images and then color them in an image editor. Or you can just print them and color them by hand.



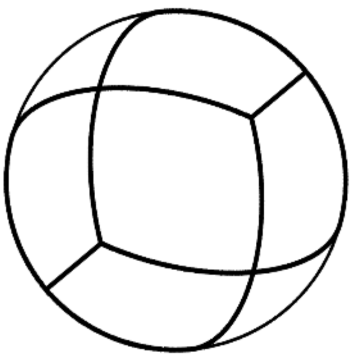
Standard view 1



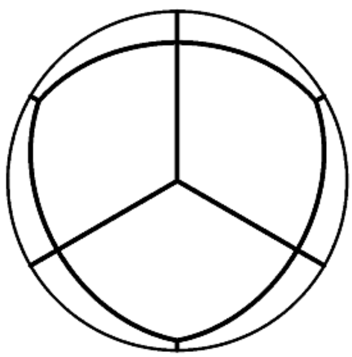
Standard view 2



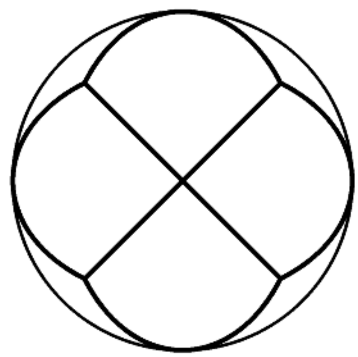
Diamond view



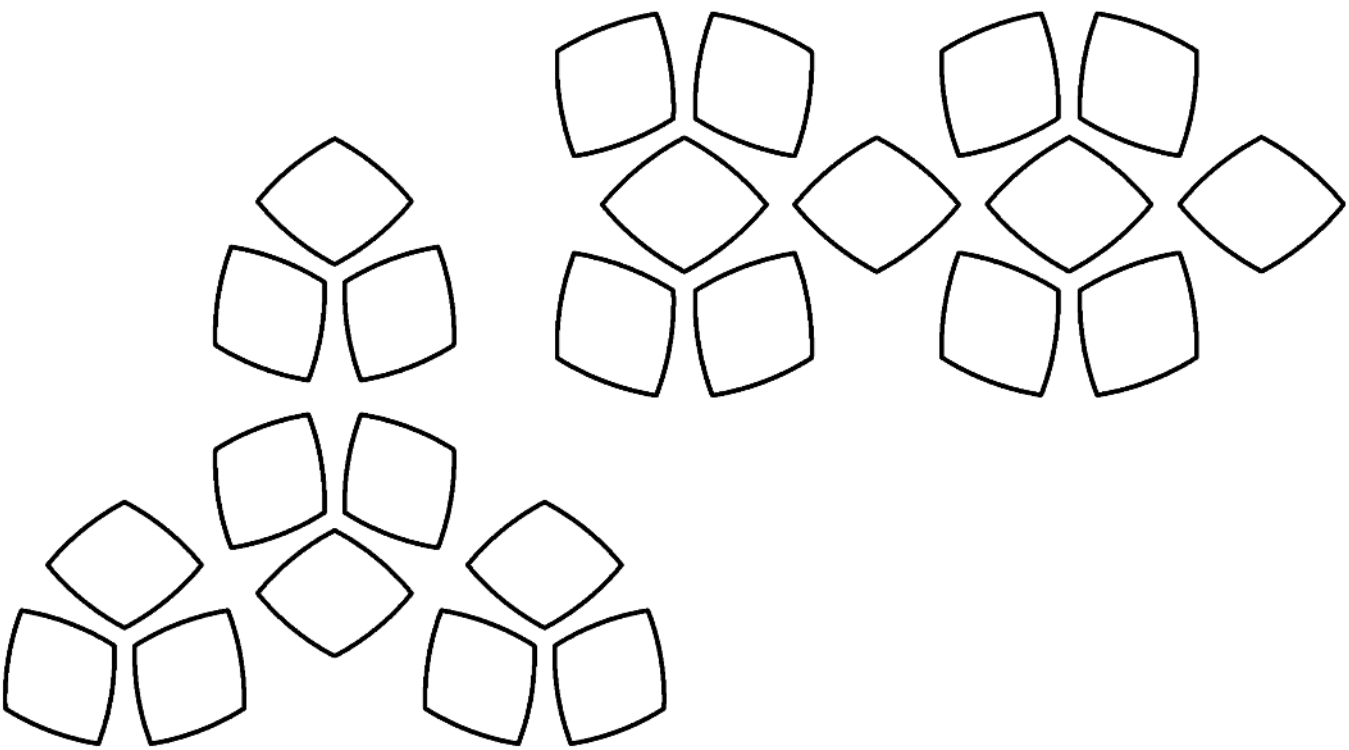
Belt view



3-Way Vertex view



4-Way Vertex view



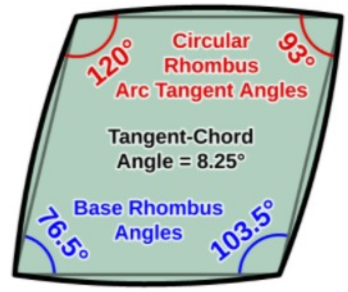
Sizing Formulas for Drawing the Pattern

Back to Chapter Index

The next section has a table of pre-calculated pattern measurements for all ⅛″ diameter increments from 1¾″ – 3″. Following that are the drawing instructions. If you do not need to create a custom size, skip to that. I provide printable measuring tapes at the end of the **General Information and Techniques** chapter in case you care to measure your beanbags. The "Mathematics" section has explanations of all the formulas, and expresses their values in higher precision.

Design summary

The panel shape is a rhombus (diamond) with circular edges. On the right is a generic diagram of the shape showing the angles of the base rhombus and the wider angles formed by the edge arcs. Drawing this shape involves drawing a guide rectangle, the corners of which are the locations of the four circles that intersect to form the circular rhombus.



Adjusting for the influence of fabric attributes on beanbag size

The bag I made with thick corduroy was 0.861 – 4.597% larger than the mathematical prediction depending on whether I filled it loosely or over-filled it (this is almost exactly the sizing of the rhombic triacontahedron, though over twice that of the regular dodecahedron, oddly). Moderately filled it was 2.189% larger. I target halfway between the min and max inflations when sizing my patterns, which is 2.73%. This makes my adjustment factor **1.0273**.

I use the adjustment factor to adjust the pattern size to produce a more accurate finished size when using my fabric and stitching techniques. If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may need to figure out your own adjustment factor. For help, see the **General Information and Techniques** chapter under "Adjusting/Scaling a Pattern to Produce an Accurate Ball Size".

The bag I made with my design testing fabric (fairly thin, stiff, tightly-woven, non-stretch), had an average and moderately filled inflation of almost zero (-0.006 & 0.009%). When loosely filled and tightly filled, it was 1.878% smaller and 0.778% larger, respectively. So if you are using a fabric like this, I recommend that you use the Base value in the measurement tables rather than the Adjusted value.

As I understand it, based on past beanbag experiments with denim and the more recent ones with corduroy and felt, the bag size is affected by fabric attributes as follows. The folding of the fabric at the seams will cause thick, firm fabrics like denim to significantly shrink the bag size unless the fabric has some stretch (denim stretches a little). Folding thin fabric doesn't consume as much of its size, and my design testing fabric is both thin and has no significant stretch, and so ended up almost the exact predicted size. Corduroy is a softer, more loosely woven fabric than denim and flexes and compresses more easily, and so not as much of the panels' size is consumed by the folding, and it stretches about like denim, and so it ends up larger.

Sizing formulas

Below are the formulas to calculate the pattern dimensions (*Diameter* refers to your target ball size). The value in orange is the adjustment factor. **Don't forget to multiply the final result by 25.4 if you need to convert inches to millimeters.**

- **Guide Rectangle Long Side = *Diameter* × 2.3471 ÷ 1.0273**
- **Guide Rectangle Short Side & Rhombus Arc Radius = *Diameter* × 1.7049 ÷ 1.0273**
- **Guide Arc Radius** (for hand drawing) = $\sqrt{2\left(\frac{Short\ Side}{2}\right)^2}$ (This is the hypotenuse of a right triangle with legs equal to half of Short Side)

Table of Pre-Calculated Pattern Measurements

Back to Chapter Index

The table below has stitching pattern measurements for each ⅛″ diameter increment from 1¾″ to 3″. The values in the **Adjusted** columns account for my 1.0273 adjustment factor. The adjusted values decrease the **Base** pattern size so that you will get a more accurate finished size when using corduroy or something similar (a soft, flexible, moderately thick fabric). If you are using a firm denim or a thin, but non-stretch fabric, use the Base value instead. I attempt to explain why in the "Adjusting for the influence of fabric attributes on beanbag size" topic above.

To make a cutting pattern, simply increase the Rhombus Arc Radius by the desired seam allowance. The rest of the measurements remain the same.

Finished Diameter	**Guide Rectangle Long Side** (mm)		**Guide Rectangle Short Side & Rhombus Arc Radius** (mm)		**Long Minus Short*** (mm)		**Guide Arc Radius*** (mm)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
1¾″ (44.5mm)	104.328	**101.556**	75.781	**73.767**	28.547	**27.789**	53.585	**52.161**
1⅞″ (47.6mm)	111.780	**108.810**	81.194	**79.036**	30.586	**29.774**	57.413	**55.887**
2″ (50.8mm)	119.232	**116.064**	86.607	**84.305**	32.626	**31.759**	61.240	**59.613**
2⅛″ (54.0mm)	126.684	**123.318**	92.020	**89.574**	34.665	**33.743**	65.068	**63.339**
2¼″ (57.2mm)	134.136	**130.572**	97.433	**94.843**	36.704	**35.728**	68.895	**67.064**
2⅜″ (60.3mm)	141.588	**137.826**	102.845	**100.112**	38.743	**37.713**	72.723	**70.790**
2½″ (63.5mm)	149.040	**145.080**	108.258	**105.381**	40.782	**39.698**	76.550	**74.516**
2⅝″ (66.7mm)	156.492	**152.334**	113.671	**110.651**	42.821	**41.683**	80.378	**78.242**
2¾″ (69.9mm)	163.944	**159.588**	119.084	**115.920**	44.860	**43.668**	84.205	**81.968**
2⅞″ (73.0mm)	171.396	**166.842**	124.497	**121.189**	46.899	**45.653**	88.033	**85.693**
3″ (76.2mm)	178.848	**174.096**	129.910	**126.458**	48.938	**47.638**	91.860	**89.419**

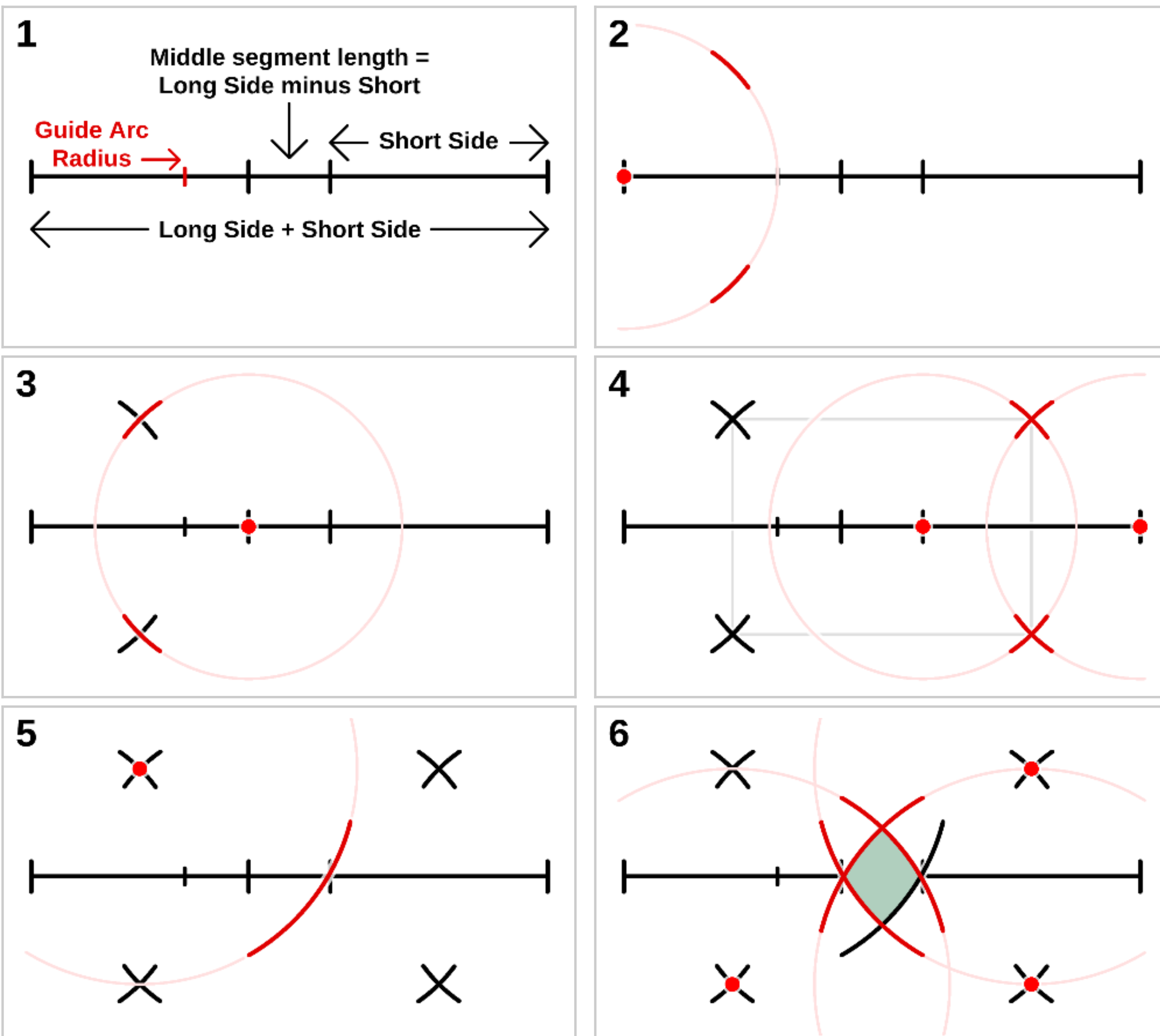
* Only used for drawing manually.

How to Draw the Panel Shape

Back to Chapter Index

The panel shape is a circular-edged rhombus quadrilateral or diamond. It is formed by drawing a guide rectangle and drawing circles centered at each of its corners. There are instructions for drawing the shape manually, followed by instructions for drawing it with a CAD program (Sketchup in particular). For the manual drawing instructions, you don’t actually draw a rectangle, just its corners using circular arc intersections.

There is a separate set of illustrations for each method. Their numbers correspond to the step numbers. To conserve your template material, I recommend that you draw the pattern on paper and then glue or tape the pattern to your template material before cutting it out.



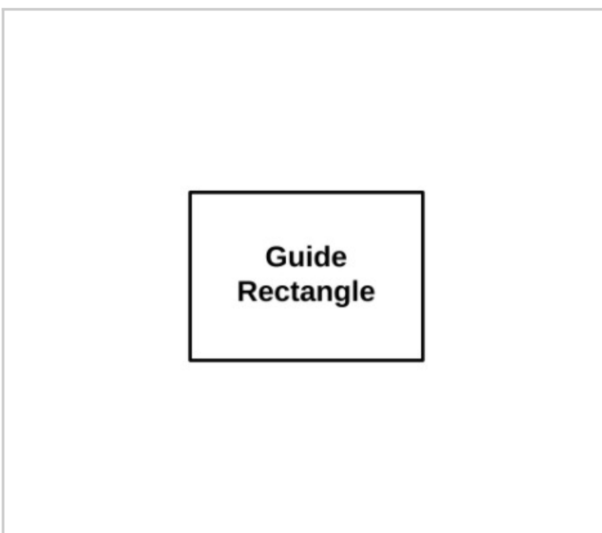
Manual directions

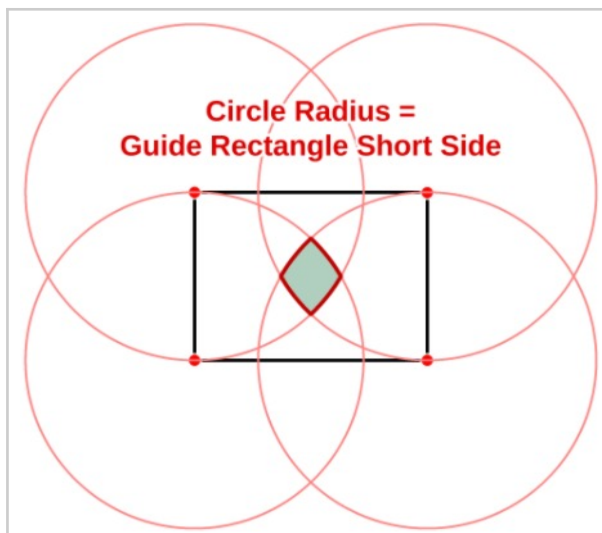
(The terms in bold refer to columns in the pattern measurement table above.

1. Draw a horizontal line the length of **Long Minus Short** and mark each end of it. Then continue the line on both ends by the length of **Guide Rectangle Short Side** and mark the ends of those

lines. The total length of the line is **Guide Rectangle Long Side** + **Guide Rectangle Short Side**. Make another mark the distance of **Guide Arc Radius** from the left end of the line.

2. Place the compass on the left end of the line and extend it to the **Guide Arc Radius** mark. Draw a small arc above and below the line, between the compass point and the near end of the middle segment.
3. Position the compass on the left end of the middle segment and draw two arcs that cross the first two, forming X-shaped intersections.
4. Repeat this procedure at the other end of the line. This results in four points marked by Xs, which are the four corners of the guide rectangle (shown in faint gray).
5. Adjust the compass to **Rhombus Arc Radius**, which is the shorter distance between the arc intersections, and also the distance between the endpoints of the line and the short middle segment, place it on one of the four arc intersections, and draw a quarter arc or so across the initial line.
6. Repeat the previous step for each of the four arc intersections. The intersection of the four new arcs forms the circular rhombus in the middle.
7. To draw a cutting pattern, draw everything the same, but for the final four arcs, increase the compass radius by the desired amount of seam allowance.





SketchUp directions (Protractor method)

(The terms in bold refer to columns in the pattern measurement table above.)

1. Draw a rectangle with sides equal to **Guide Rectangle Long Side** and **Guide Rectangle Short Side**.
2. Draw circles centered at each of the rectangle's corners with radius equal to the guide rectangle's short side (simply extend the radius to the other end of the short side). The intersection of the four circles forms the circular rhombus in the middle.

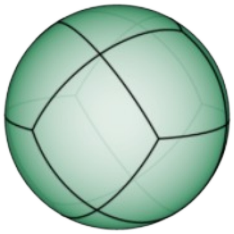
3. To draw a cutting pattern, simply increase the radius of the circles by the desired seam allowance.

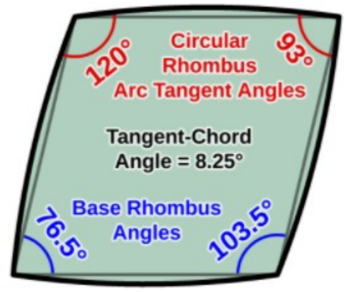
Back to Chapter Index

Mathematics Behind the Relationship Between the Pattern Parameters and the Ball Size

Back to Chapter Index

This section describes the math involved in drawing patterns to produce specified beanbag sizes, and creating the pattern sizing formulas. *(The numbers in tiny, right-justified typeface are my computer calculator's unrounded values which I display rounded to six places for brevity.)*





My method of designing this shape was to decide upon target vertex sums on the ball (the sum of angles of the three or four panel corners that form each vertex), and then calculate a rhombus and arc that would produce those sums (see the "How I Developed This Design" section for more information on my process). The result is a rhombus with 76.5° & 103.5° corners and an arc whose tangent forms an 8.25° angle to the rhombus' edges. This arc widens the corners by 16.5°, resulting in a 372° angle sum at the ball's 4-way vertices and a 360° sum at the 3-ways. For the theory behind my design methods, and the significance of the tangent-chord angle, read the "Curved-Edge Faces" section of Chapter 5 of the *01 – Homemade Juggling Beanbag Guide – Index & Supplementary Chapters* document. The "How I Developed This Design" section has a brief summary.

My first step was to calculate the base rhombus.

Base rhombus calculations

To calculate the rhombus angles and tangent-chord angles that will produce specific vertex sums, three equations are needed. I will define ***a* to be the acute angle, *b* to be the other angle, and t to be the tangent-chord angle** (the angle between the rhombus edge and the tangent to the arc that spans it). I will create three equations defining the necessary properties of the angles, solve two of them for the variables of the third, substitute them into the third, and solve that.

$\mathbf{a + b = 180°}$	(property of a rhombus)
$\mathbf{4a + 8t = 372°}$	(4 acute corners with a tangent-chord angle on either side of each)
$\mathbf{3b + 6t = 360°}$	(same as above, but for the 3 obtuse corners)

Solve the first equation for b: $b = 180 - a$
Solve the second equation for t: $t = 46.5 - 0.5a$

Substitute both into the third equation: $3(180 - a) + 6(46.5 - 0.5a) = 360$

Solve for a: $540 - 3a + 279 - 3a = 360$ ► $6a = 459$ ► $\mathbf{a = 76.5°}$
So $\mathbf{b} = 180 - 76.5 = \mathbf{103.5°}$
and $\mathbf{t} = 46.5 - 0.5(76.5) = \mathbf{8.25°}$

Calculating the circle radius and sagitta

Here, I will recalculate the arc's tangent-chord angle using a more direct method that requires one of the rhombus' angles and corresponding vertex sum (just to show how it is done). Then, using my edge arc radius formula from Chapter 5, I will use the tangent-chord angle to calculate the radius needed to produce that arc (in terms of the rhombus side length, s), which is the first step toward calculating the guide rectangle, whose corners are the circle centers for the arcs that form the circular rhombus. I will also need the Sagitta, or the height of the arc's apex over the rhombus edge, so I will calculate that, too.

Tangent-Chord Angle $= \dfrac{372 - 4(\textit{Rhombus}\ \text{Acute}\ \textit{Angle})}{8} = \dfrac{372 - 4(76.5)}{8}$ **= 8.25°**

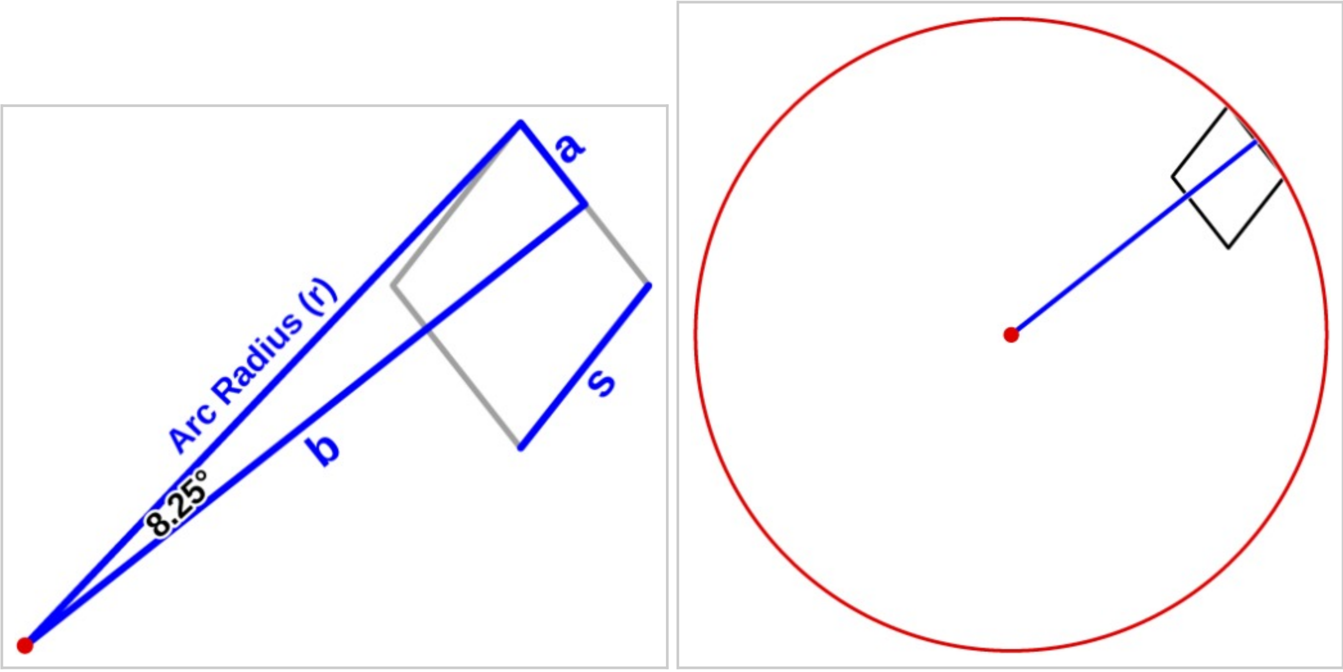
or $\dfrac{360 - 3(\textit{Rhombus}\ \text{Obtuse}\ \textit{Angle})}{6} = \dfrac{360 - 3(103.5)}{6}$ **= 8.25°**

Arc Radius, *r* = $\dfrac{0.5s}{\sin 8.25°} \approx \dfrac{0.5s}{0.143493}$ **≈ 3.484500*s*** (s = base rhombus side length) 3.4844997119833499454194200520516

Sagitta, *g* = $r - \dfrac{0.5c}{\tan 8.25°}$ **≈ 0.036060*s*** 0.036059739646286235744813156922 6

Calculating the circle positions relative to the rhombus

To draw the circular rhombus initially, I draw a base rhombus with a simple edge length such as 100mm, calculate the circle radius needed for that rhombus, and then calculate the distance between where the circle center would lie and the center of the rhombus' edges, and draw a line to those points. That is side b in the illustration below. Side a is half of the rhombus' side length. The circle radius line then completes a right triangle by adding the hypotenuse. I can calculate the length of side b because I know the other sides and the angles.



$r \approx 3.484500s$

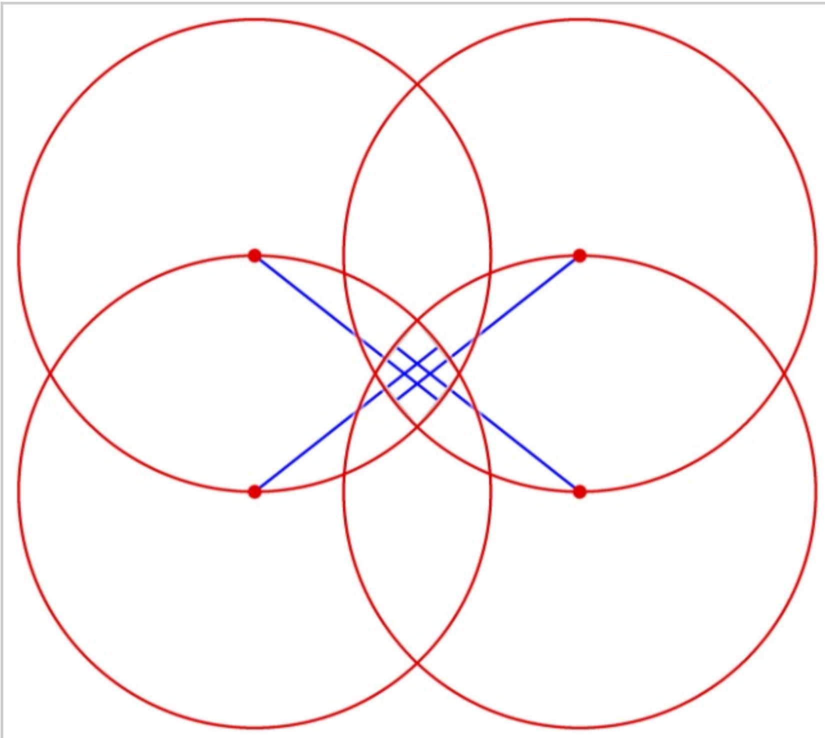
$a = 0.5s$

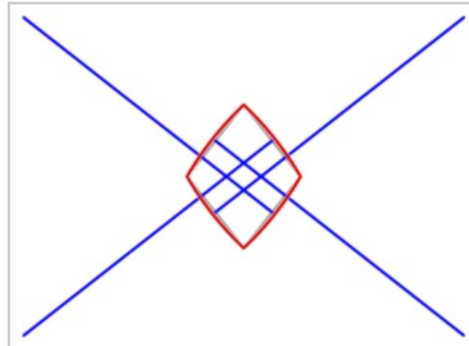
$b = \dfrac{a}{\tan 8.25°}$ or $\sqrt{r^2 - a^2}$ ►

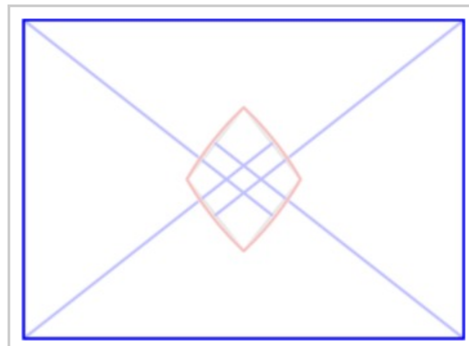
$b = \dfrac{0.5s}{\tan 8.25°}$ or $\sqrt{(3.484500s)^2 - (0.5s)^2} \approx 3.448440s$ 3.4484399723370637096746068951 29

Now that I know the perpendicular distance from each rhombus edge to the corresponding circle center, I can draw four sticks, one from each edge, and draw a circle centered at the end of each stick, and that forms my circular rhombus where the circles intersect.

The endpoints of those four sticks form the corners of a rectangle as shown on the bottom right. The short side of the rectangle happens to be equal to the circle radius, but I will perform the calculations to prove it.





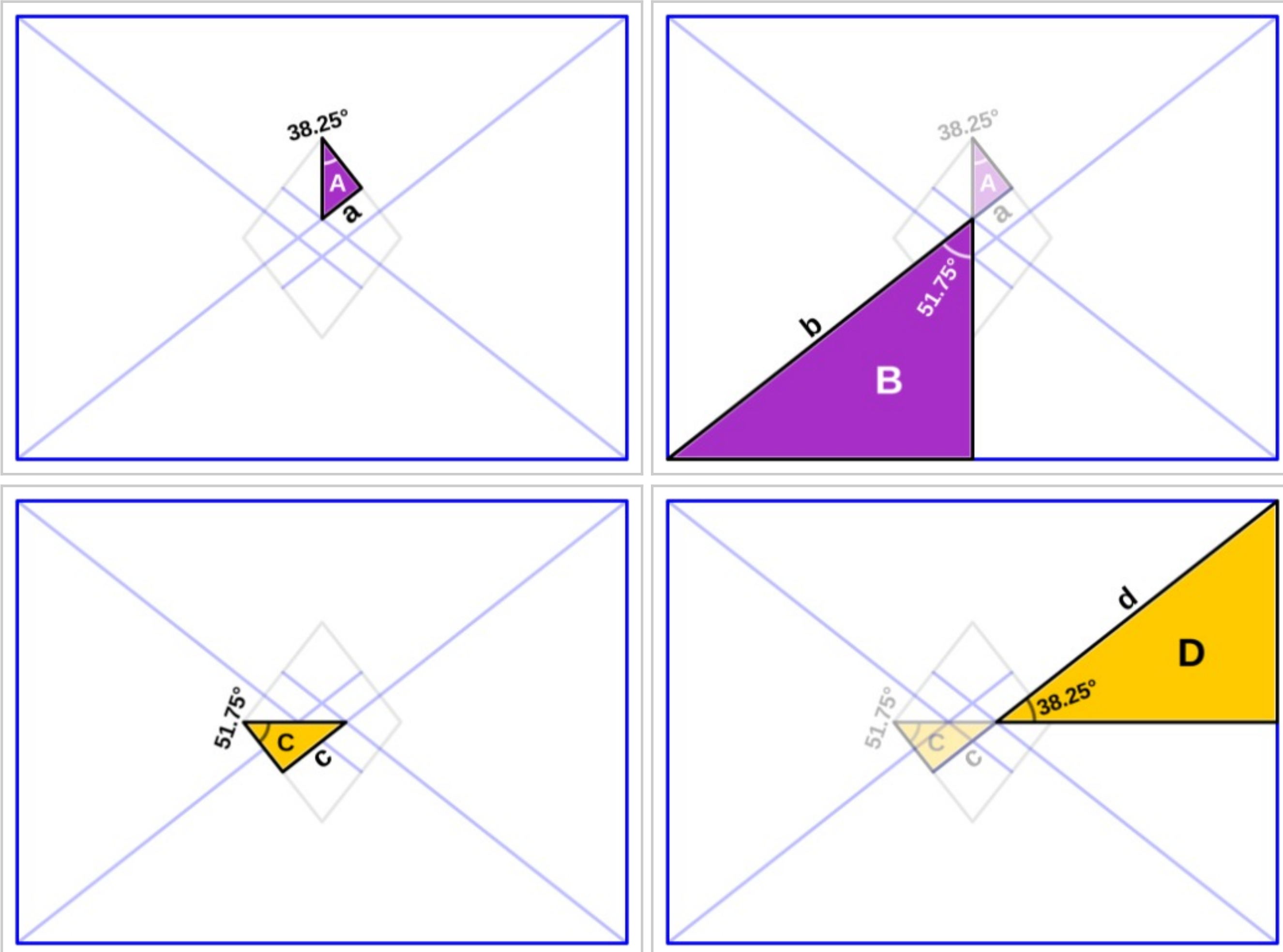


What I need in order to draw the circular rhombus of a desired size is the relationship between it and the guide rectangle that defines its circle centers. Then, instead of drawing an arbitrary rhombus and building the circular rhombus onto that, I will be able draw a guide rectangle and four circles centered at its corners, and that will produce the correctly-sized circular rhombus directly. That is what the next topic covers.

Calculating the ratio between the rhombus edge length and the guide rectangle

Calculating the guide rectangle is complicated. More complicated than for the designs based on Platonic or Archimedean solids. I will first define four triangles based on the guide rectangle and the circle center sticks, which will enable me to calculate the rectangle's dimensions in terms of the rhombus edge length.

The rectangle's side lengths are at this point unknown, and so is their relationship to the rhombus and the circle center sticks. The things I do know are the arc radius, sagitta, and tangent-chord angle, and the rhombus' angles. I also know that the circle center sticks' length is equal to the arc radius minus the sagitta. That is important to the formulas that follow. Using those facts and the triangles illustrated below, I can solve for the unknown lengths in terms of the rhombus' side length, s.



For the rectangle's long side I first need to calculate side *a* of triangle A, and then calculate side *b* (the hypotenuse) of triangle B by subtracting side *a* and the arc's sagitta (apex height), *g*, from the arc radius, *r*. Then I can use that to calculate the rectangle's long side. Then I need to repeat that procedure for the short side.

Triangle A side $a = 0.5s(\tan 38.25°) \approx 0.394168s$ 0.39416821729254625329463284504431

Triangle B side $b = r - g - a \approx 3.054272s$ 3.0542717550445174563799740500847

Guide Rectangle Long Side, $G_L \approx 2((\sin 51.75°)(3.054272s)) \approx \mathbf{4.797143s}$ 4.7971426414946136350942451452215

Triangle C side $c = 0.5c(\tan 51.75°) \approx 0.634247s$ 0.63424697637266229255294858938314

Triangle D side $d = r - g - c \approx 2.814193s$ 2.8141929959644014171216583057459

Guide Rectangle Short Side, $G_S \approx 2(\sin 38.25°)(2.814193s) \approx \mathbf{3.484500s}$ 3.4844997119833499454194200520516

Now I can draw a circular rhombus of any size I want in terms of the rhombus' side length. But what I need is to be able to draw a circular rhombus that will produce a desired ball size. That is what the next three topics cover.

Calculating the circular rhombus' dimensions

To draw a circular rhombus sized for a desired ball size, I need to know the relationship between the guide rectangle and the resulting ball size produced by the circular rhombus it forms. First, I need to know the dimensions of the circular rhombus (still in terms of the rhombus' sides, *s*).

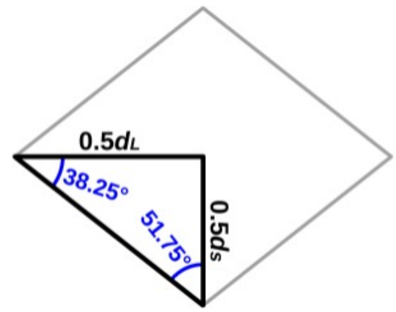
I will define the following variables to define the circular rhombus:

s = side length of the angular rhombus within the circular one
d_L = long diagonal
d_s = short diagonal
e = edge arc length

The diagonals can be calculated in terms of the side length by creating a right triangle that bisects both of the rhombus' angles and applying the trig functions to it:

$\boldsymbol{d_L} = 2(\sin 51.75°)s$ or $2(\cos 38.25°)s \approx \mathbf{1.570634s}$ 1.5706338617614898549406805578949

$\boldsymbol{d_s} = 2(\sin 38.25°)s$ or $2(\cos 51.75°)s \approx \mathbf{1.238188s}$ 1.2381878986196679738831217124922

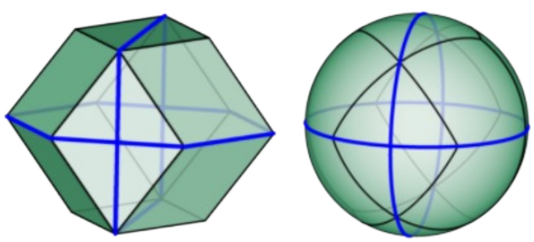


To calculate the edge arc length I can use the arc length formula from the end of Chapter 5. θ in this case is the tangent-chord angle.

$$\boldsymbol{e} = \frac{2\theta° r\pi}{180} = \frac{2(8.25°)(3.484500s)\pi}{180} \approx \mathbf{1.003464s}$$ 1.0034638805219089065042875046128

Calculating the ball circumference in terms of the angular rhombus' side length

Next, I need the circumference of the ball expressed in terms of the rhombus side length. The circumference can be calculated two different ways:



Circumference *A* (vertical) = $4d_L = 4(1.570634s) \approx \mathbf{6.282535s}$ 6.2825354470459594197627222315796

Circumference *B* (horizontal) = $2d_s + 4e = 2(1.238188s) + 4(1.003464s) \approx \mathbf{6.490231s}$ 6.4902313193269715737833934434356

B is 3.3% larger than *A*, and *B* occurs six times on the polyhedron versus three for *A*, so I will use a weighted average of the two:

Weighted Average Circumference ≈ 6.420999s 6.4209993618999675224431697061503

Guide rectangle and arc radius expressed in terms of the ball size

Now that I have defined the arc radius, guide rectangle, and ball circumference all in in the same terms (that of the inner rhombus edge), I can define them in terms of each other. In order to draw a guide rectangle and circles to produce a desired ball size, I need to express them in terms of the ball size. To calculate them in terms of the ball Circumference, C, I divide the expressions I calculated earlier by the circumference expression above. For the Diameter, D, I multiply the circumference expressions by π.

Guide Rectangle Long Side $\approx \dfrac{4.797143}{6.420999}C \approx \mathbf{0.747102}\boldsymbol{C}$ 0.7471021831833890341797851160980 7

$\approx \mathbf{2.347091}\boldsymbol{D}$ 2.347090730169830957502220849818 5

Guide Rectangle Short Side $\approx \dfrac{3.484500}{6.420999}C \approx \mathbf{0.542672}\boldsymbol{C}$ 0.5426724899957457462048613579312 9

$\approx \mathbf{1.704856}\boldsymbol{D}$ 1.704855907875915402832372401623 7

Arc Radius $\approx \dfrac{3.484500}{6.420999} \approx \mathbf{0.542672}\boldsymbol{C}$ 0.5426724899957457462048613579312 7

$\approx \mathbf{1.704856}\boldsymbol{D}$ 1.704855907875915402832372401623 7

Cutting pattern calculations

To make a cutting pattern, simply increase the arc radius by the desired seam allowance. The guide rectangle remains the same.

Side note: Isovertex rhombus calculations

“Isovertex” is a term I coined to describe a polyhedron with two or more different types of vertices, whose faces have been modified to produce equal sums of angles at all vertex types. This produces a better sphere, though not a closed polyhedron. I am including these calculations for the benefit of anyone who is interested. I had already written this Mathematics section for a pattern based on the isovertex rhombus, so I might as well tack this part on here.

There are two ways to calculate the isovertex rhombus angles. I originally used the algebraic method, but I later discovered the weighted average method.

Calculating the Isovertex Rhombus Angles – Algebraic Method

I will define ***a* to be the acute angle** and ***b* to be the other angle**. I will create two equations defining the necessary properties of the angles, perform a substitution, and solve.

$\boldsymbol{a + b = 180}$ (property of a rhombus)

$\boldsymbol{3b = 4a}$ ► $b = \tfrac{4}{3}a$ (definition of an isovertex face forming 3-way and 4-way vertices)

Substitution into the first equation: $a + \tfrac{4}{3}a = 180$

Solve: $\tfrac{7}{3}a = 180$ ► $a = \dfrac{3(180)}{7}$ ► $\boldsymbol{a} = \mathbf{77.142857°}$ 77.142857142857142857142857142857

So $\boldsymbol{b}$ = 180 – 77.142857 = **102.857143°** 102.857142857142857142857142857142857 14

Calculating the Isovertex Rhombus Angles – Weighted Average Method

The two angles on the normal rhombus are

Acute angle = $\arccos\left(\dfrac{1}{3}\right) \approx 70.528779°$ 70.528779365509308630754000660038

Obtuse angle = $\arccos\left(-\frac{1}{3}\right)$ or 180 – 70.528779 ≈ 109.471221° 109.471220634490691369245999339996

So the two vertex sums on the polyhedron are

4-way vertex sum ≈ 282.115117 282.11511746203723452301600264015
3-way vertex sum ≈ 328.413662 328.41366190347207410773799801989

To calculate the count of each vertex type on the polyhedron, take the count of each type of corner on the face (2*a* and 2*b*), multiply each by the number of faces (24 of each), and divide each by the number meeting at the corresponding vertex type (24*a*/4 and 24*b*/3). There are 6 4-way and 8 3-way vertices. So

Weighted average ≈ $\frac{282.115117(6)+328.413662(8)}{14}$ **≈ 308.571429** 308.57142857142857142857142857143

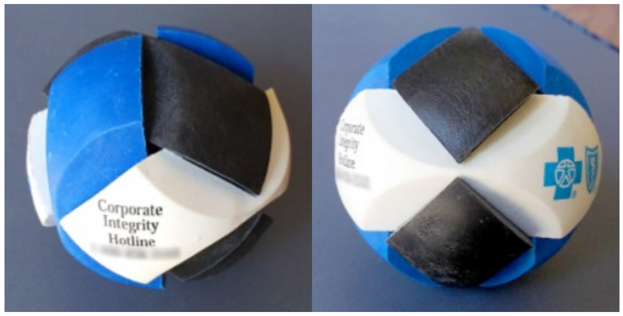
Then simply divide that by 4 to get the acute angle and by 3 to get the obtuse angle:

Acute angle ≈ $\frac{308.571429}{4}$ **≈ 77.142857°** **Obtuse angle** ≈ $\frac{308.571429}{3}$ **≈ 102.857143°**

Back to Chapter Index

How I Developed This Design

[Back to Chapter Index]

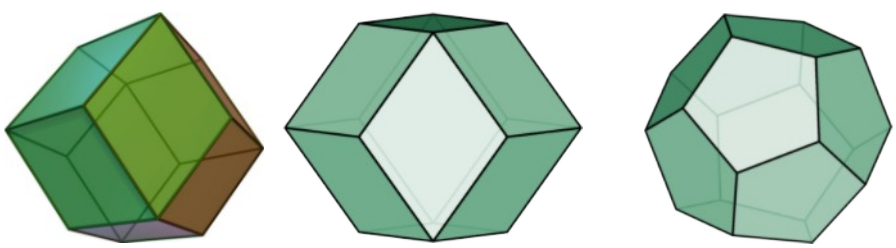


Initial inspiration

In July, 2022 I corresponded with a fan of my Homemade Juggling Beanbag Guide, Uri Yurman, in response to his donation. (I later learned that Uri is a juggling performer and well-known beanbag maker in Israel ([Facebook], [Instagram], [Twitter]), and owner of Uri Kaduri, his on-again, off-again beanbag company.) Uri mentioned that he was designing a modified rhombic dodecahedron beanbag pattern.

I had seen this polyhedron before, but I had never paid much attention to it. I did not like the look of it, and I did not think it could possibly make a good sphere, even with curved edges[3]. Wikipedia's illustration, shown below on the left, has a perspective distortion that makes the shape look particularly angular and poorly suited to converting into a sphere. My own illustration in the middle shows how wide the 4-way vertices are compared to the faces. The regular dodecahedron (on the right) is much more nearly spherical, having uniform and blunter vertices.

I was surprised Uri was interested in this shape, and I privately doubted he could make a good beanbag with it (I did not know he was an experienced professional at this point and took him for a beginner hobbyist). We discussed ideas for designing the rhombus and its curvature, though, and I sent him some experimental patterns I drew up based on my experiences with other designs.



Left: Wikipedia's rhombic dodecahedron illustration by [CC BY-SA 3.0] at [https://commons.wikimedia.org/w/index.php?curid=613268].

Middle: Rhombic dodecahedron illustration by me.

Right: Regular, Platonic dodecahedron illustration by me.

Three weeks later I was visiting my cousin who had just moved into his first house, and in one of his boxes I found the ball pictured under the section header above. I don't know what it is, but it appears to be a puzzle. It is composed of six interlocking pieces, each of which is shaped like an elongated wedge with the ends spherically beveled to present a pair of spherical diamond faces to the surface of the ball. The pieces are made of flexible plastic, so perhaps they can be disassembled. I did not try.

The design of the ball intrigued me and I wondered if it was based on a polyhedron. After studying it for a minute, I realized that it was a spherical rhombic dodecahedron. Quite a surprising coincidence. It showed me what a spherical form of this polyhedron looked like; it was quite attractive. I was suddenly interested in basing a beanbag on this shape. (To be fair, [Wikipedia's Rhombic Dodecahedron page] does

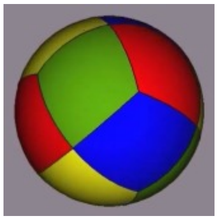
---

3 I was right in predicting that the rhombic dodecahedron is poorly suited to forming a sphere. It needs severe modification to work well, and it was difficult to find a panel shape that allowed twelve rhombi to produce a uniform sphere.

actually depict a spherical form, but I probably never scrolled down far enough to find it, and it is an unimpressive illustration, anyway.)

The color arrangement of the puzzle ball also demonstrated a quality of the rhombic dodecahedron that Uri had said was his motivation for basing a beanbag design on it, but which was difficult to see in Wikipedia's illustration. Each color of the puzzle ball lies on a ring of four diamonds that encircles the ball perpendicularly to the other two. So by orienting the grain of woven fabric along the same diagonal on all panels, the ball will be composed of three matching, orthogonal rings and will therefore have a very balanced stretch.

I told Uri about the puzzle ball discovery and how it had inspired me to base a beanbag design on this shape, and he sent me a link to a video that was part of his inspiration. It displays a four-color, revolving, 3D depiction of a spherical rhombic dodecahedron, and had shown Uri how beautiful this design could be. I used this CG ball to map the colors for my "Four-Color Patchwork Ball" color arrangement.



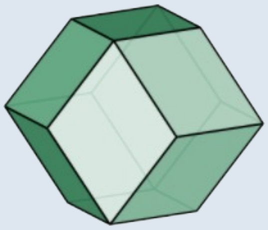
Screen capture above from "Spherical rhombic dodecahedron" by 1ciekaw: https://www.youtube.com/watch?v=F7UZmCuxvkk.

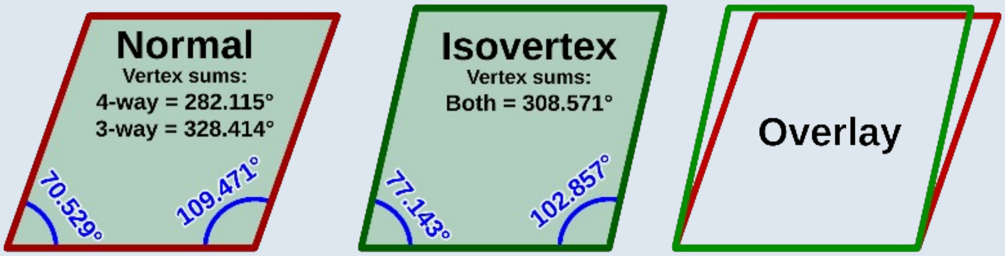
Developing the optimal panel shape

Background Notes: *The "Curved-Edge Faces" section of Chapter 5 has a full explanation of why the sum of angles forming a polyhedral vertex is important, and why I made the choices I did. In short, a sum of 360° will result in a perfectly flat vertex that neither forms a point nor puckers inward. That is optimal when trying to form a smoothly spherical ball. This is achieved by rounding the edges of the faces, which both widens the corners and gives the finished seams a curved shape around the ball.*

Sometimes, a higher apex on the edge arcs is needed in order for the finished seams to have the right curvature. This produces a wider angle at the corners and a vertex sum greater than 360°, but up to a point that still produces a perfectly good vertex shape. Bézier curves can be designed to produce both the correct corner angle and apex height, but designing them is usually not worth the effort as the improvement over circular arcs is imperceptible, or nearly so.

"Isovertex" is a term I coined to describe a polyhedron with two or more different types of vertices, whose faces have been modified to produce equal sums of angles at all vertex types. This produces a better cloth sphere, though not a true polyhedron, because the faces no longer fit together without bending. In the case of the rhombic dodecahedron, four acute rhombus corners, which are 70.529°, meet at the 4-way vertices producing a sum of 282.116°, and three obtuse corners, 109.471°, meet at the 3-way vertices producing a sum of 328.413°. The isovertex version of the rhombus, which can be solved algebraically, has angles of 77.143° and 102.857°, which both produce sums of 308.571° (which is the weighted average of the two vertex sums).





Comparison of the normal and isovertex rhombus.

Skip to the [Conclusion](#) subsection for a tabular summary of all my experiments, and the conclusion of my correspondence with Uri.

First, I'll mention that I made two balls using the uncurved normal rhombus and the uncurved isovertex rhombus, just to see how they would turn out. **The normal rhombic dodecahedron makes a very angular shape**, with sharp 4-way vertices that protrude far out and 3-way vertices that are pulled flat, resulting in a severe octahedron shape. However, **the isovertex rhombus, uncurved, actually makes a pretty good ball**. It is a little misshapen and not up to my standards, but is quite round and smooth enough for juggling. The curved-edge version is unmistakably superior, though.

This was the most difficult shape to make into a sphere that I have worked with. Not only does the base polyhedron utterly fail to produce even a rough approximation of a sphere (unlike the regular dodecahedron, which produces a very good cloth sphere without any modification), but **figuring out the curvature to use is much more complicated than with shapes having matching angles**.

A single arc across a rhombus edge will add the same angular increase to both corners, but in order to produce both a target 4-way vertex sum and a target 3-way vertex sum, I would need to add differing angles to each corner. A Bézier curve could do that, but not an arc. But Bézier curves are difficult to design, and cannot be easily defined mathematically and reproduced, which is the primary purpose of this guide. So I used arcs, and for my first few experiments I had to change the base rhombus, using algebra to solve for one that, after the arcs are added to it, would produce the two sums I wanted. (And anyway, I think that even a Bézier curve on the normal rhombus would not produce as good a ball as the circular arc on my modified rhombus.)

It turns out that making both vertices have the same angle sum does not work as well as I want. The 4-way vertex sums need to be higher than the 3-way. With equal sums the 4-way vertices tend to bulge out from the sphere relative to the 3-way vertices.

I started out by creating a rhombus with edge arcs that formed 372° sums at both of the ball's vertex types (using a rhombus with 74.5° and 105.5° corners and a 9.25° **tangent-chord angle** for the arc, which is **the angle between the arc's tangent at the corner and the rhombus edge it spans, which is the arc's chord**). I figured that would be about the upper limit since that is what the 14-panel design needed (the regular dodecahedron needed only 366° vertices to make a good ball). That ball looked a little too flat at the vertices, especially at the 3-ways (indicating over-wide angles forming them, or else too high an arc apex), but what bothered me even more was the unbalanced vertex characteristic described above, which resulted in a ball that was not as uniformly spherical as the one based on the regular dodecahedron.

I later made a ball with both vertex sums equal to 360° (using a rhombus with 75° and 105° corners and a 7.5° tangent-chord angle for the arc). That had the same issue of slightly protruding 4-way vertices, but they were also too acute. So the angles forming them needed to be wider.

I made those at the end of August and the beginning of September, 2022. I burned out for a few months after that and did no more work on the design. I had little interest in adding the rhombic dodecahedron to my guide at that point, anyway, as the panel structure of the finished ball did not appeal to me as much as it had on the puzzle ball. Also, my depression has been worsening over the past couple years or so, and so I have little emotional energy to drive me to do things or give me any interest in things. I was very bored, though, and needed a project.

I did work on the color arrangements, since that was easy and fun, and gave me something to do. As I figured out more arrangements and saw how versatile this design is, I became more motivated. In January, 2023, I felt sufficiently motivated to perform further experimentation. Over the course of that month I made four more balls, and then five more, plus the two polyhedral versions, in early February.

I first tried converting the isovertex rhombus into a circular version that would create a 360° 3-way vertex sum. This produces 4-way vertex sum of 377.143°. It is not possible for a single arc on the isovertex rhombus to produce the same sum at both vertices, and I decided I did not want vertices of less than 360°, which will technically be slightly angular, even if imperceptibly. That also reduced the number of angle combinations I needed to try. That ball's 4-way vertices were slightly too flat, meaning the angles at the acute corners of the circular rhombus were a bit too wide.

So I then made a ball with a circular rhombus producing 366° 4-way vertices and 360° 3-way vertices (the base rhombus has 75.75°/104.25° corners and a tangent-chord angle of 7.875°). The 4-ways on that were too narrow and bluntly pointed. They stuck outward from the sphere while the 3-way vertices were slightly inverted, resulting in a non-uniform ball.

So I widened the acute angle to form a 372°/360° ball (76.5°/103.5°, 8.25°). That one was pretty well balanced. There was still a hint of the characteristics of the previous ball, but I was willing to live with them if I could not improve on the design. I also made one with the fabric grain oriented along the short diagonal instead of the long diagonal as I always had before, just to be sure that was not a factor I needed to account for. It turned out exactly like the other one. I later made a felt version, which turned out as well as the ones made with my design testing fabric, which is fairly thin, stiff, tightly-woven, and non-stretch. I was nearly satisfied with this design, and since I hadn't communicated with Uri for a few months, I sent him an update, and this latest pattern for him to try.

I almost made a second 377°/360° ball with the circular isovertex rhombus, just to make sure the first was not a fluke, since over time it was balancing out and not showing the defects I had initially observed, and was looking better than the 372°/360° ball. But after more careful examination, I could see that the 4-way vertices were indeed too flat. So instead, I decided for the first time to try a design that resulted in a vertex sum of less than 360°. Since I did not want to increase the 4-way vertex sum beyond 372°, but I did want to balance the two types of vertices better, rounding the 4-ways and bringing the 3-ways outward a little in relation to them, I made a pattern that used the isovertex rhombus, but rounded so as to produce a 372° sum at the 4-ways. This made the 3-ways 356.143°.

That was noticeably more balanced than all the previous ones. The difference between it and the 372°/360° ball is very small, but there does seem to be an improvement. I also liked that the base rhombus is the isovertex rhombus.

The seam curvature seemed like it might be just a tiny bit too steep, though. So after the success of using a 3-way vertex sum of less than 360°, I decided to go a step lower and try making the 4-way vertices 366°, which makes the 3-ways 351.643°. There was no consistently observable difference between the two balls. Sometimes one looked a little more uniform and round, and sometimes the other did. More often than not, though, the 4-way 366° vertices looked a tiny bit acute. My instinct told me to stick with the first, the 372°/356.143°.

After that, I made the corduroy ball both for the chapter photo and pattern adjustment factor, and to make sure the design worked for a fabric with a bit of stretch. Unfortunately, the 4-way vertices on that ball were too flat and the 3-ways were a little pointed, resulting in a minor rounded cube shape. I knew that might be at least in part due to the non-uniform stretch of the corduroy, so I made a ball with felt. That ended up much better, but the 3-ways were still slightly more acute than I liked.

Conclusion

I burned out again for a month or two after making the corduroy ball. I considered trying a 368°/357° design, but couldn't get motivated to make the ball. I also doubted it would make a significant difference. Eventually, after comparing and examining the different balls occasionally and considering the options, I decided that the 372°/360° ball was probably the best I was going to get. It has slightly wider 3-way vertices, and their very slight invertedness relative to the 4-ways would compensate for their protruding nature with stretchier fabrics.

I may someday revisit this project and see if there is a design that produces a better balanced ball, perhaps using a Bézier curve, but this design is very good – certainly good enough for anyone who isn't such an obsessive perfectionist as I – so it will suffice quite well if I don't. As I got more serious about publishing this chapter, I decided I first wanted to try one final experiment: a 372°/366° ball. I got as far as making the panels for it, but I ran out of steam again and couldn't assemble it. I suspect it will have too much of the unbalanced vertex issue, anyway.

Update from 4/26/2023 – Almost a week after publishing this chapter, I looked back through my email thread with Uri and found one in which he had told me part of his more advanced method of designing the panel shape. It involved mathematically projecting the rhombic dodecahedron onto a sphere, using the coordinates of the vertices to calculate the lengths of the rhombus diagonals along the surface of the sphere, and then using those to calculate the rhombus' angles with trigonometry.

His resulting rhombus angles were 76.16826893° and 103.8317311°. Those are very close to my chosen angles of 76.5° and 103.5°, and, in fact, closer to that rhombus than to any of my other experiments. The fact that our two very different design methodologies, his purely mathematical and mine more empirical, both arrived at almost the same rhombus helps confirm that my design is probably very close to the best possible shape. I no longer feel the need to try my 372°/366° experiment (which used a 75.5°/104.5° rhombus). But the curvature of my pattern might still benefit from some tweaking.

Uri has shifted his attention to other pursuits and has not finished his rhombic dodecahedron design or made any experimental beanbags. I was disappointed when he told me this because I wanted to see what shape he came up with and compare the resulting ball to mine. He intended to use Bézier curves for his pattern. I enjoyed my correspondence with him, though, and it was good to have a friend with whom to share the project experience.

I began this project on August 29, 2022 and took eight and a half months to complete it due to all periods during which I was too burned out to continue the project. I made a total of 11 balls with 7 designs and 3 fabrics, plus the 2 straight-edged balls. **On the next page is a summary of all my experiments.**

Chronological Summary of Design Experiments (For my reference, and for anyone who wants to continue my work)			
Ball Vertex Sums (4-way, 3-way)	**Rhombus Angles (Acute, Obtuse)**	**Tangent-Chord Angle**	**Remarks**
372°, 372°	74.5°, 105.5°	9.25°	4-way vertices bulge out from the sphere relative to the 3-ways. A little too flat at the vertices, especially the 3-ways.
360°, 360°	75°, 105°	7.5°	4-way vertices bulge out from the sphere relative to the 3-way vertices, and are too acute.
377.143°, 360°	77.143°, 102.857° (Isovertex)	8.572°	4-way vertices slightly too flat.
366°, 360°	75.75°, 104.25°	7.875°	4-way vertices too narrow and bluntly pointed, and bulged out from the sphere relative to the 3-ways.
372°, 360°	**76.5°, 103.5°**	**8.25°**	Good shape, well balanced. Just a hint of the unbalanced vertex issue.
372°, 356.143°	77.143°, 102.857° (Isovertex)	7.929°	Design testing fabric ball showed very slight improvement over previous design, but the corduroy ball's 4-way vertices were too flat and 3-ways were a little pointed resulting in minor rounded cube shape. Felt ball was better, but still had slightly pointed 3-ways.
366°, 351.643°	77.143°, 102.857° (Isovertex)	7.179°	No significant difference from previous design.

Back to Chapter Index