The Complete Homemade Juggling Beanbag Guide

14-Panel Spherical Equidistant Cuboctahedron Chapter

Small file size version (150dpi patterns & images)

By Joshua Clifton
Contact me on Reddit under Joshuacliftojm
or at juggle@joshuaclifton.com

Published 11/26/2020 Last edited 5/24/2025

This is part of a multi-document guide. Hyperlinks with the icon[1] open other guide documents[2], if they are saved to the same folder (**CTRL+Click** opens them in a new tab).

Visit my website to download those, and check back occasionally for revisions and corrections:

www.joshuaclifton.com/juggle

Compare the Last Edited date above on the right with the one on the web page to see if I have submitted changes.

Please contact me with your thoughts! Feedback on this project would be helpful and encouraging. You may also request custom patterns or other help with your project.

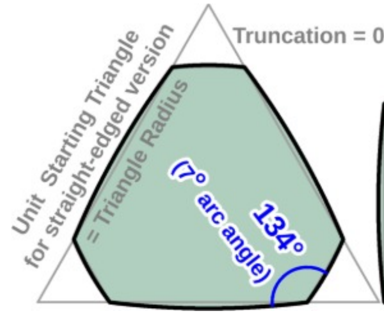
If this guide is useful to you, please **consider donating at my website** linked on the left. I am not monetizing the guide, and I am in need of income.

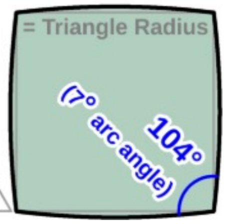
My website also provides blank **color arrangement diagrams** for experimenting with new arrangements in an image editor.

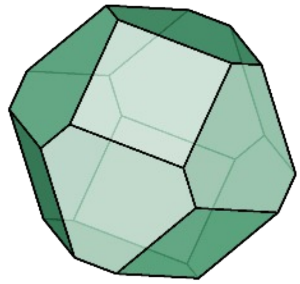
1 Icon from https://freesvg.org/vector-illustration-of-external-link-icon

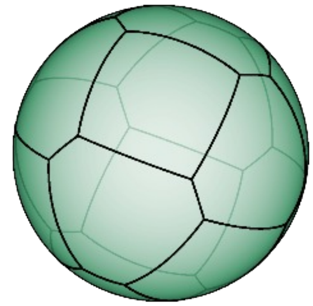
2 **If the linked PDF does not open at the specified location**, keep it open, switch to the previous PDF's tab, and click the link again. **Cross-document links may not work in mobile PDF readers.** In that case you must open the document yourself and find the linked topic.

14-PANEL SPHERICAL EQUIDISTANT CUBOCTAHEDRON INSTRUCTIONS









“Diamond Ring” bags I made years ago with the straight-edged patterns

My original, straight-edged, denim beanbag

Chapter Index

Design Notes

The two different panel shapes in this structure add a visual complexity that the lower panel-count structures lack. This is a more complex and difficult design to make than those, but if you enjoy the process, I think the extra work is worth the beauty of this design. Being related to the octahedron (and the cube), it has octahedral symmetry, allowing it to **support a checkered color arrangement**. This was part of my motivation for creating this design. In 2023 I created patterns for the true cuboctahedron and the related truncated octahedron designs (both with curved panel edges). They are together in their own instructional chapter document.

A true cuboctahedron is composed of triangles and squares, but my design is modified so that both face shapes are the same distance from each other, and this truncates the triangles into semiregular hexagons and makes them more nearly the same size as the squares, producing a more uniform feel and appearance.

I created the straight-edged version of this design in April, 2013 to emulate the 14-panel beanbags I often see in online juggling stores. For the Second Edition of this guide I was able to design **curved**

edges for the panels, making the bag **more smoothly spherical**. See the “How I Developed This Design” section for more discussion. The 14-panel design needs the curves more than the dodecahedron when using a stiff fabric. It can look and feel a little angular without them.

This design can be modified to produce a structure similar to a Super Mario Bros. mushroom (and the Star Wars BB-8 droid). I discuss this variation in the second to last section titled “14-Panel Polka Dot Variation”.

Super Mario Bros. mushroom image from https://mario.fandom.com/wiki/Super_Mario_3D_Land/Gallery. BB-8 droid image © & TM LucasFilm Ltd.

I also modified it in 2016 to create a Poké Ball. I discuss this in the last section titled “14-Panel-Derived Poké Ball”.

Poké Ball image from https://pokemon-fano.fandom.com/wiki/Poke_Ball

Supplies

Back to Chapter Index

- **For the templates**
  - Cardboard or Template Plastic, Scissors or X-Acto Knife, Glue Stick or Double-Sided Adhesive Tape (to attach the pattern to the template material). **For drawing the pattern by hand:** Paper, Protractor, Compass (for the circular panel shapes), metric Ruler, fine-point Pencil.
- **For the beanbag**
  - Fabric, Needle and durable Thread, Scissors, Fabric Marker or soft Pencil, beanbag Filler, Funnel.
- **For your information**
  - Unless you are experienced with this sort of thing, I recommend that you browse through the **General Information and Techniques** chapter (in the ***01 – Homemade Juggling Beanbag Guide – Index & Supplementary Chapters*** document) before starting. You may find some tips there that will improve your experience and your beanbags.

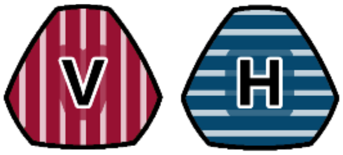
Printing and Drawing the Patterns

Later in this chapter I provide ready-to-print patterns. (When printing them, be sure to tell the Print Dialog to print only the page(s) you want so you don’t print the entire document.) After those are sizing formulas, tables of pre-calculated pattern measurements, and instructions for drawing both the circular and straight-edged patterns. Click the hyperlinks or look to the Chapter Index to locate those sections.

Color Arrangements

Back to Chapter Index

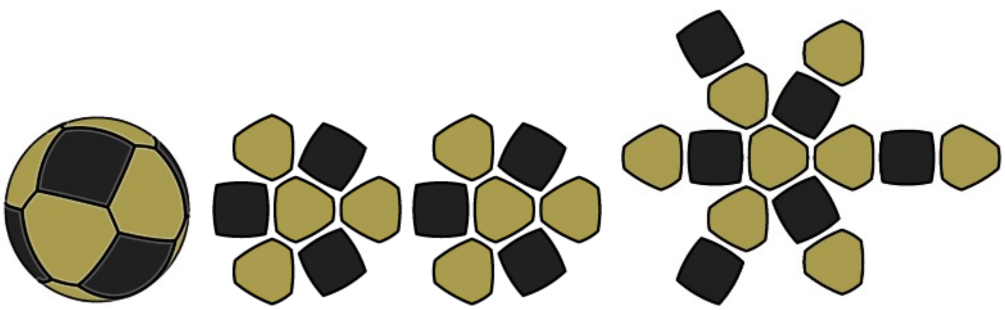
Following is a collection of color arrangement ideas, grouped by the number of colors they contain. The 2-color arrangements of the dodecahedron are possible with this design, so I have included them. I include **two different assembly layout diagrams** for each arrangement: the one for my **dual-hemisphere assembly method** (assemble two separate hemispheres and then sew them together around the equator) and a more **general purpose layout** in case you don't care for my method.



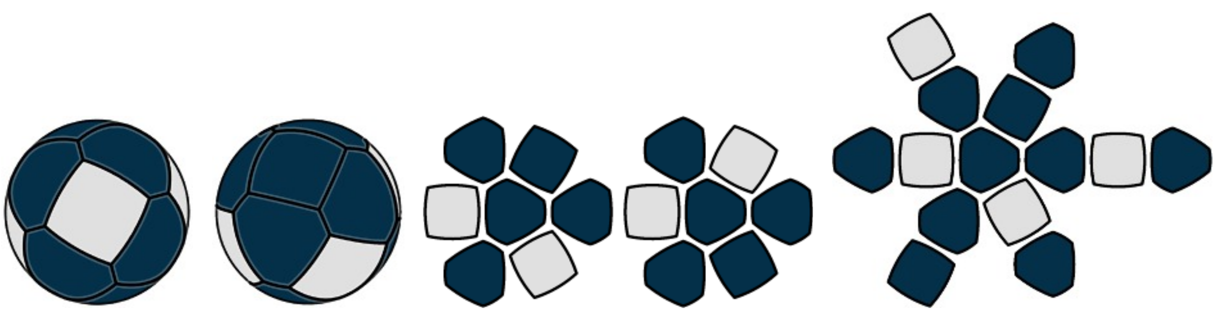
The **V** and **H** labels (on diagrams having hexes not all of the same color) indicate how to orient each hex pattern relative to woven fabric's grain or corduroy cords. This will make the grainline conform to my recommended layout in the "Making the Panels" section, which calls for half of the hexes to be oriented one way and the other half the other so that the direction of stretch is balanced on the ball. **For the unlabeled arrangements, just make four hexes of each orientation.**

A great method of playing with color arrangements is to make a single-color bag and then stick colored thumbtacks into the panels. Having that for reference will also help you keep track of what you are doing as you assemble a bag. **I also provide printable blank color arrangement diagrams** for the ball views and the assembly layout after the printable patterns. Look at the chapter index to locate them.

2 colors



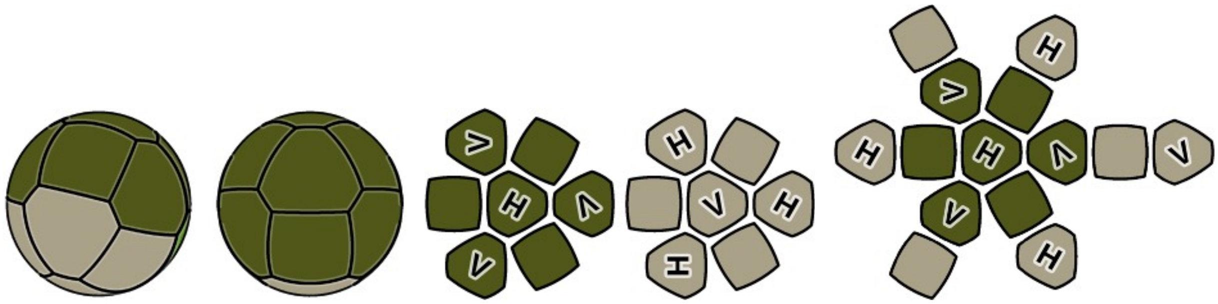
#1: Soccer Ball. The squares one color and the hexagons another, contrasting color.



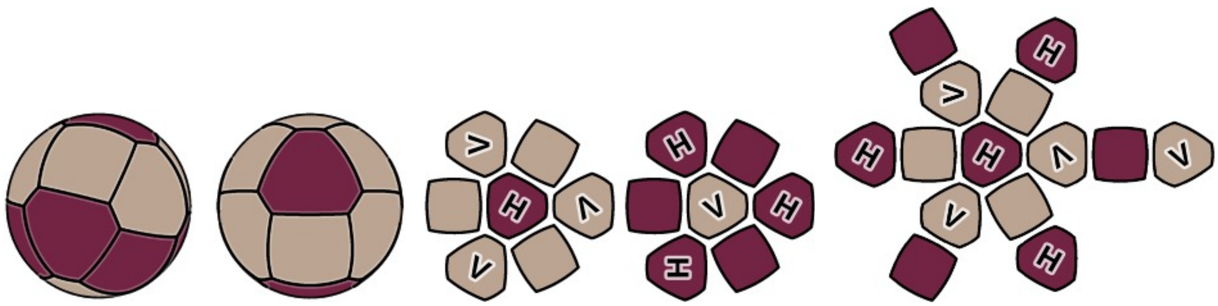
#2: Diamond Ring. A ring of four diamonds/squares of color A around the middle surrounded by a contrasting color B on the two "caps" above and below this ring. Each cap is composed of a square surrounded by four hexes.

My sister chose this arrangement when I made a set of juggling bags for her (pictured on the right). At that time I didn't think this was a very attractive arrangement, and I was surprised she chose it. But when I saw the finished bags, I was impressed by their simple elegance.

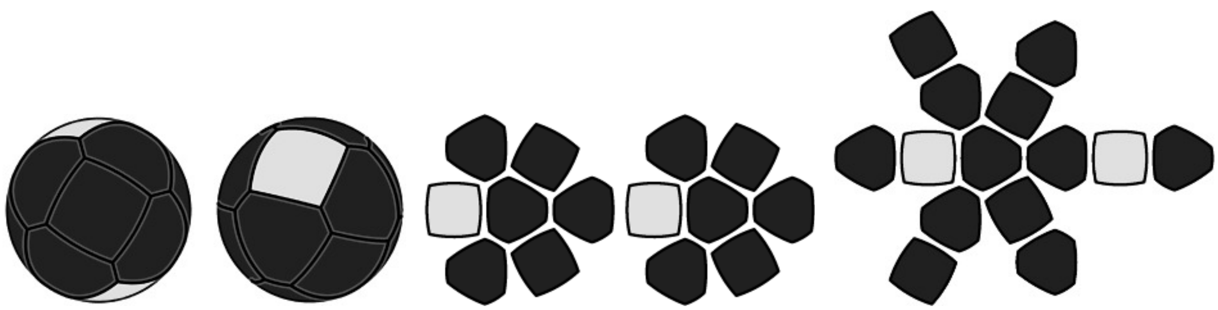
All of the 2-color arrangements of the dodecahedron design.



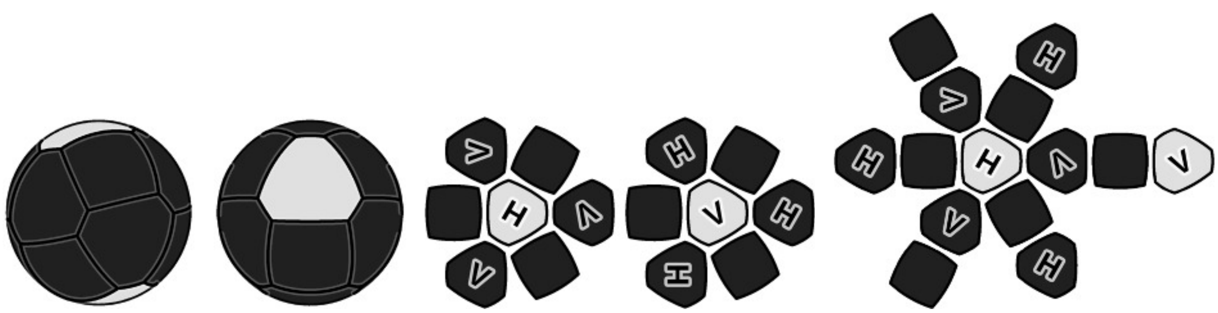
#3: Hemispheres. Each hex-centric, somewhat triangular-looking hemisphere a different color.



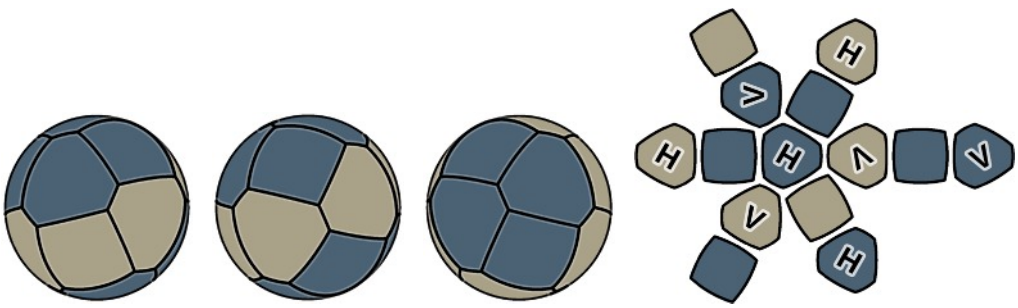
#4: Alternating Hex-centric Rings.

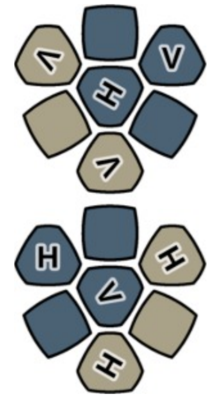


#5a: Square-centric Billiard Ball. Color A on two opposite square panels surrounded by color B on the remaining 12 panels.

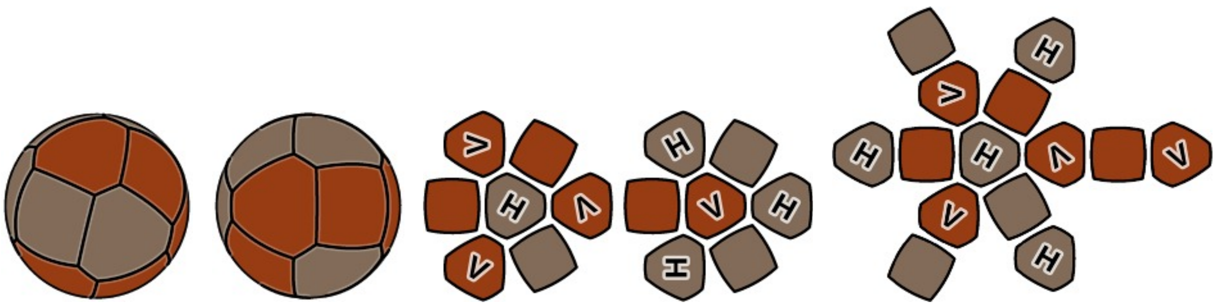


#5b: Hex-centric Billiard Ball. Same as the above arrangement but with color A on two hexes.



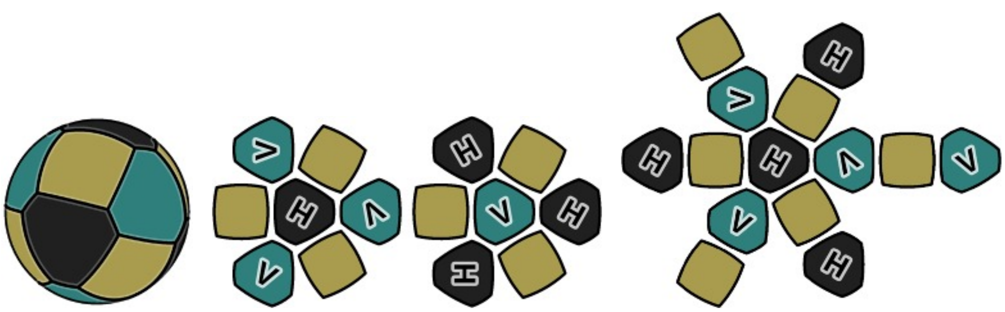


#6: Belt. A belt of color A around two caps of color B, each composed of two squares and two hexagons. (I rotated the dual-hemisphere diagram to produce a cleaner layout.)

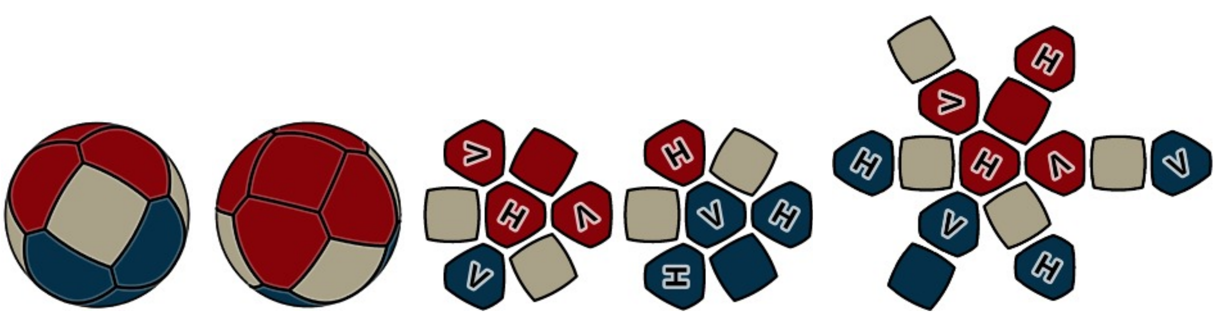


#7: Double Helix. Two intertwining spirals or serpentine curves of contrasting colors. Note that this is a chiral arrangement (its mirror image is not the same). Laying the panels out as shown above with the fronts facing up will yield the clockwise helices shown in the ball illustrations, while laying them out with the backs facing up will flip the arrangement to its mirror image.

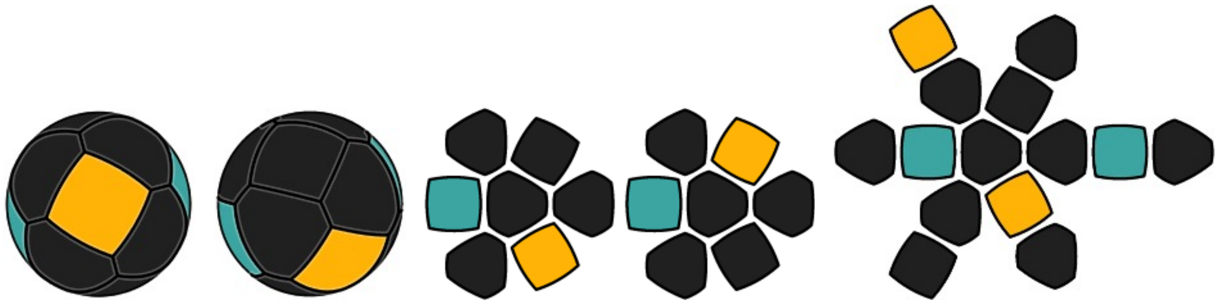
3 colors



#8: Checker Ball. The squares are color A and the hexagons alternate between colors B and C so that no panel has a neighbor of the same color. My denim beanbag shown at the beginning of the chapter uses this arrangement.

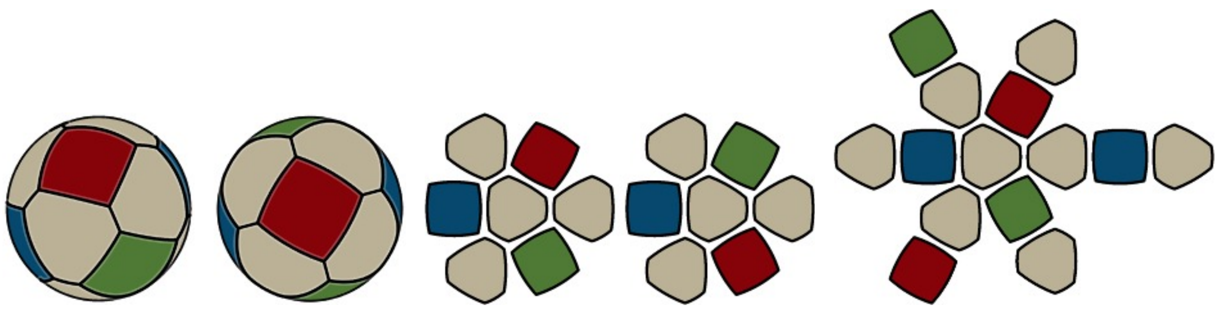


#9: Diamond Ring with Dual Caps. Based on the 2-color Diamond Ring arrangement, this has the same ring of four diamonds/squares of color A around the middle, but with color B on the five-panel "cap" on one hemisphere and color C on the other. Each cap above and below the ring of diamonds is composed of a square surrounded by four hexes.

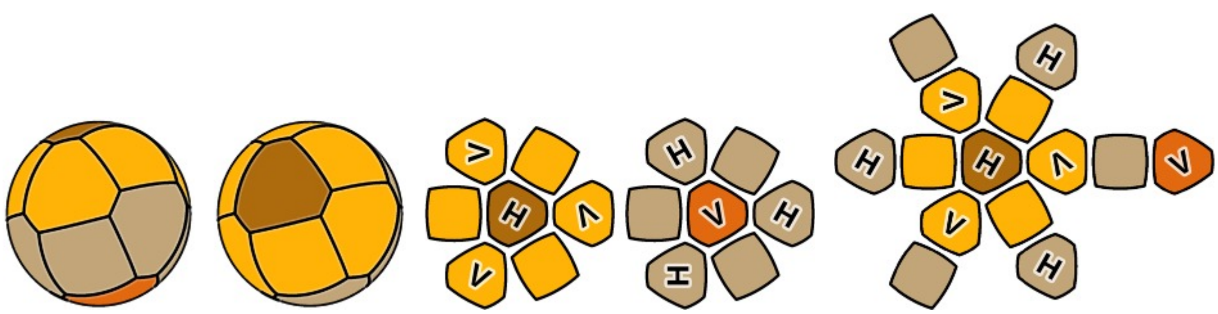


#10: Bi-Color Diamond Ring. Same as the 2-color Diamond Ring arrangement, but the ring has two alternating colors. Each color is opposite its match.

4 colors

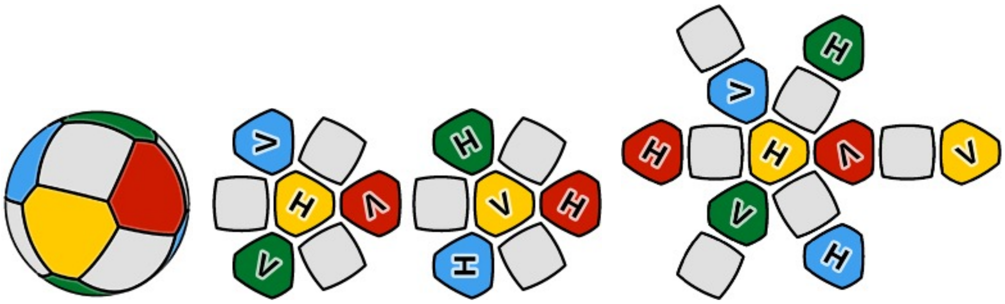


#11: Soccer Ball (4-color variation). Color A on the hexagonal panels and colors, B, C, and D each on a pair of opposite square faces. This results in color A framing each of the other colors. My corduroy bag shown at the beginning of the chapter uses this arrangement.

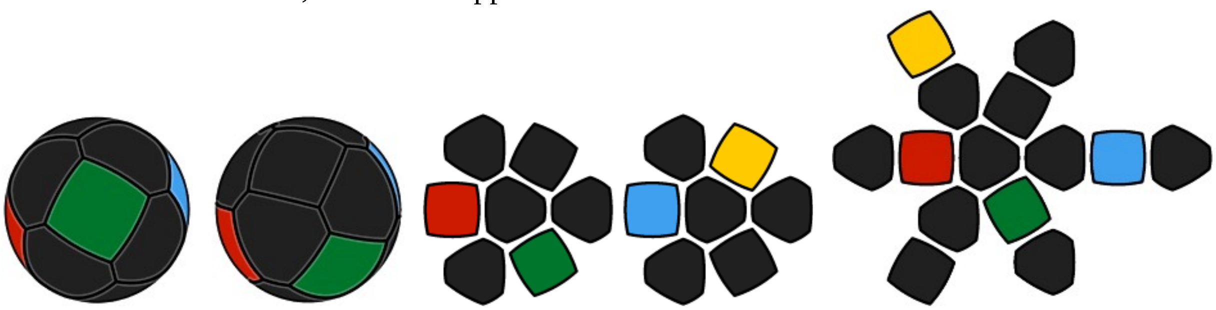


#12: Concentric Rings, from the dodecahedron design. The rings are hex-centric. Each hemisphere can be a different theme, if you like, such as the Caffè Mocha and Mint on the right.

5 colors

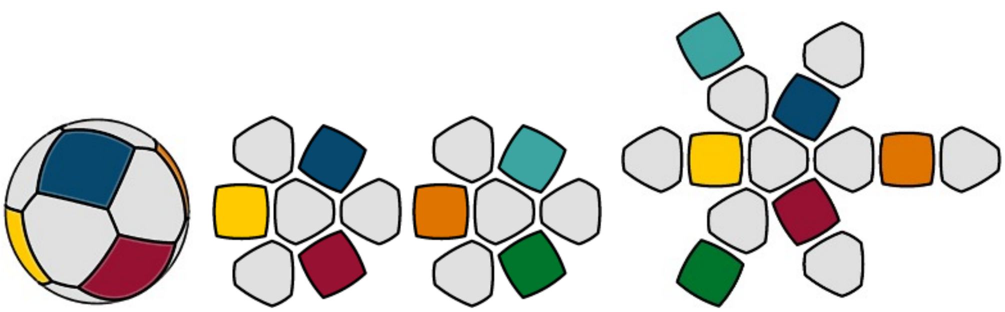


#13: Checker Ball (5-color variation). Similar to the 3-color Checker Ball arrangement, but the hexes are in four colors, each color opposite its match.

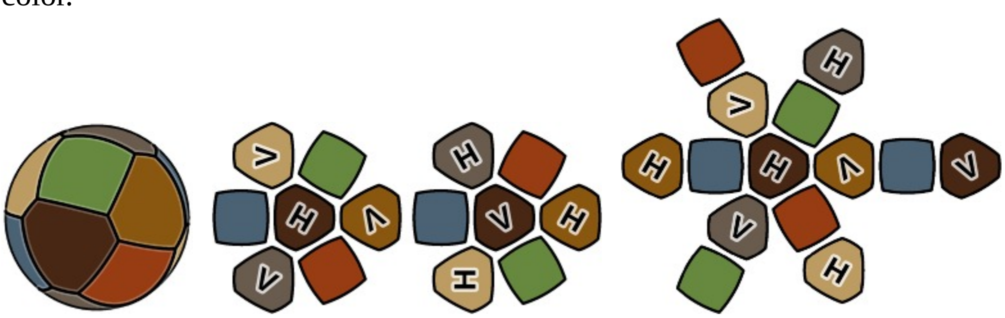


#14: Quad-Color Diamond Ring. Same as the 2-color Diamond Ring arrangement, but each of the diamonds/squares of the ring is a unique color.

7 colors



#15: Soccer Ball (7-color variation). The hexagons all one color and each square a unique color.



#16: Patchwork Ball. Each color on a pair of opposite panels.

Cutting Out the Templates

Back to Chapter Index

To make exterior type templates, simply use scissors to cut along the patterns (or an X-Acto knife and steel ruler for the straight-edged shapes). If you want to make stencil (interior) types, you will need to use an X-Acto knife. If you lack the skill to cut curves with a knife, you can convert the curves into three or more straight cuts that approximate the curve.

If you use a thick marker to trace the patterns, remember to **stitch on the side of the pattern lines where the edges of the template were** (inside the lines for exterior templates, outside the lines for stencil or combo), so you don't change the size of the ball. If the marker soaks through the fabric you're using, however, you will need to stitch inside the patterns to hide the lines within the seams. In that case, when using stencil or combo templates, cut out the templates' interiors slightly outside the lines, shifting the edges outward by the width of the marker lines. Then the edges of the patterns they produce will be correctly positioned for stitching inside them. For combo templates, shift the outer edges by the same amount to maintain the same seam allowance.

I recommend keeping the inner part that you cut out of stencil or combo templates for use in drawing the front stitching patterns. Step 2 of the Assembly instructions explains why.

Continued on the next page.

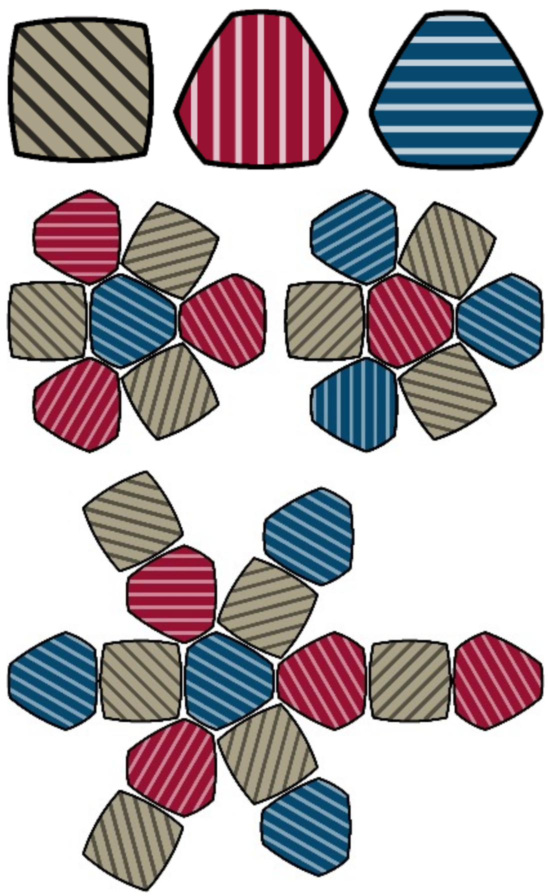
Making the Panels

Back to Chapter Index

Depending on the type of template you're using (exterior or interior) and whether it is translucent or not, you must be careful which pattern, cutting or stitching, you trace first so that the **second template doesn't hide the lines of the first** and prevent you from aligning the two. **Do not necessarily use them in the sequence below**.

1. You will need **6 squares and 8 hexes**, and **you will be tracing the patterns onto the back of the fabric (the side that will be inside the bag)**. If you use cutting templates, first trace those.

   If you are using something like **corduroy, denim, or a striped fabric, or other woven fabric**, I recommend orienting the **squares diagonally** to the lengthwise/straight grain (or cords) of the fabric, and the **hexes half in a vertical orientation and half in a horizontal** as shown on the right.

   You can then arrange the lines of the fabric as shown in the panel layouts, so that on the ball the lines of the squares/diamonds are oriented the same way as I recommend on the cube (each square is surrounded by squares with perpendicular orientations), and the lines of the hexes are arranged as I recommend on the octahedron, with no adjacent panels having matching orientations. This will **produce a balanced look, and it will balance the fabric's direction of stretch** so the ball is not lopsided or otherwise non-spherical.

2. Use the smaller, stitching templates to trace the stitching patterns within each cutting pattern, being sure to center them well (centering them allows you to align the patterns more easily as you sew, but is not otherwise important).

3. Cut out the panels.

Continued on the next page.

Assembly

Back to Chapter Index

Illustrated instructions are on the next page and written instructions begin on the page after that.

I assemble this design by forming two separate hemispheres and then sewing them together. This gets all the panels joined to each other as soon as possible, and in a simple and clear manner, which reduces the risk of losing track of the arrangement during assembly. **If you prefer simply to attach all panels to a central hex**, use that layout type from my color arrangements section instead of the one in Illustration A. You can then start with the stitching path I use for the first hemisphere (Illustrations A & B, or A_{ALT}), and continue from there to attach the rest of the panels. The remaining illustrations and some of the written instructions will not apply.

For the dual-hemisphere method I describe **two different approaches to sewing the hemispheres**: my original approach which uses as few as 6 threads, and an alternate one based on my 32-panel assembly method that uses few as 3 threads. **The alternative method is much simpler at the cost of more duplicate stitching.**

For the **original method**, the stitching for each hemisphere begins at the intersection indicated by the spot in the center of the arrows in Illustration A (on the next page) and proceeds around the "hub" seams and then out the "spoke" seam. Additional threads are used for the remaining spoke seams on one hemisphere (the first is shown in Illustration B). I join the hemispheres together by using the spoke threads from one to continue up into the "equator", and then into the spoke seam of the other hemisphere. Up to three equatorial seams will be sewn from the outside (indicated by the dashed lines).

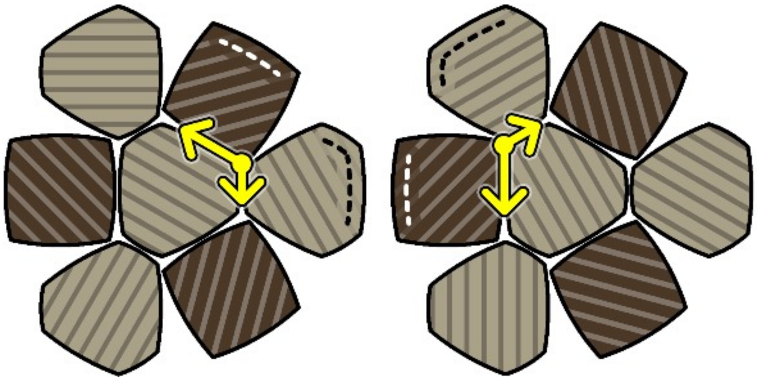
In the **alternative method** (Illustrations A_{ALT} and B_{ALT}) you sew out and back at each spoke seam as you proceed around the hubs. This results in complete hemispheres which are much simpler and easier to join together because you have only the equator to sew around, not the spoke seams as well. The duplicate stitches can be extra long (at least if you are using the backstitch), and so are not terribly tedious.

With so many panels and seams, **it is easy to make a mistake** as I have and misalign the panels or join four corners together instead of three. It may be helpful to make a cardboard model with colored panels or labels to use as a reference. A card stock like index cards or file folders works well for this. Just make straight-edged templates (rather than circular), cut a few layers at a time to produce the panels faster, and tape them together.

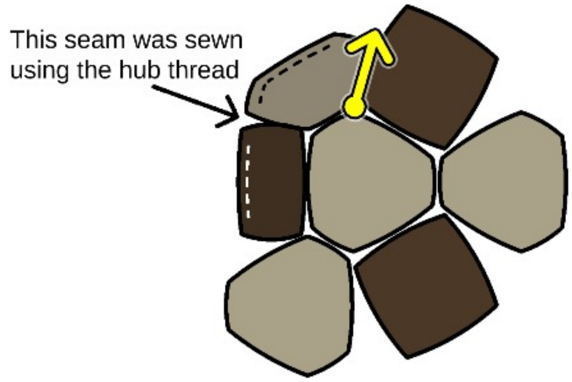
Helpful Hints: While assembling the bag, remember the following points.

- Every intersection will have three panel corners. Two will be hexes and one will be a square.
- No square will join to another square.
- The hexes' short edges will always join to another hex, and the long edges will always join to a square.

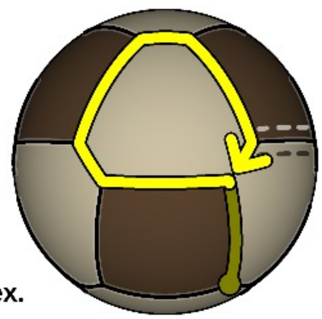
A **Sew around each hub panel starting between the panels with the front stitching patterns, then out the spoke seam.**



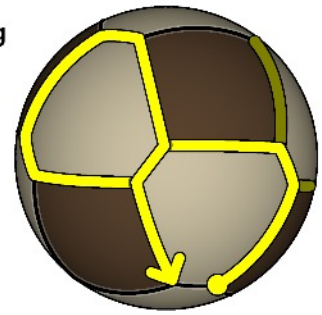
B **Sew a spoke seam adjacent to the front pattern.**



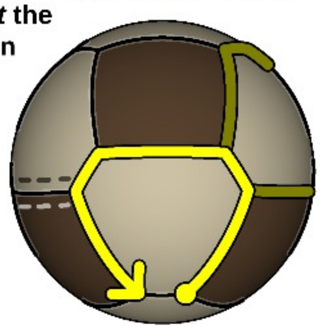
C **Continue into the equator seams, joining the hemispheres, then up the spoke seam. Cross the short, hub edge of the hex at the top (double-stitching it) and then continue around the hex.**



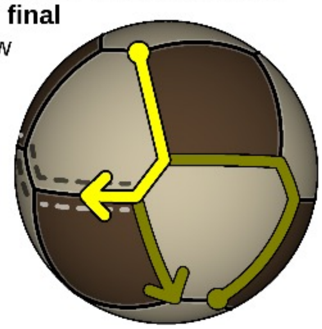
D **The next spoke thread follows a figure eight around two hexes, double-stitching three short seams.**



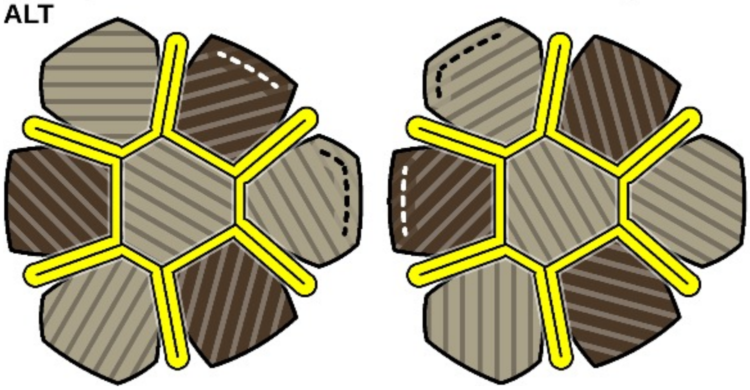
E **Sew around the final hex panel that has two unsewn spoke seams (the hex *without* the front pattern on it).** Do not yet enter the seams with the front stitching patterns.



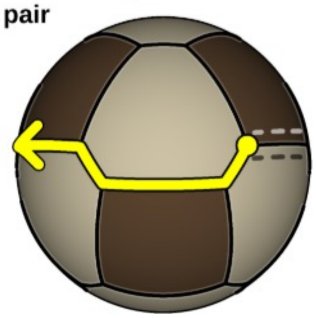
F **Sew the final spoke seam on the other hemisphere and then begin sewing the final seams.** Sew as much as you don't need to turn the bag out through.



A ALT **Alternatively, sew around the hub panels, but at each spoke seam sew out and back before continuing around.**



B ALT **After sewing both hubs and all spoke seams, place the hemispheres together at a pair of edges adjacent to the front stitching patterns and sew around the equator. Skip C – F.**



Note that in the ball illustrations the ball is still inside out and so the front stitching patterns (the dashed lines) won't actually be visible. I show them just for positional reference.

1. ***Illustration A:* Lay the panels out as shown** and **arrange them according to your color pattern**. My illustrations assume the fronts face up, but it doesn't matter (except for chiral color arrangements). The **hatching lines in Illustration A serve as a guide** if you are using a woven fabric, or something like corduroy or a striped fabric and want to **orient the lengthwise/straight grain of the fabric** as I prefer to, both **for aesthetics and for a balanced fabric stretch**. Further explanation of this is in the "Making the Panels" section.

2. Use the stitching template to **draw stitching lines on the fronts** of the six outer panel edges shown with dashed lines to form three seams in a row around the equator. My stitching pathway leaves these three seams partially unsewn so the bag can be turned out between them. They will then be **sewn from the outside following the front stitching lines**. (If you use a thin or flexible fabric and don't need such a large opening, just skip marking the upper pair or two of edges.) Be sure to align the template as well as possible with the stitching patterns on the backs.

   If you want to **hide the stitching lines within the seams**, sketch them a millimeter or two nearer to the panel edges and stitch slightly inside them (toward the middle of the panels). **If you use a Stencil or Combo type template,** use the inner portion that you cut out of the template to draw these patterns, since the main template will cover the area near the edge.

   I have found it helpful to **add marks along the front stitching lines for each stitch** so that I can more easily keep the exterior stitches even with each other and not get a skewed seam. I space the stitch marks $\frac{1}{8}$″ (3mm) apart. If you **make these marks on your template first**, you can more easily transfer them onto these and future panels.

3. ***Illustration A (stitching):* Start with a central hex panel and sew a panel to each of its sides** beginning at the corner between the panels with the front stitching patterns and proceeding in either direction. **Sew the panels with their front faces together** so the bag will be inside out.

   Illustration A_{ALT}:* Alternatively, sew out and back at each spoke seam**, joining the outer panels to each other. The duplicate stitches can be up to twice as long if you're using the backstitch and are careful how tightly you pull them (if you pucker the fabric, wiggle it straight again). **If you take that route, you will be skipping to Step 11** (after completing the other hemisphere), beginning at the second paragraph labeled "Illustrations F and B_{ALT}***". Just join the two completed hemispheres, starting at one end of the front stitching patterns and sewing around to the other end (see the "Crossing seam intersections…" topic in the **General Information and Techniques** chapter for advice). Then follow the relevant instructions in Step 11.

   When you finish the two hemispheres in the alternative method, be sure to **join the hemispheres so as to form your intended color pattern** and to make the front stitching lines on each half meet each other. Joining the first couple of equatorial edges is **easier if you flip the hemispheres right side out** so the front faces of the panels are exposed and can be placed together.

4. ***Continuing Illustration A:*** When you have attached all six panels and the thread has **reached the starting point again**, **sew the adjacent sides of the two outer panels together**, connecting the two segments of the front stitching lines, and then tie off the thread and trim it. You are done with this hemisphere for now. The spoke seams (the adjacent edges of the outer panels) will be sewn with threads that continue from the other hemisphere.

5. **Construct the other hemisphere in the same way.**

6. ***Illustration B:*** On either hemisphere, **sew a spoke seam** that is adjacent to the one that is already sewn, starting at the center panel and sewing outward. If you orient the panels as they are in the illustrations, your path can match mine. But if you don't, just modify the path as needed, using mine as a general guide. Either way, the next step continues this thread, so leave it hanging.

7. ***Illustration C:*** **Continue the thread into the other hemisphere** by sewing across a long and short equatorial seam, which will join the two hemispheres, and then up the spoke seam of the other hemisphere. **Be sure to select the correct panel from the other hemisphere** to join so as to form your intended color pattern and to make the front stitching lines on each half meet each other. **Aligning the panel edges correctly can be somewhat confusing**. Refer to the Helpful Hints if you are confused. Joining the first couple of equatorial edges is **easier if you flip the hemispheres right side out** so the front faces of the panels are exposed and can be placed together.

8. If your panels are oriented as mine are, you can continue across the short, hub seam of the hex on the second hemisphere, double-stitching it, and continue around the hex, closing the spoke seam on the second hemisphere that is adjacent to the front patterns. (If you are using the backstitch, you can make the duplicate stitches up to twice as long without causing the fabric to ripple as long as you're careful how tightly you pull them. You could probably make one long stitch across the short seam, depending on the size of the ball you're making, and make the retreating stitch reach halfway across or less. I make two stitches across, though, just to be extra careful about bunching the seam. If you do bunch it up, just wiggle it straight again.) Tie and trim this thread.

9. ***Illustration D:*** Start a thread at the hub end of the next spoke seam on the first hemisphere (the one on the right in my illustrations), sew up the spoke seam, then along five equatorial seams (short-long-short-long-short), and then up the second hemisphere's spoke seam. Continue around the hex in the opposite direction (double-stitching the short hub seam), down the spoke seam, across the short equatorial seam you just stitched, and then down the spoke seam adjacent to the one you started with, forming most of a figure eight around the two hex panels. Tie the thread and trim it.

10. ***Illustration E:*** Sew around the final hex panel that has two unsewn spoke seams, closing the last equatorial seams not having the front patterns marked on them.

11. ***Illustration F:*** Sew the final spoke seam on the other hemisphere and continue across the short equatorial seam, restitching it.

    Illustrations F and B_{ALT}: Then proceed into the final equatorial seams with the front patterns marked on them. **Sewing a few starter stitches** makes it easier to continue from the outside. Sew as much of the final opening as you don't need for turning the bag right side out. **First, though, I recommend that you tie the thread at, or a short distance before, the front pattern seams** (leaving enough of an opening to allow the panels to spread and the bag to be turned out) so it does not loosen behind that point when you turn the bag out.

    To **reduce the number of stitches you need to make from the outside**, you can make extra stitches and then loosen them to allow the panels to spread enough to turn the bag out. Then you can pull them tight again from the outside. If you want to do this, or if you want to be able to loosen the last several stitches enough to push a funnel between them, **your final thread will need several inches of extra length**.

12. At this point, **consider ironing the seam allowances flat**; see the **General Information and Techniques** chapter under "Better Seams by Ironing". The crowded seam allowances at the nearly 6-way intersections are a little difficult to fold out. For the best results, I recommend that you first fold down the two short seams, then at each end, fold the outside allowances of the two adjacent long seams over them, then fold one side of the corner in the middle over the other. The photos below show what this looks like for one intersection.

13. **Turn the bag right side out through the opening**. A good method for this is to use the back end of a pen or other slender tool to push the fabric through the opening from the opposite side and then either invert the bag around the tool, or remove the tool and work the bag through with your fingers. **Be gentle so as not to pop any stitches**.

14. **Pull out the last stitch so that the thread is on the outside** where you can get to it. Continue sewing the opening closed following the front stitching lines. For help, see the "Stitching Techniques" section of the **General Information and Techniques** chapter under "Backstitch from the exterior Approaches". Fill the bag at some point during this final sewing with a funnel. I find it helpful to **put some filler in first to prevent the bag from collapsing** while I sew, and to hold the seam allowances in place and give me something to push the needle against.

    You can sew the entire opening closed before fully filling the bag, which prevents the filler from spilling back out while you sew. Just loosen the last several stitches enough to push the funnel between them, or at least to push some filler in with your fingers. Then re-tighten the stitches (see "Tips on finishing the bag").

Back to Chapter Index

Ready-to-Print Patterns

Back to Chapter Index

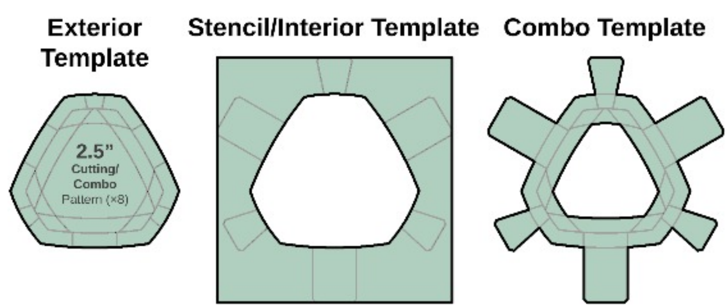
The pattern pages are 8.27″×11″ (210mm×279mm) to fit both “Letter” and “A4” sizes. ***Make sure the print is not being scaled to fit the printer margins*** *(select Default/None scaling/Actual size/Ignore printer margins). To verify correct sizing,* ***compare the centimeter grid to a ruler*** *and adjust the next print if necessary. (Note that PDF viewers and printers can both contribute to slight size inaccuracy.)*

On the following pages are patterns for beanbag diameters from 2″ – 3″ in ¼″ increments, and 7″ patterns for scaling to larger sizes. The patterns are reduced by 5.81% from the mathematical calculation to account for the inflation in size I observed in my corduroy bag. **If you use a dense/stiff or completely non-stretch fabric, I recommend enlarging the pattern to about 103%.**

To make the templates, I recommend cutting out the portions of the paper with the patterns you want and gluing or taping them to your template material, and then cutting along the patterns.

The cutting patterns have 4mm, 6mm, and 8mm allowances so you can choose the amount that works best for your fabric and preference (the lighter, 6mm pattern is a hair under ¼″), and they include **tabs for the optional combo type template** (stitching pattern on the inside, cutting pattern on the outside, with the tabs to help you hold it down). The hex’s small tabs may not be needed.

The examples on the right show the **three ways you can cut out the Cutting/Combo templates** (using the 8mm allowance). Remember that the cutting patterns have slightly different proportions from the stitching patterns (they are parallel, not proportional), so you should not use them as stitching patterns.



To produce other pattern sizes or correct an incorrectly sized beanbag, adjust the size scaling in the print dialog. For example, to reduce my 2.5″ patterns to the 2.3″ size recommended by the Juggling Store for small hands and numbers juggling, divide 2.3 by 2.5, multiply the result by 100, and that is your scale (92% in this case). If your beanbag ends up not being the expected size, see the **General Information and Techniques** chapter under “Adjusting/Scaling a Pattern to Produce an Accurate Ball Size” for help with correcting it.

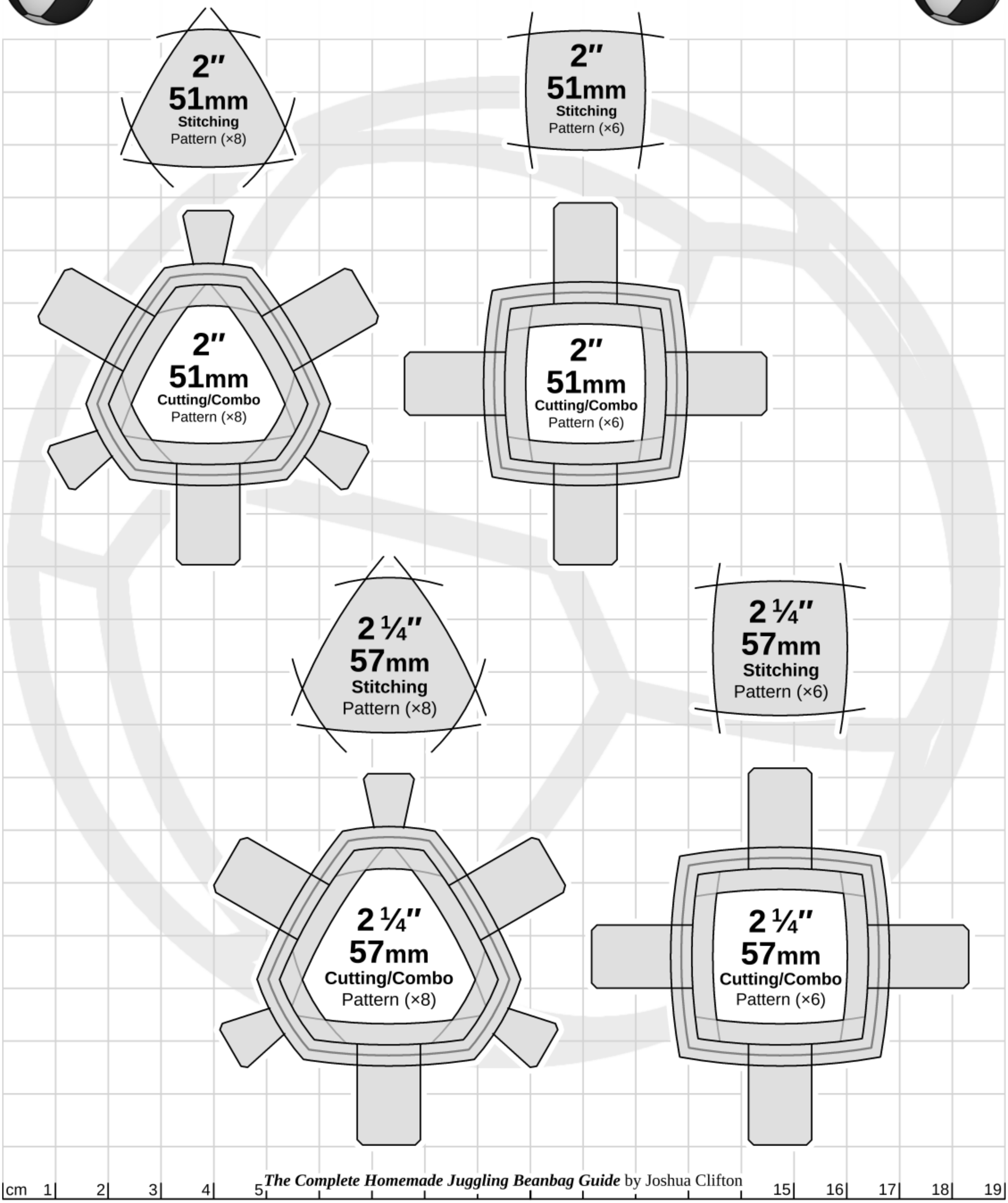
The table below provides the scaling for the ⅛″ increments between my ¼″ sizes. The centimeter grid can be used to verify correct scaling.

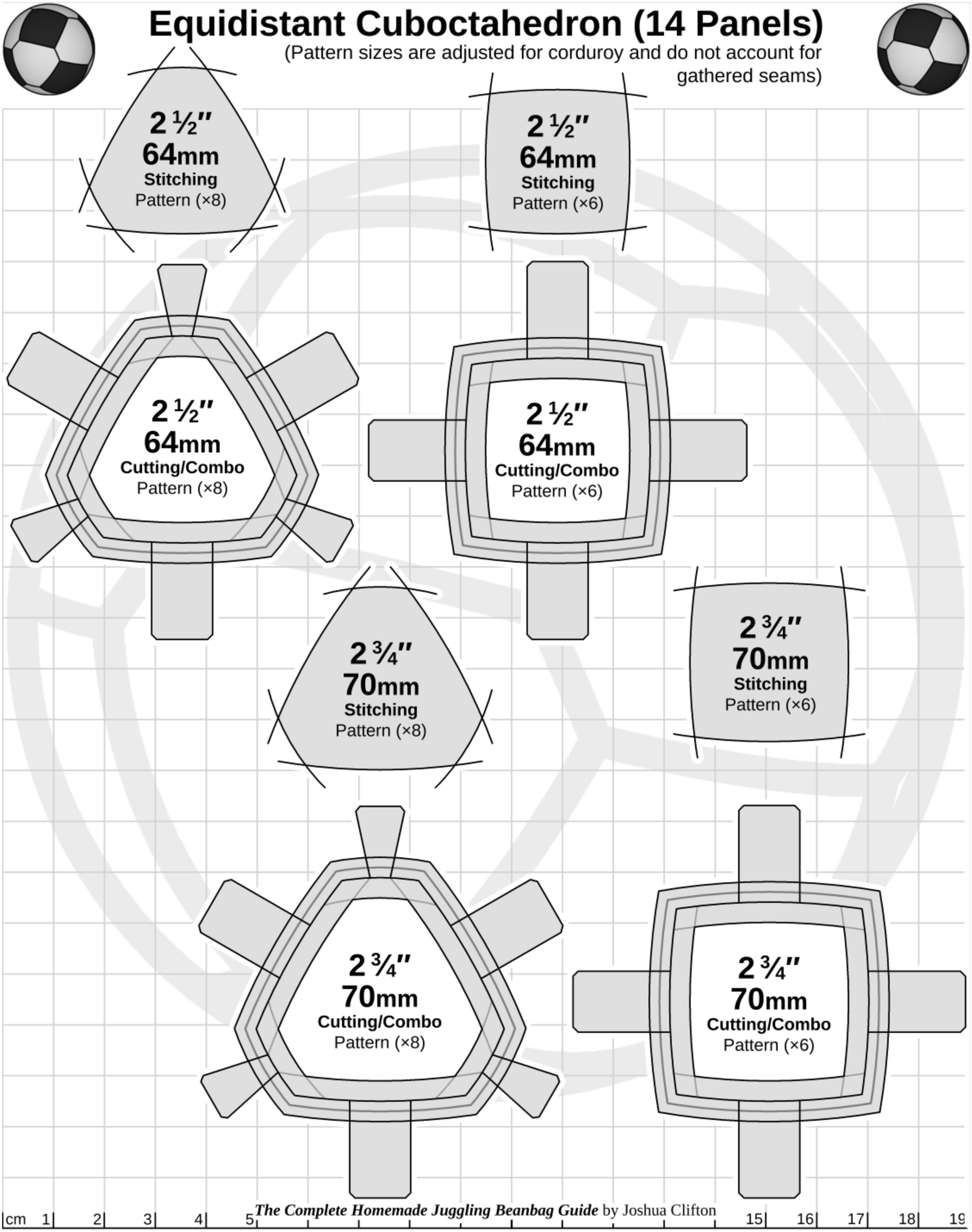
Target Diameter	Print this pattern size	At this scale
1¾″ (44.5mm)	2″	87.5%
1⅞″ (47.6mm)	2″	93.8%
2⅛″ (54.0mm)	2¼″	94.4%
2⅜″ (60.3mm)	2½″	95%
2⅝″ (66.7mm)	2¾″	95.4%
2⅞″ (73.0mm)	3″	95.8%

If you cut out these patterns as straight-edged shapes (from corner to corner), the resulting beanbag will be about 98% the size of the circular-edged version. So to produce the same size, scale the printout to 102%.

Equidistant Cuboctahedron (14 Panels)

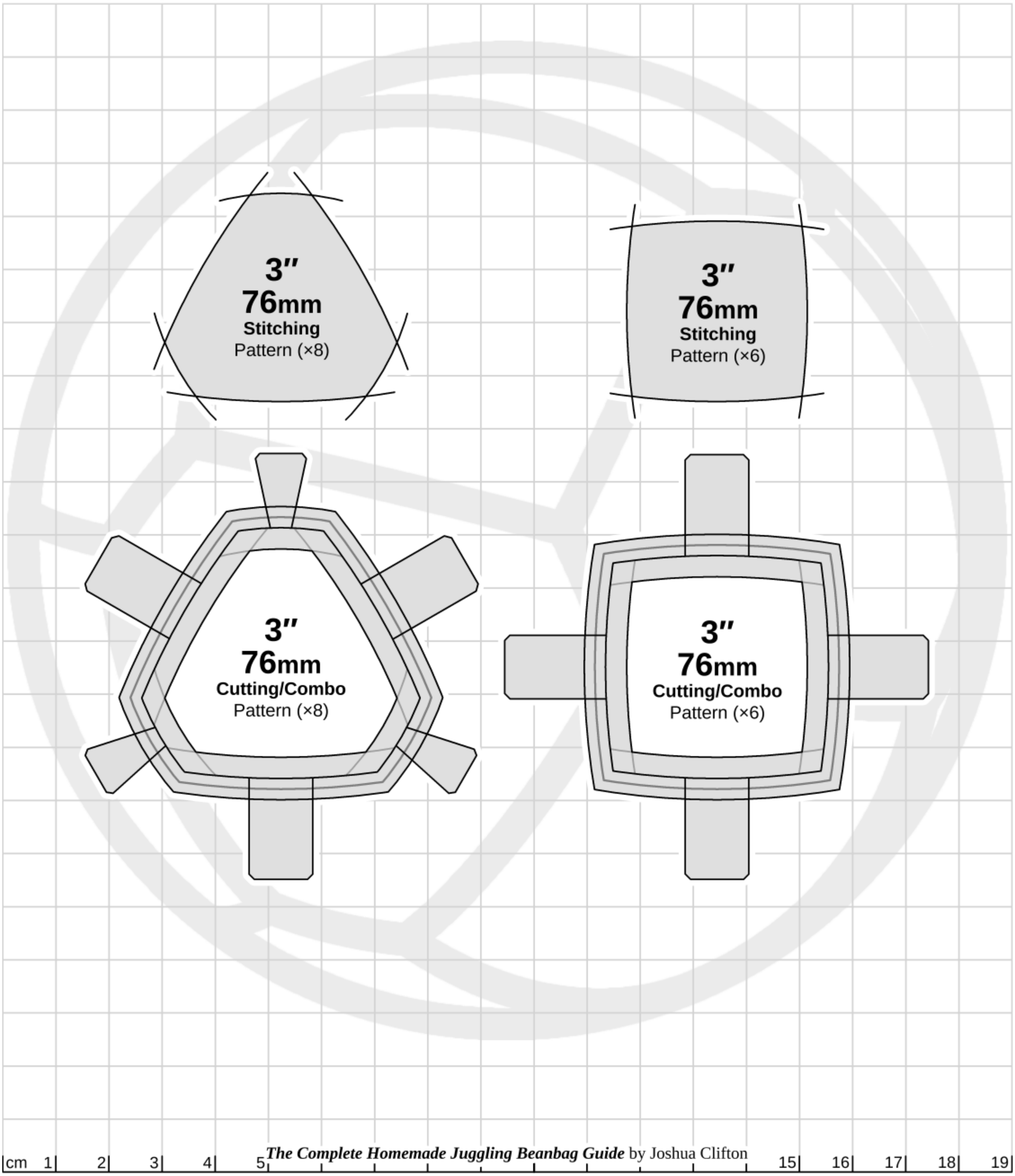
(Pattern sizes are adjusted for corduroy and do not account for gathered seams)




Equidistant Cuboctahedron (14 Panels)
(Pattern sizes are adjusted for corduroy and do not account for gathered seams)
2 ½" 64mm Stitching Pattern (×8)
2 ½" 64mm Stitching Pattern (×6)
2 ½" 64mm Cutting/Combo Pattern (×8)
2 ½" 64mm Cutting/Combo Pattern (×6)
2 ¾" 70mm Stitching Pattern (×8)
2 ¾" 70mm Stitching Pattern (×6)
2 ¾" 70mm Cutting/Combo Pattern (×8)
2 ¾" 70mm Cutting/Combo Pattern (×6)
cm 1 2 3 4 5 15 16 17 18 19
The Complete Homemade Juggling Beanbag Guide by Joshua Clifton

Equidistant Cuboctahedron (14 Panels)

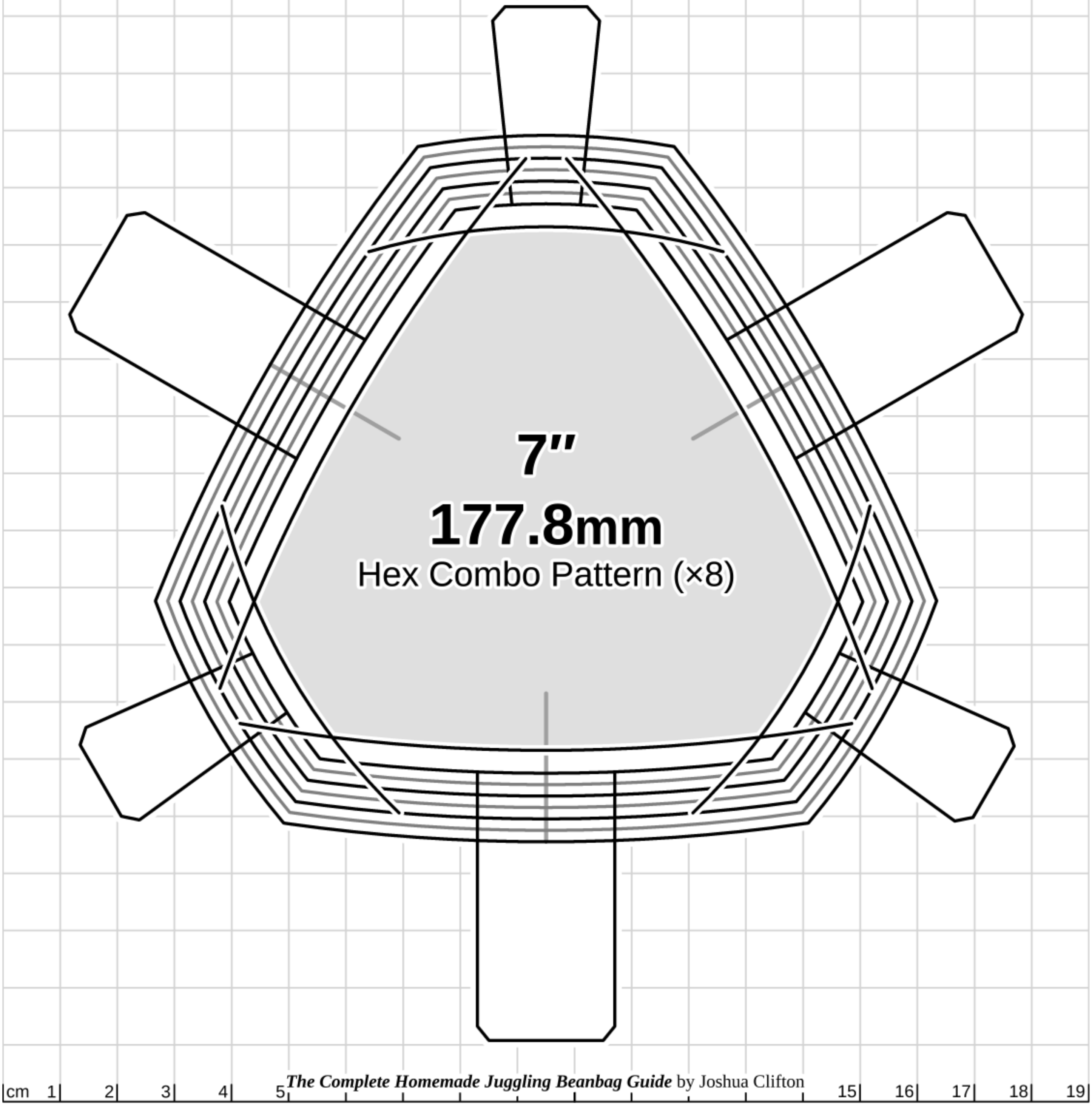
(Pattern sizes are adjusted for corduroy and do not account for gathered seams)



Equidistant Cuboctahedron (14 Panels)

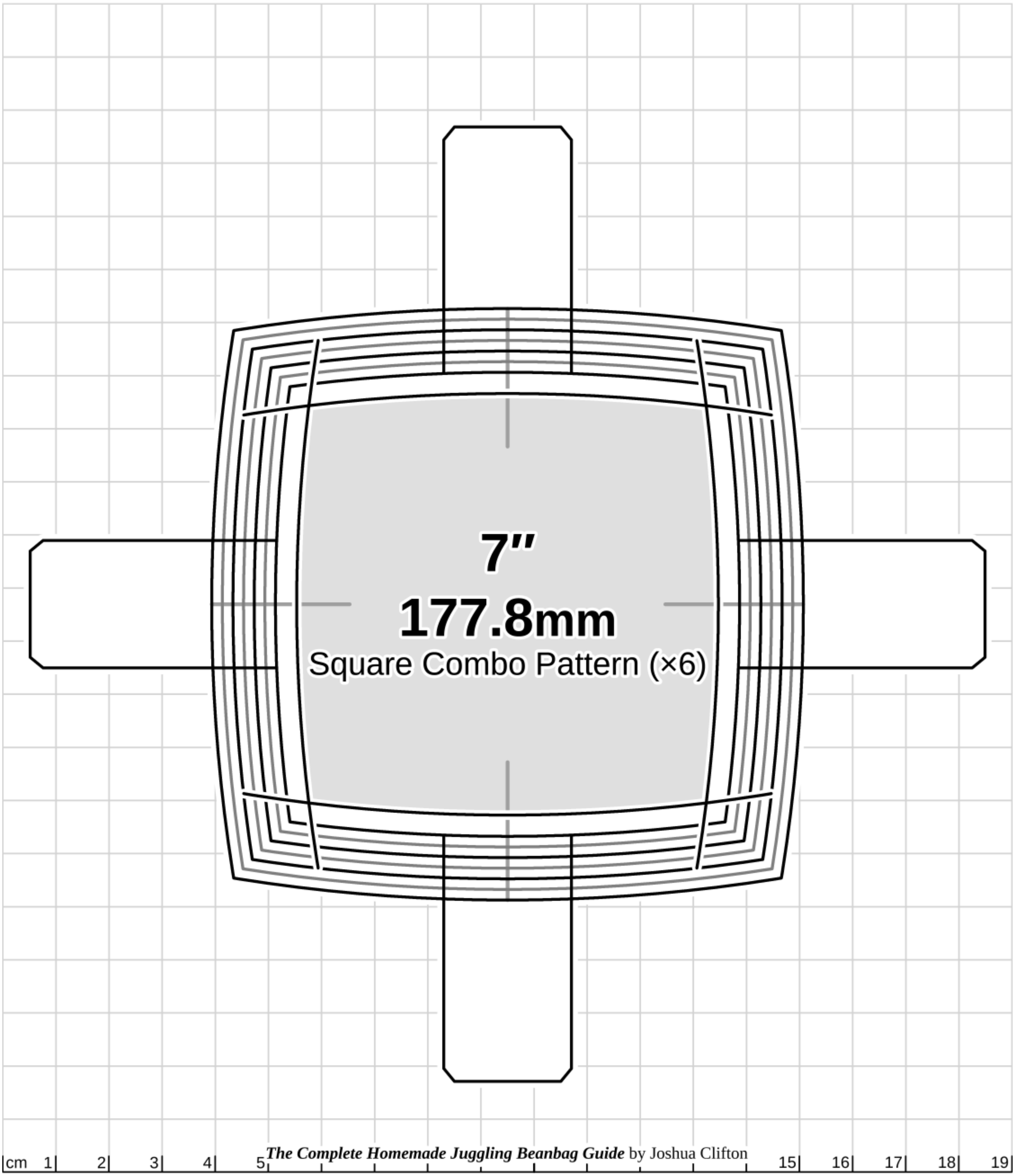
(Pattern sizes are adjusted for corduroy and do not account for gathered seams)

Extra large and versatile patterns for scaling to larger sizes in the Print Dialog (the square is on the next page). Print each pattern twice if you want both a stitching template and a cutting template (or cut out combo templates). The inner patterns (filled with gray) are the stitching patterns. Each dark pattern outside of those marks a 4mm seam allowance interval (at 100% scaling). Use those or the lighter, half-intervals between them to cut out the amount of allowance you want for the cutting templates.



Equidistant Cuboctahedron (14 Panels)

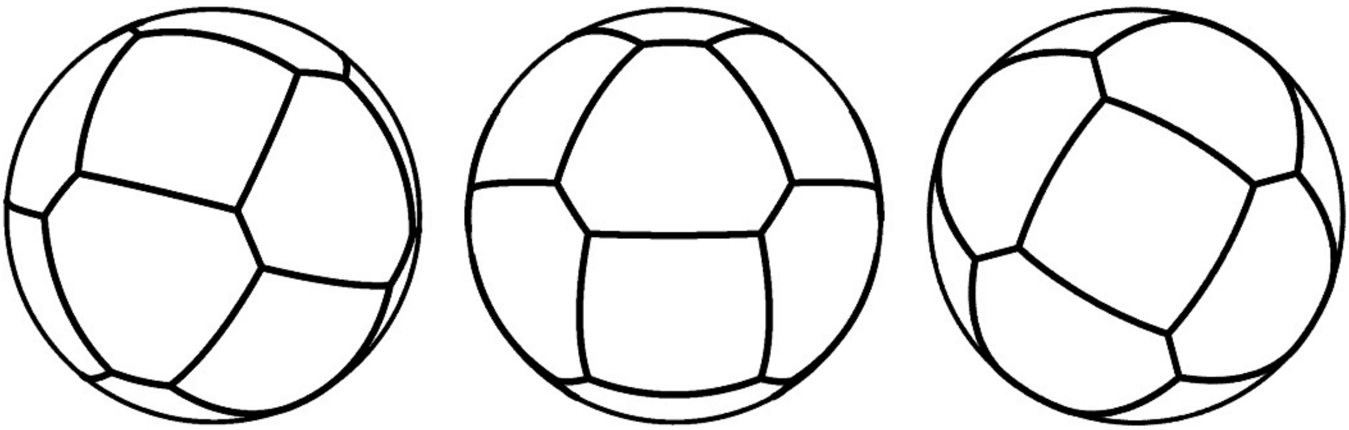
(Pattern sizes are adjusted for corduroy and do not account for gathered seams)

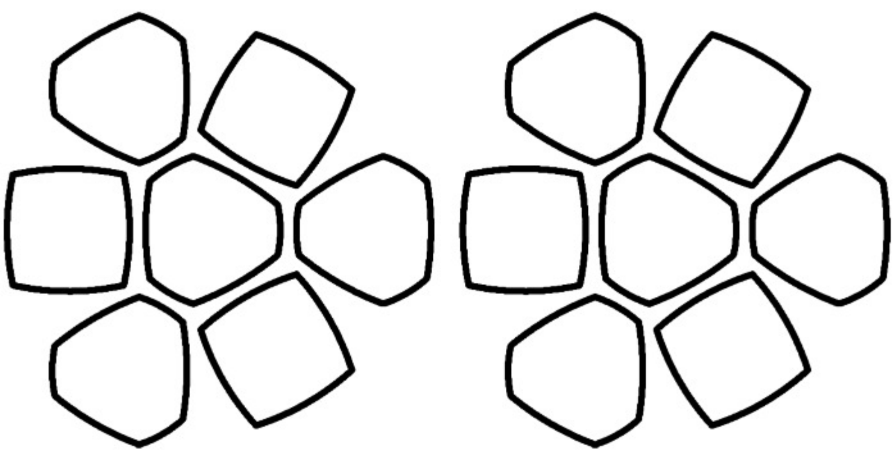


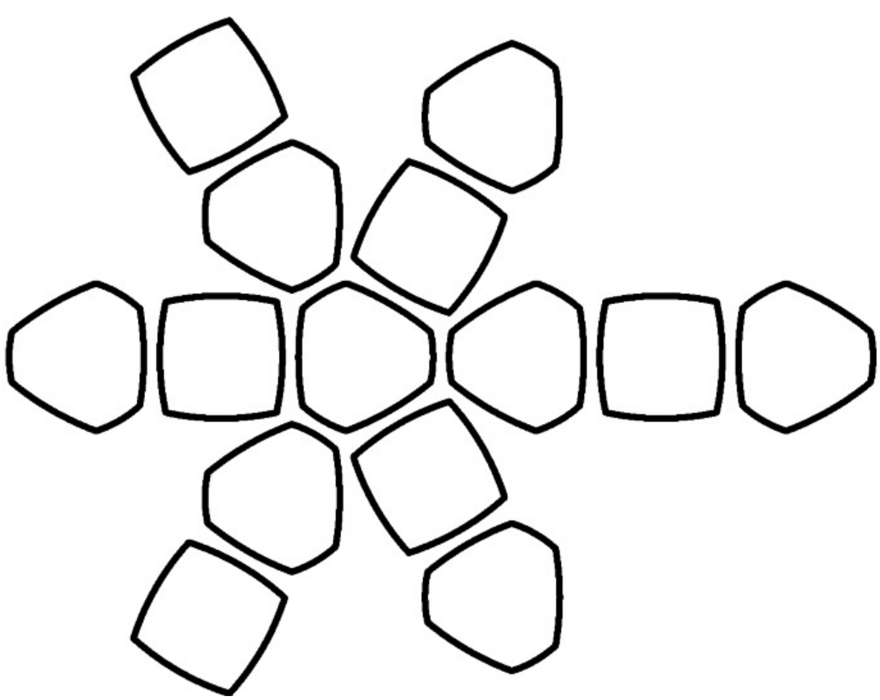
Blank Color Arrangement Diagrams

Back to Chapter Index

These are the ball and assembly layout diagrams I used for my color arrangement illustrations. You can use these to experiment with your own arrangements. I also offer PNG format diagrams for download on my website that you can use in an image editor. If they are unavailable, you can capture a screenshot of this page or export the image and then color it in an image editor. Or you can just print it and color it by hand.





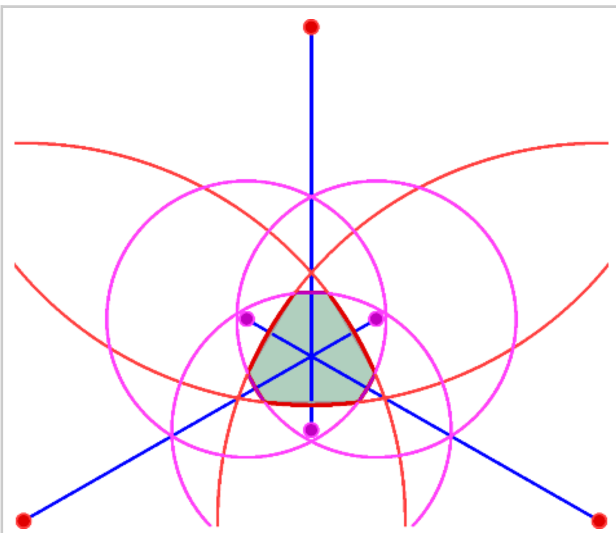


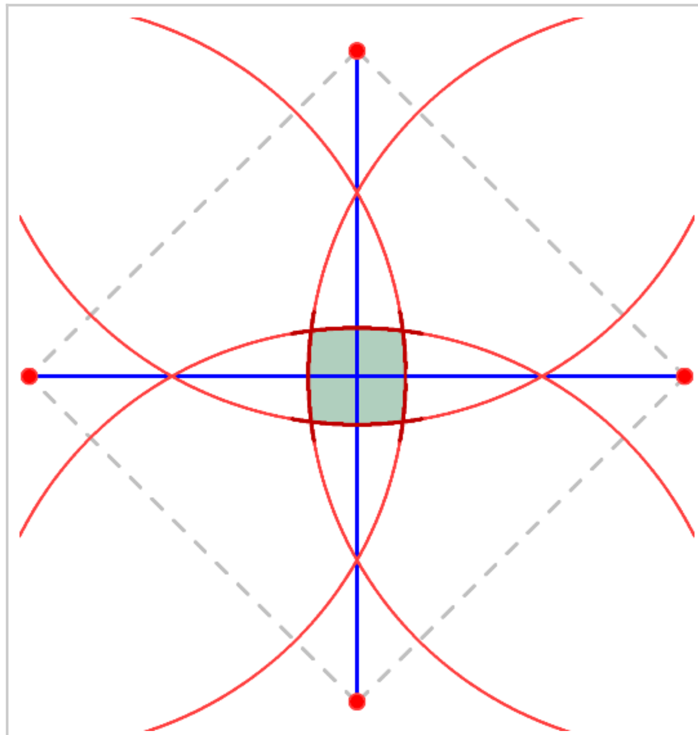
Sizing Formulas for Drawing the Circular Patterns

Back to Chapter Index

The drawing instructions that follow include, for each panel shape, a table of pre-calculated panel framework and arc measurements for all ⅛″ diameter increments from 1 ¾″ – 3″. If you do not need to create a custom size, skip to those. The "Mathematics" section has explanations of all the formulas and ratios, and expresses their values in higher precision. Sections on the simpler, straight-edged shapes follow the circular-edge sections. I provide printable measuring tapes at the end of the **General Information and Techniques** chapter in case you care to measure your beanbags.

Design summary





The circular patterns are drawn using a framework, the endpoints of which are the circle centers for the arcs that form the panel shapes. To draw the framework and arcs to produce a desired bag size, you need the relationship between framework's legs and arc radii, and the resulting bag circumference. I provide the formulas that define those relationships below, and they are fully explained and illustrated in the "Mathematics" section of this chapter.

Adjusting for the influence of fabric attributes on beanbag size

The bag I made with thick corduroy was **3.85 – 7.77%** larger than the mathematical prediction depending on whether I filled it loosely or over-filled it. I target halfway between the min and max inflations when sizing my patterns, which is **5.81%**. This makes my adjustment factor **1.0581**.

I use the adjustment factor to adjust the pattern size to produce a more accurate finished size when using my fabric and stitching techniques. If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may need to figure out your own adjustment factor. For help, see the **General Information and Techniques** chapter under "Adjusting/Scaling a Pattern to Produce an Accurate Ball Size".

The bag I made with my design testing fabric which is fairly thin, stiff, tightly-woven, and non-stretch, was 1.28 – 4.30% larger, but that was just for analyzing the shape characteristics of the bag. Though I did not make a straight-edged 14-panel bag with corduroy, the one I made with my design testing fabric inflated 1.95% less than the circular one with the same fabric. I am guessing the corduroy one would have the same difference in inflation, so I am assuming 3.86% in the straight-edged panel sizing table. (Just as a matter of record, my straight-edged denim bag from years ago measured 1.75 – 4.78% larger.)

As I understand it, the bag size is affected by fabric attributes as follows. The folding of the fabric at the seams will cause thick, firm fabrics to significantly shrink the bag size unless the fabric has some stretch. Folding thin fabric doesn't consume as much of its size, but my design testing fabric, though fairly thin, has no stretch at all, and so ended up producing about the same size bag as the denim, which stretches a little. Corduroy is a softer, more loosely woven fabric than denim and flexes and compresses more easily, and so not as much of the panels' size is consumed by the folding. My denim and design testing fabric bags have very prominent seams while the corduroy bag has much more subtle seams.

Sizing formulas

Below are the formulas to calculate the pattern construction elements (*Diameter* and *Circumference* refer to your target ball size). The value in orange is the adjustment factor. **Don't forget to multiply the final result by 25.4 if you need to convert inches to millimeters.**

Hex Panel

- **Framework Long Leg = *Diameter* × 1.5905 ÷ 1.0581**
  = *Circumference* × 0.5063 ÷ 1.0581
- **Framework Short Leg = *Diameter* × 0.3595 ÷ 1.0581**
  = *Circumference* × 0.1144 ÷ 1.0581
- **Angle between Framework Sticks = 120°/60°**
- **Long Arc Radius = *Long Leg* × 1.1485**
- **Short Arc Radius = *Short Leg* × 1.8597**
- For double-checking: **Panel Height = *Long Leg* × 0.3429**

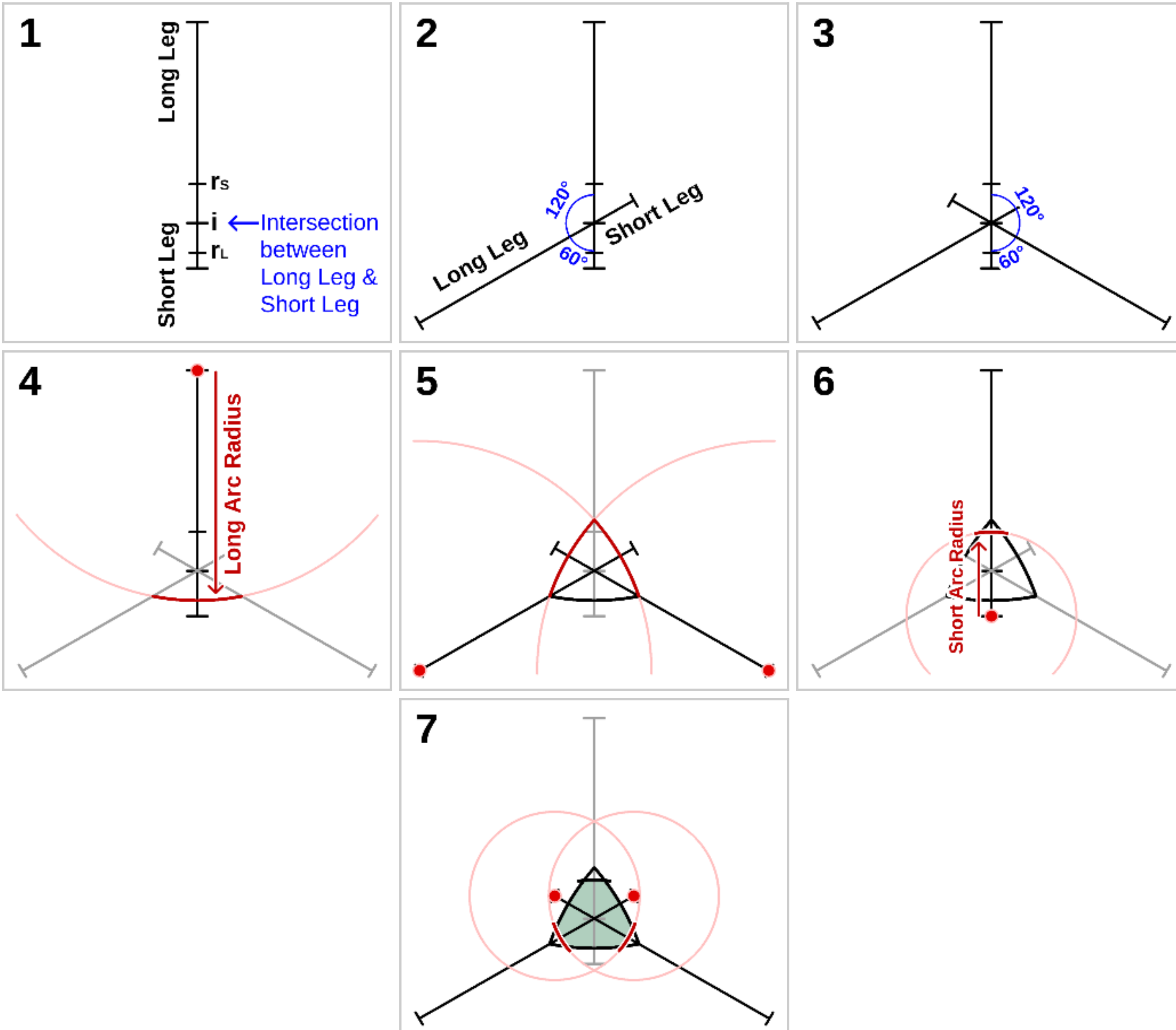
Square Panel

- **Circle Center Distance (Guide Square Diagonal) = *Diameter* × 3.1810 ÷ 1.0581**
  = *Circumference* × 1.0125 ÷ 1.0581
- **Framework Long Leg (Half Circle Center Distance) = [same as Hex's Long Leg]**
- **Guide Arc Radius (Guide Square Side) = *Diagonal* ÷ $\sqrt{2}$ ≈ *Diagonal* × 0.7071**
- **Pattern Arc Radius = [same as Hex's Long Arc Radius]**
- For double-checking: **Panel Width = *Guide Square Diagonal* × 0.1485**
  = *Long Leg* × 0.2971

How to Draw the Circular Hexagon

Back to Chapter Index

Following the pattern measurement table are manual and SketchUp directions for drawing the circular hex panel shape. The illustration numbers correspond to the steps of the manual directions. To conserve your template material, I recommend that you draw the pattern on paper and then glue or tape the pattern to your template material before cutting it out.



Circular Hexagon pattern measurement table

The table below has stitching pattern measurements for each ⅛″ diameter increment from 1¾″ to 3″. The values in the **Adjusted** columns account for my 1.0581 adjustment factor. The adjusted values decrease the **Base** pattern size so that you will get a more accurate finished size when using corduroy or something similar (a soft, flexible, moderately thick fabric). If you are using a firm denim or a thin, but non-stretch fabric, use the Base value instead. I attempt to explain why in the "Adjusting for the influence of fabric attributes on beanbag size" topic in the previous section.

To draw the cutting pattern, use the same framework but increase the Arc Radii by the desired seam allowance (I use 8mm) and center the new arcs at the same points. The cutting pattern will be larger than, but parallel to, the stitching pattern. Alternatively, you could just draw a simpler, straight-edged pattern around the circular one since the curves are so shallow.

Finished Diameter	**Long Leg Length** (mm)		**Short Leg Length** (mm)		**Long Arc Radius** (mm)		**Short Arc Radius** (mm)		**Pattern Height** (mm) (for double-checking)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
1¾″ (44.5mm)	70.698	**66.816**	15.982	**15.104**	81.198	**76.740**	29.721	**28.089**	24.240	**22.909**
1⅞″ (47.6mm)	75.747	**71.588**	17.123	**16.183**	86.998	**82.221**	31.844	**30.095**	25.971	**24.545**
2″ (50.8mm)	80.797	**76.361**	18.265	**17.262**	92.798	**87.703**	33.967	**32.101**	27.703	**26.182**
2⅛″ (54.0mm)	85.847	**81.133**	19.406	**18.341**	98.598	**93.184**	36.089	**34.108**	29.434	**27.818**
2¼″ (57.2mm)	90.897	**85.906**	20.548	**19.419**	104.398	**98.666**	38.212	**36.114**	31.166	**29.454**
2⅜″ (60.3mm)	95.947	**90.678**	21.689	**20.498**	110.198	**104.147**	40.335	**38.120**	32.897	**31.091**
2½″ (63.5mm)	100.997	**95.451**	22.831	**21.577**	115.998	**109.628**	42.458	**40.127**	34.629	**32.727**
2⅝″ (66.7mm)	106.046	**100.223**	23.972	**22.656**	121.798	**115.110**	44.581	**42.133**	36.360	**34.364**
2¾″ (69.9mm)	111.096	**104.996**	25.114	**23.735**	127.598	**120.591**	46.704	**44.139**	38.091	**36.000**
2⅞″ (73.0mm)	116.146	**109.769**	26.255	**24.814**	133.397	**126.073**	48.827	**46.146**	39.823	**37.636**
3″ (76.2mm)	121.196	**114.541**	27.397	**25.893**	139.197	**131.554**	50.950	**48.152**	41.554	**39.273**

Manual directions for the Circular Hex

(The terms in bold refer to columns in the pattern measurement table above.)

1. Draw a vertical line of length **Long Leg Length** and mark the ends of it. Continue the line at the bottom by the distance of **Short Leg Length** and mark the endpoint of that line. The intersection is labeled *i* in Illustration 1. Mark a point that is located **Long Arc Radius** (r_L) distant from the *top* of the line (the outer endpoint of the Long Leg) as shown in Illustration 1. The mark will be located on the Short Leg. Measure another point located **Short Arc Radius** (r_s), distant from the *bottom* of the line (the opposite endpoint of the Short Leg). This mark will be on the Long Leg.

2. Place a protractor on the line, center it at point *i*, and measure a 120° angle out from the upper segment (or 60° from the lower segment). Draw a line at that angle (down and to the left) of length **Long Leg Length** with its endpoint at point *i*. Continue the line beyond *i* (up and right) by the length of **Short Leg Length**. This forms two framework sticks that intersect where their respective long and short legs meet as shown in Illustration 2.

3. Repeat the previous step for the other side of the vertical line, resulting in the framework shown in Illustration 3.

4. Place a compass at the top end of the vertical line (the first Long Leg), extend it to the **Long Arc Radius**, point r_L, and draw an arc between the other two lines as shown in Illustration 4.

5. As shown in Illustration 5, draw the same arc from the outer endpoints of the other two framework lines, forming a circular triangle.

6. Place the compass at the bottom end of the vertical line (the first Short Leg), extend it to the **Short Arc Radius**, point r_S, and draw an arc through the top corner of the triangle as shown in Illustration 6.
7. As shown in Illustration 7, draw the same arc from the inner endpoints of the other two framework lines, completing the circular hexagon panel shape. Its height, from each long arc to opposite short arc, should equal **Pattern Height**. Any error you make will be compounded several times in the juggling bag, so be as precise as you can.
8. To draw a cutting pattern, draw everything the same but increase the Long Arc Radius and Short Arc Radius by the desired seam allowance (I use 8mm) and then draw the six arcs from the same points using the new radii. Or just draw the simpler, straight-edged pattern shape around the circular one since the curves are so shallow.

SketchUp directions for the Circular Hex

(The terms in bold refer to columns in the pattern measurement table above.)

1. Draw a vertical line of length **Long Leg Length**, then continue that line on the bottom by the length of **Short Leg Length**. I will refer to the point between these two segments as point *i* as shown in Illustration 1.
2. Use the Protractor tool centered at point *i* to measure a 120° angle from the long leg segment (or 60° from the short leg segment) and draw a line of length **Long Leg Length** at that angle (down and to the left), and a line of length **Short Leg Length** in the opposite direction as shown in Illustration 2.
3. Repeat this on the other side, resulting in the framework shown in Illustration 3.
4. Draw circles of radius **Long Arc Radius** centered at each outer endpoint of the framework, forming a circular triangle where they intersect (Illustrations 4 and 5).
5. Draw circles of radius **Short Arc Radius** centered at each inner endpoint of the framework, completing the circular hexagon panel shape (Illustrations 6 and 7). Its height, from each long arc to opposite short arc, should equal **Pattern Height**.
6. To draw a cutting pattern, draw everything the same but increase the Long Arc Radius and Short Arc Radius by the desired seam allowance (I use 8mm) and then draw the six arcs from the same points using the new radii.

Back to Chapter Index

How to Draw the Circular Square

Back to Chapter Index

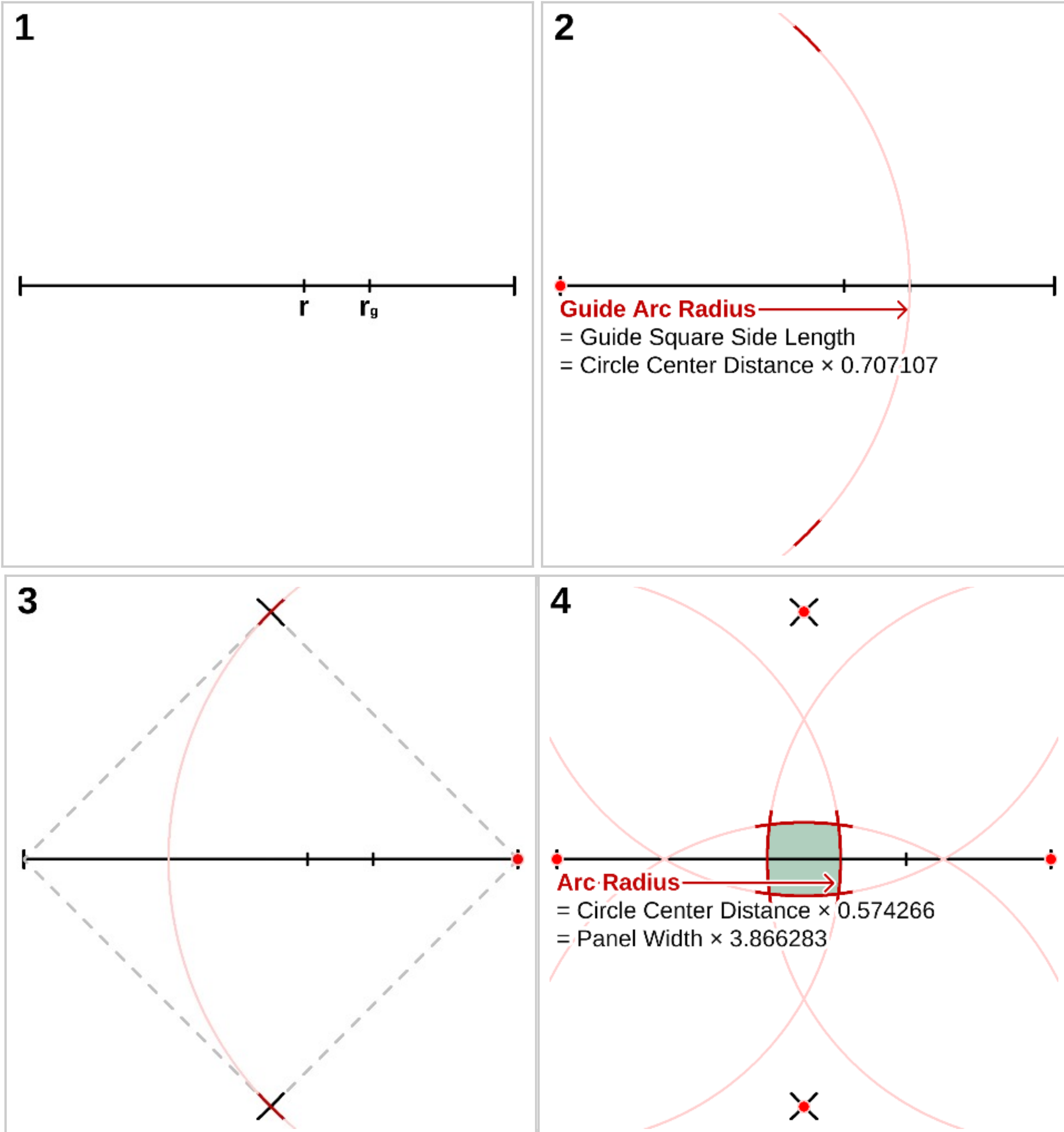
Following the pattern measurement table are manual and SketchUp directions for drawing the circular square panel shape. The illustration numbers correspond to the step numbers. To conserve your template material, I recommend that you draw the pattern on paper and then glue or tape the pattern to your template material before cutting it out.

Circular Square pattern measurement table

The table below has stitching pattern measurements for each ⅛″ diameter increment from 1¾″ to 3″. The values in the **Adjusted** columns account for my 1.0581 adjustment factor. The adjusted values decrease the **Base** pattern size so that you will get a more accurate finished size when using corduroy or something similar (a soft, flexible, moderately thick fabric). If you are using a firm denim or a thin, but non-stretch fabric, use the Base value instead. I attempt to explain why in the "Adjusting for the influence of fabric attributes on beanbag size" topic earlier in this chapter.

To draw the cutting pattern, use the same framework, but increase the Pattern Arc Radius by the desired seam allowance (I use 8mm) and center the new arcs at the same points. The cutting pattern will be larger than, but parallel to, the stitching pattern. Alternatively, you could just draw a simpler, straight-edged pattern around the circular one since the curves are so shallow.

Finished Diameter	**Circle Center Distance** (guide square diagonal) (mm)		**Framework Long Leg** (half circle dist) (mm)		**Guide Arc Radius** (guide square side length) (mm)		**Pattern Arc Radius** (mm)		**Pattern Width** (mm) (for double-checking)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
1¾″ (44.5mm)	141.395	**133.631**	70.698	**66.816**	99.982	**94.492**	81.198	**76.740**	21.002	**19.848**
1⅞″ (47.6mm)	151.495	**143.176**	75.747	**71.588**	107.123	**101.241**	86.998	**82.221**	22.502	**21.266**
2″ (50.8mm)	161.595	**152.721**	80.797	**76.361**	114.265	**107.990**	92.798	**87.703**	24.002	**22.684**
2⅛″ (54.0mm)	171.694	**162.266**	85.847	**81.133**	121.406	**114.740**	98.598	**93.184**	25.502	**24.102**
2¼″ (57.2mm)	181.794	**171.812**	90.897	**85.906**	128.548	**121.489**	104.398	**98.666**	27.002	**25.519**
2⅜″ (60.3mm)	191.893	**181.357**	95.947	**90.678**	135.689	**128.239**	110.198	**104.147**	28.502	**26.937**
2½″ (63.5mm)	201.993	**190.902**	100.997	**95.451**	142.831	**134.988**	115.998	**109.628**	30.002	**28.355**
2⅝″ (66.7mm)	212.093	**200.447**	106.046	**100.223**	149.972	**141.737**	121.798	**115.110**	31.503	**29.773**
2¾″ (69.9mm)	222.192	**209.992**	111.096	**104.996**	157.114	**148.487**	127.598	**120.591**	33.003	**31.190**
2⅞″ (73.0mm)	232.292	**219.537**	116.146	**109.769**	164.255	**155.236**	133.397	**126.073**	34.503	**32.608**
3″ (76.2mm)	242.392	**229.082**	121.196	**114.541**	171.397	**161.986**	139.197	**131.554**	36.003	**34.026**



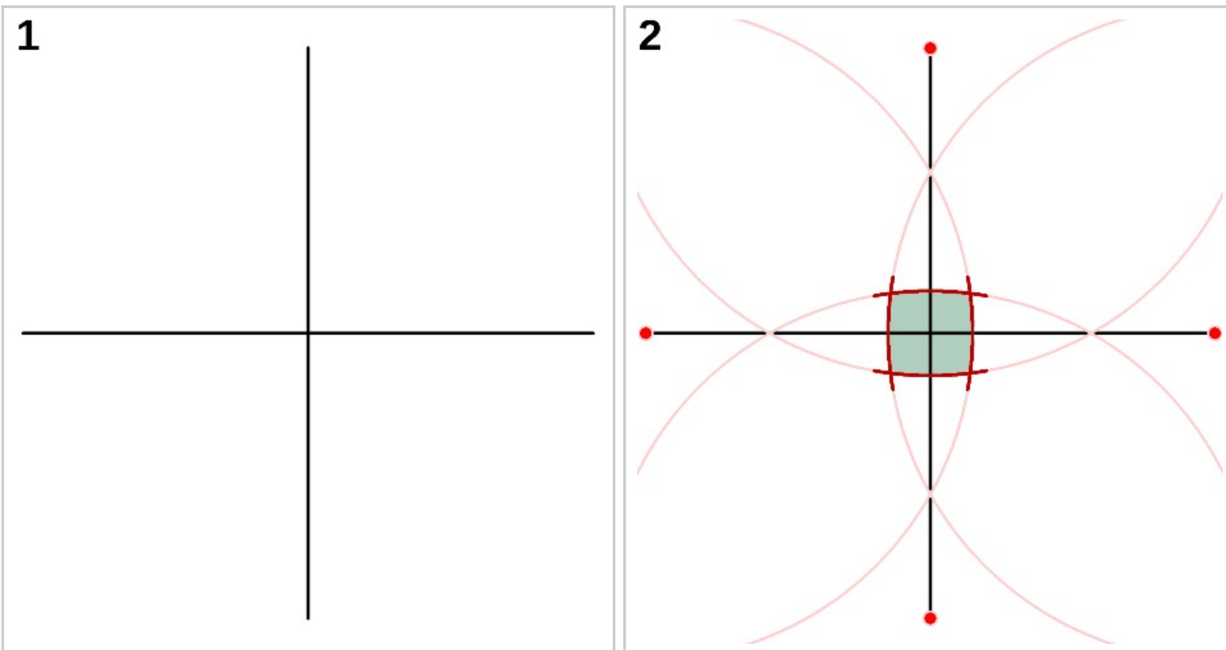
Illustrations for the manual directions. The numbers correspond to the step numbers.

Manual directions for the Circular Square

(The terms in bold refer to columns in the pattern measurement table above.)

1. Draw a horizontal line of length **Circle Center Distance** and mark each end of it. Also mark two points along it: r, which is the distance of **Pattern Arc Radius** from one end, and r_g, which is the distance of **Guide Arc Radius** from the same end. You will use these marks to extend the compass to the correct radii.
2. Extend the compass to the **Guide Arc Radius** mark (r_g), and from that endpoint of the line draw partial arcs above and below the middle of the line.

3. Place the compass on the opposite endpoint and draw two more arcs to produce two X-shaped intersections. You have now formed the four corners of the Guide Square, depicted in Illustration 3 by the dashed gray lines, which are the compass points to use in forming the panel shape.
4. Extend the compass to the **Pattern Arc Radius** mark (*r*) and draw four arcs, two centered at the endpoints of the line, and two centered at the arc intersections you made in the previous step. This forms the circular square, whose width and height should equal **Pattern Width**. Any error you make will be compounded several times in the juggling bag, so be as precise as you can.
5. To draw a cutting pattern, increase the Pattern Arc Radius by the desired seam allowance (I use 8mm) and then draw four arcs centered at the same four points. Or just draw the simpler, straight-edged pattern shape around the circular one since the curves are so shallow.



SketchUp directions for the Circular Square

(The terms in bold refer to columns in the pattern measurement table above.)

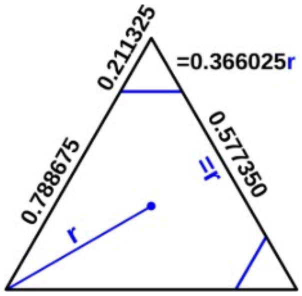
1. Draw two perpendicular lines of length **Circle Center Distance** and center them on each other. You can use the **Framework Long Leg** value to draw each half of the perpendicular line from the center of the first line.
2. Draw circles of radius **Pattern Arc Radius** centered on all four ends of the lines. The intersection of the four circles forms the circular square. Its width and height should equal **Pattern Width**.
3. To draw a cutting pattern, draw everything the same but increase the Pattern Arc Radius by the desired seam allowance (I use 8mm).

Back to Chapter Index

Sizing Formulas for Drawing the Straight-Edged Patterns

Back to Chapter Index

The drawing instructions that follow include, for each panel shape, a table of pre-calculated pattern measurements for all ⅛″ diameter increments from 1 ¾″ – 3″. If you do not need to create a custom size, skip to those. The “Mathematics” section has explanations of all the formulas and ratios, and expresses their values in higher precision.



The key element of the polyhedral design is the hexagon, which is derived from an equilateral triangle. The square is based simply on the hexagon’s long side, which happens to be the same as the triangle’s circumradius. For a full discussion of this design, see the “How I Developed This Design” section.

To draw a pattern sized to produce a desired bag size, you need the relationship between the starting triangle’s side length or circumradius and the resulting bag circumference. To draw the panel shape manually you need the side length, and to draw it in SketchUp using the polygon tool, which uses a defined circumradius, you need the triangle’s circumradius (center to corner). I provide the formulas that define those relationships below, and they are fully explained and illustrated in the “Mathematics” section of this chapter.

Though I did not make a straight-edged bag with corduroy, the one I made with my design testing fabric inflated 1.95% less than the circular one with the same fabric, so I am guessing the corduroy one will have the same difference in inflation, so I am assuming an adjustment factor of 1.0386 in the straight-edged panel sizing table.

I use the adjustment factor to adjust the pattern size to produce a more accurate finished size when using my fabric and stitching techniques. If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may need to figure out your own adjustment factor. For help, see the **General Information and Techniques** chapter under “Adjusting/Scaling a Pattern to Produce an Accurate Ball Size”.

Below are the formulas to calculate the pattern construction elements (*Diameter* and *Circumference* refer to your target ball size). The value in orange is the adjustment factor. **Don’t forget to multiply the final result by 25.4 if you need to convert inches to millimeters.**

Sizing formulas

Hex Panel

- **Starting Triangle Side Length = *Diameter* × 0.7930 ÷ 1.0386**
  = *Circumference* × 0.2524 ÷ 1.0386
- **Starting Triangle Circumradius = *Diameter* × 0.4578 ÷ 1.0386**
  = *Circumference* × 0.1457 ÷ 1.0386
- **Triangle Truncation to Form the Hexagon = *Triangle Side* × 0.2113**
  = *Triangle Circumradius* × 0.3660
- For double-checking: **Resulting Hex Long Side = Triangle Circumradius**
- For double-checking: **Hex Height = *Triangle Side Length* × 0.6830**

Square Panel

- **Square Side Length = [same as Starting Triangle Circumradius above]**
- **Square Diagonal** (for drawing by hand) **=** ***Side Length*** **×** $\sqrt{2} \approx$ ***Side Length*** **× 1.4142**

How to Draw the Straight-Edged Hexagon

Back to Chapter Index

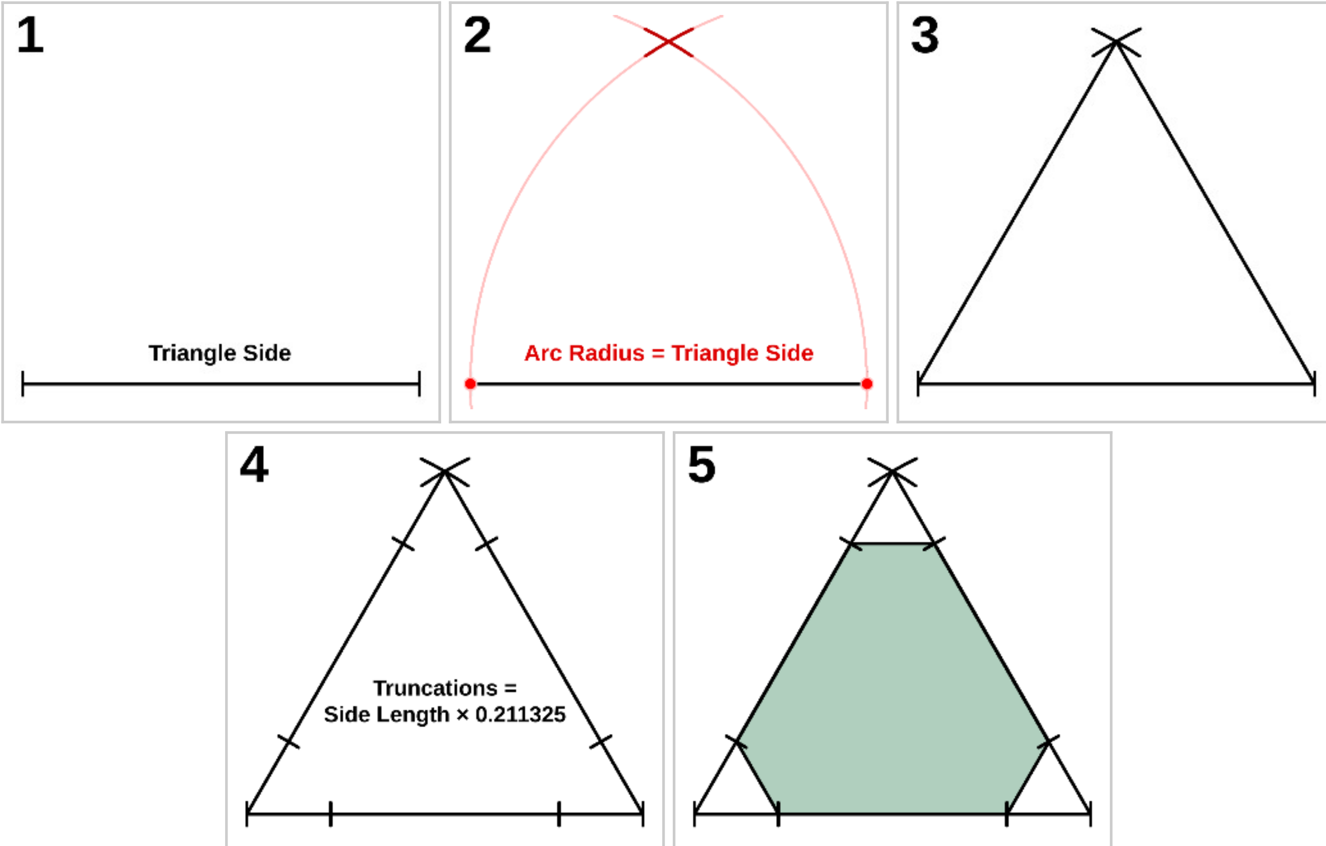
Following the pattern measurement table are manual and SketchUp directions for drawing the hex panel shape. To conserve your template material, I recommend that you draw the pattern on paper and then glue or tape the pattern to your template material before cutting it out.

Straight-Edged Hexagon pattern measurement table

The table below has stitching pattern measurements for each ⅛″ diameter increment from 1¾″ to 3″. The values in the **Adjusted** columns account for the 1.0386 estimated adjustment factor. The adjusted values decrease the **Base** pattern size so that you will get a more accurate finished size when using corduroy or something similar (a soft, flexible, moderately thick fabric). If you are using a firm denim or a thin, but non-stretch fabric, use the Base value instead. I attempt to explain why in the "Adjusting for the influence of fabric attributes on beanbag size" topic earlier in this chapter.

To draw the cutting pattern, increase the Triangle Side Length by the desired allowance × 3.4641, or increase the Triangle Circumradius by the allowance × 2. Increase the Truncation by the allowance × 1.1547.

Finished Diameter	**Triangle Side Length** (mm)		**Triangle Circumradius** (mm)		**Truncation** (mm)		**Pattern Height** (mm) (for double-checking)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
1¾″ (44.5mm)	35.249	**33.939**	20.351	**19.595**	7.449	**7.172**	24.076	**23.181**
1⅞″ (47.6mm)	37.767	**36.364**	21.805	**20.995**	7.981	**7.685**	25.795	**24.837**
2″ (50.8mm)	40.285	**38.788**	23.259	**22.394**	8.513	**8.197**	27.515	**26.493**
2⅛″ (54.0mm)	42.803	**41.212**	24.712	**23.794**	9.045	**8.709**	29.235	**28.148**
2¼″ (57.2mm)	45.321	**43.636**	26.166	**25.193**	9.577	**9.221**	30.955	**29.804**
2⅜″ (60.3mm)	47.838	**46.061**	27.620	**26.593**	10.109	**9.734**	32.674	**31.460**
2½″ (63.5mm)	50.356	**48.485**	29.073	**27.993**	10.642	**10.246**	34.394	**33.116**
2⅝″ (66.7mm)	52.874	**50.909**	30.527	**29.392**	11.174	**10.758**	36.114	**34.771**
2¾″ (69.9mm)	55.392	**53.333**	31.981	**30.792**	11.706	**11.271**	37.833	**36.427**
2⅞″ (73.0mm)	57.910	**55.757**	33.434	**32.192**	12.238	**11.783**	39.553	**38.083**
3″ (76.2mm)	60.428	**58.182**	34.888	**33.591**	12.770	**12.295**	41.273	**39.739**



Illustrations for the manual directions. The numbers correspond to the step numbers.

Manual directions for the Straight-Edged Hex

(The terms in bold refer to columns in the pattern measurement table above.)

1. Draw a horizontal line the length of **Triangle Side Length** and mark each end of it.
2. Place a compass on one end of the line, extend it to the other end, and draw a small arc above the center of the line. Draw the same arc from the other end of the line. The resulting X-shaped intersection marks the third corner of the triangle.
3. Draw a line from each endpoint mark to the X, forming an equilateral triangle.
4. Measure a distance equal to **Truncation** inward from each end of each side as shown in Illustration 4.
5. Join the truncation marks to form the hexagonal panel shape. Its height, from each long edge to opposite short edge, should equal **Pattern Height**. Any error you make will be compounded several times in the juggling bag, so be as precise as you can.
6. To draw a cutting pattern, multiply the desired allowance by 3.4641 and add that to the **Triangle Side Length**, then multiply the allowance by 1.1547 and add that to the **Truncation**. Or, just draw the cutting pattern around the stitching pattern, using its edges as guides.

SketchUp directions for the Straight-Edged Hex

(The terms in bold refer to columns in the pattern measurement table above.)

1. Use the polygon tool (in the Shapes tool drop-down, or in Draw menu -> Shapes) set to 3 sides and draw a triangle with circumradius = **Triangle Circumradius**, which will result in a triangle with sides of length **Triangle Side Length**.
2. Draw lines of length **Truncation** inward from each end of each side (their endpoints will match the locations of the truncation marks in Illustration 4).
3. Join the pairs of truncation endpoints across each of the triangle's corners, forming the hexagonal panel shape as shown in Illustration 5. Erase the triangle's corners. The resulting panel's height, from each long edge to opposite short edge, should equal **Pattern Height**.
4. To draw a cutting pattern, multiply the desired allowance by 2 and add that to the **Triangle Circumradius** (or by 3.4641 and add that to the **Triangle Side Length**), and multiply it by 1.1547 and add that to the **Truncation**. Or, just draw the cutting pattern around the stitching pattern, using its edges as guides.

Back to Chapter Index

How to Draw the Straight-Edged Square

Back to Chapter Index

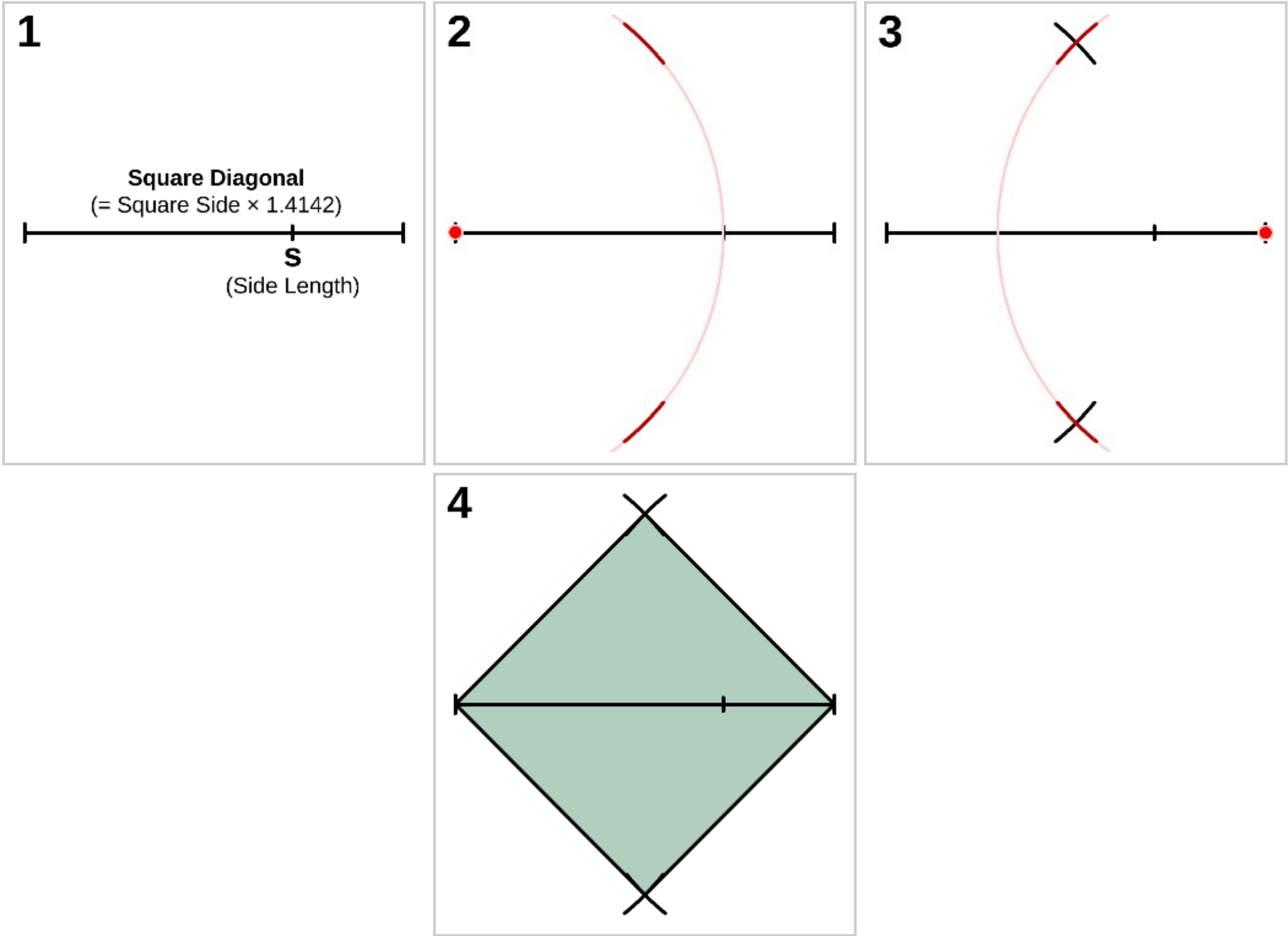
Following the pattern measurement table are manual and SketchUp directions for drawing the square panel shape. To conserve your template material, I recommend that you draw the pattern on paper and then glue or tape the pattern to your template material before cutting it out.

Straight-Edged Square pattern measurement table

The table below has stitching pattern measurements for each ⅛″ diameter increment from 1¾″ to 3″. The values in the **Adjusted** columns account for the 1.0386 estimated adjustment factor. The adjusted values decrease the **Base** pattern size so that you will get a more accurate finished size when using corduroy or something similar (a soft, flexible, moderately thick fabric). If you are using a firm denim or a thin, but non-stretch fabric, use the Base value instead. I attempt to explain why in the "Adjusting for the influence of fabric attributes on beanbag size" topic.

To draw the cutting pattern, increase the Square Diagonal by the desired allowance × 2.8284, and increase the Square Side Length by the allowance × 2.

Finished Diameter	**Square Diagonal** (mm)		**Square Side Length** (mm)	
	Base	**Adjusted**	Base	**Adjusted**
1¾″ (44.5mm)	28.781	**27.711**	20.351	**19.595**
1⅞″ (47.6mm)	30.837	**29.691**	21.805	**20.995**
2″ (50.8mm)	32.893	**31.670**	23.259	**22.394**
2⅛″ (54.0mm)	34.948	**33.649**	24.712	**23.794**
2¼″ (57.2mm)	37.004	**35.629**	26.166	**25.193**
2⅜″ (60.3mm)	39.060	**37.608**	27.620	**26.593**
2½″ (63.5mm)	41.116	**39.588**	29.073	**27.993**
2⅝″ (66.7mm)	43.171	**41.567**	30.527	**29.392**
2¾″ (69.9mm)	45.227	**43.546**	31.981	**30.792**
2⅞″ (73.0mm)	47.283	**45.526**	33.434	**32.192**
3″ (76.2mm)	49.339	**47.505**	34.888	**33.591**



Illustrations for the manual directions. The numbers correspond to the step numbers.

Manual directions for the Straight-Edged Square

(The terms in bold refer to columns in the pattern measurement table above.)

1. Draw a horizontal line the length of **Square Diagonal** and mark each end of it. Mark another point located the distance of **Square Side Length** from the left end of the line (labeled *s* in Illustration 1).
2. Place a compass on the left end of the line, extend it to point *s*, and draw a short arc above and below the center of the line as shown in Illustration 2.
3. Place the compass on the other end of the line and draw the same two arcs, forming two X-shaped intersections as shown in Illustration 3. These form the other two corners of the square.
4. Draw lines connecting each line endpoint to each arc intersection, forming the square panel as shown in Illustration 4. Measure its sides and make sure they are equal. Any error you make will be compounded several times in the juggling bag, so be as precise as you can.
5. To draw a cutting pattern, multiply the desired allowance by 2.8284 and add that to the **Square Diagonal** (the first line you drew), and multiply it by 2 and add that to the **Square Side Length** (the distance of point *s*). Or, just draw the cutting pattern around the stitching pattern, using its edges as guides.

SketchUp directions for the Straight-Edged Square

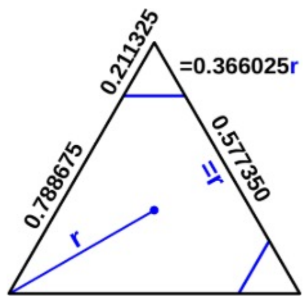
(The terms in bold refer to columns in the pattern measurement table above.)

1. Draw a square with sides of length **Square Side Length**. SketchUp snaps lines to perpendiculars, so drawing a square is easy and does not require the Protractor tool. (You could also use the Polygon tool set to 4 sides and draw a square with circumradius equal to one half of **Square Diagonal**).
2. To draw a cutting pattern, multiply the desired allowance by 2 and add that to the **Square Side Length**. Or, just draw the cutting pattern around the stitching pattern, using its edges as guides.

Mathematics Behind the Relationship Between the Pattern Parameters and the Ball Size

Back to Chapter Index

This section describes the math involved in drawing patterns to produce specified beanbag sizes, and creating the pattern sizing formulas. *(The numbers in tiny, right-justified typeface are my computer calculator's unrounded values which I display rounded to six places for brevity.)*



The key element of the design is the hexagon, which is derived from an equilateral triangle. The square is based simply on the hexagon's long side, which happens to be the same as the triangle's circumradius (center to corner).

The truncation applied to the triangle is defined as

$$1-\frac{1}{3-\sqrt{3}} \approx 0.211325$$

High precision ratios used in my calculations:
0.211324865405187117745425609749   0.788675134594812882254574390251   0.577350269189625764509148780502
0.36602540378443864676372317075287

This formula is explained in the "How I Developed This Design" section, and I also show the proof that the triangle's circumradius equals the derived hexagon's long side.

Once you have used the circumference formulas that follow to decide on a starting triangle size for your beanbag, multiply the triangle side length by 0.211325 to get the amount of truncation for each end of each side (or multiply the circumradius by 0.366025), and make the square the same width as the triangle's circumradius (which is the *triangle's side length* × $1/\sqrt{3} \approx 0.577350$).

Calculating the circumference

For the design formulas, I will define the following variables:

d = diagonal of the square
h_h = height of the hexagon
r_t = circumradius of the starting triangle (equal to w_s and s_{hl})
s_t = side length of the starting triangle
s_{hs} = short side length of the hexagon
w_s , s_{hl} = width/height of the square and hexagon's long side length (these are equal)

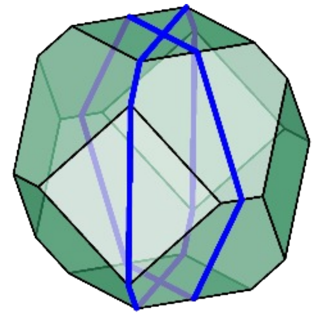
Now I will evaluate each of those in terms of the starting triangle's side length, s_t. The values in blue are from the diagram above.

$d \approx (\sqrt{2})\, 0.577350 s_t \approx 0.816497 s_t$ 0.81649658092772603273242802490202

h_h = *triangle height – truncated corner height* $\approx \frac{\sqrt{3}}{2} s_t - \frac{\sqrt{3}}{2}(0.211325 s_t) \approx 0.683013 s_t$ 0.68301270189221932338186158537649

$r_t, s_{hl}, w_s \approx 0.577350 s_t$ 0.577350269189625764509148780502

$s_{hs} \approx 0.211325 s_t$ 0.211324865405187117745425609749



There are two ways to define the circumference of the polyhedron. One is 4 × *height of the hexagon* + 2 × *width of the square* (which is the same as the long side of the hexagon) and the other is 4 × *diagonal of the square* + 4 × *short side of the hexagon*.

Circumference $A = 4h_h + 2w_s$
Circumference $B = 4d + 4s_{hs}$

So,

Circumference $A \approx 4(0.683013s_t) + 2(0.577350s_t) \approx 3.886751s_t$ 3.88675134594812882254574390251

Circumference $B \approx 4(0.816497s_t) + 4(0.211325s_t) \approx 4.111286s_t$ 4.1112857853316526019114145386041

Circumference B is greater than A by 5.78%, which is significant enough that I will use a weighted average of the two. To determine the count of each type of circumference, take one of the dimensions of either face that composes that circumference (for instance, the hex's height or the square's diagonal), calculate the sum of those on all faces, being sure to account for the number of orientations in which the dimension can be measured (3 hex heights per hex for a total of 24, or 2 square diagonals per square for a total of 12) and then divide by the number of those along each circumference path (24 hex heights ÷ 4 or 12 square diagonals ÷ 4).

Circumference A occurs 6 times on the polyhedron and B occurs 3 times, so A is weighted twice.

Weighted Average Circumference ≈ $3.961596s_t$ 3.961596159075970082334300781208

I also want the ball size defined in terms of the triangle's circumradius, which is the same as the hexagon's long side and the square's sides.

Weighted Average Circumference ≈ $\frac{3.961596}{0.577350}s_{hl}$ **≈ $6.861686r_t = 6.861686s_{hl}$** 6.8616858265892964564388467542872

Starting triangle and square expressed in terms of ball size

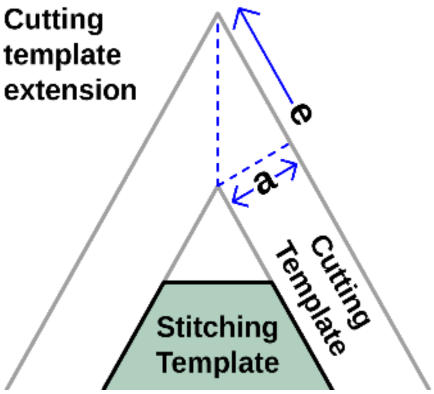
To express the starting triangle and the square in terms of a desired ball size, which is necessary to draw a pattern, I will solve the above expressions for the right-hand variables to express them in terms of the ball Circumference, C, and then the Diameter, D (by multiplying the right sides of the equations by π).

Starting Triangle Side Length ≈ $0.252424C$ 0.25242350806227732040503418584863
≈ $0.793012D$ 0.79301183852181437451713409472074

Starting Triangle Circumradius, Hex Long Side, and Square Side ≈ $0.145737C$ 0.14573678032954548038483629391552
≈ $0.457846D$ 0.45784559844112956764003880663711

Cutting pattern adjustments

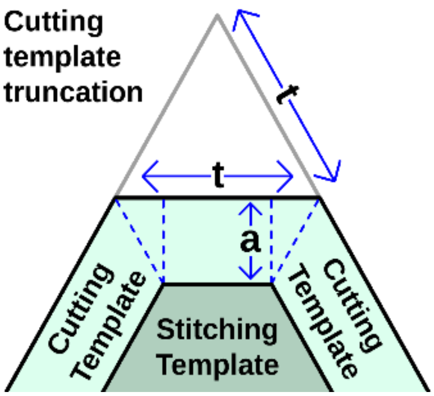
Designing the cutting patterns requires more trigonometry. For the square, simply extend the side length by twice the allowance, or the diagonal by the allowance × $2\sqrt{2}$. The diagrams below illustrate how to calculate the hex. The triangles outlined in blue dashed lines are 30-60-90 right triangles, which helps in solving them. I won't go through all the trigonometry; I'll just give the formulas.



e is the amount to extend one end of each side to get a seam allowance ***a***. You would have to double that to get the full amount by which to extend each side for the cutting pattern and then, if you're drawing it in SketchUp, convert it to a change in circumradius.

Side Length Extension, 2*e* = $2\sqrt{3}a$ **≈ 3.464102*a*** 3.4641016151377545870548926830117

Circumradius increase = $\frac{1}{\sin 30°}a$ or $2\sqrt{3}(\frac{\sqrt{3}}{3})a$ **= 2*a***



t is the amount of truncation to apply to each end of each side. It equals the short side of the stitching pattern plus the tops of the two 30-60-90 right triangles in blue dashed lines. The short side is 0.366025 × triangle circumradius which is just the stitching pattern truncation. The triangle tops are each $a/\sqrt{3}$. So the formula for *t* is

Truncation, *t* = stitching truncation + $\frac{2}{\sqrt{3}}a$

≈ stitching truncation + 1.154701*a* 1.1547005383792515290182975610039

Back to Chapter Index

The circular design calculations begin on the next page.

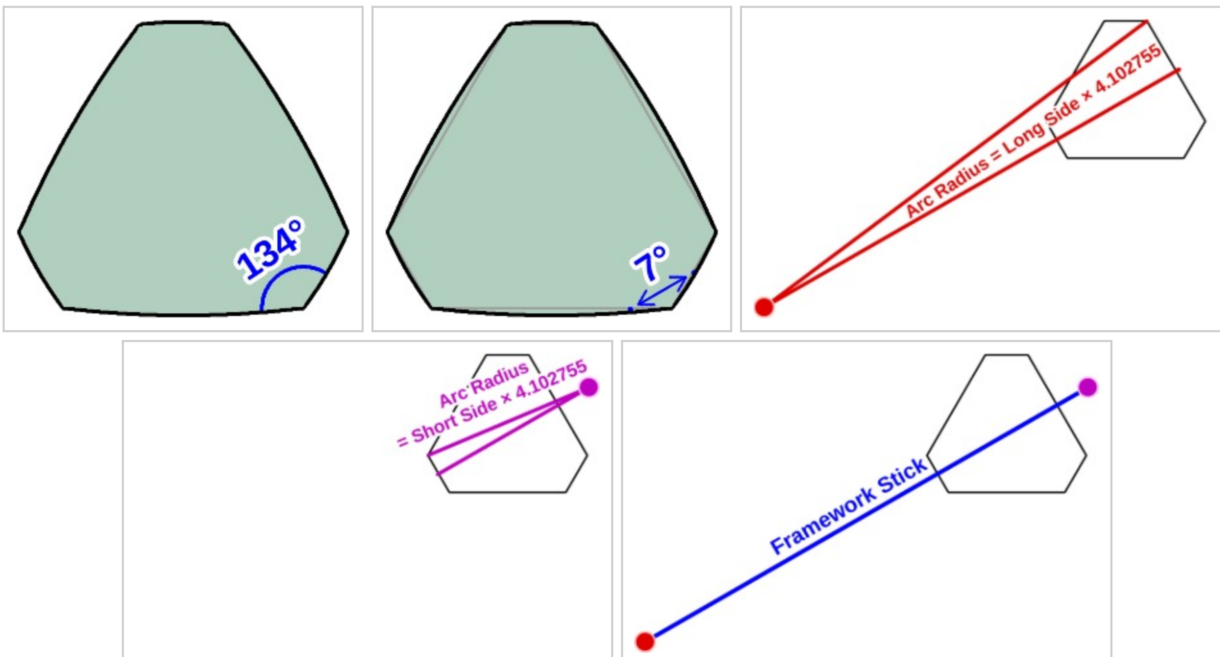
Circular panel shape calculations

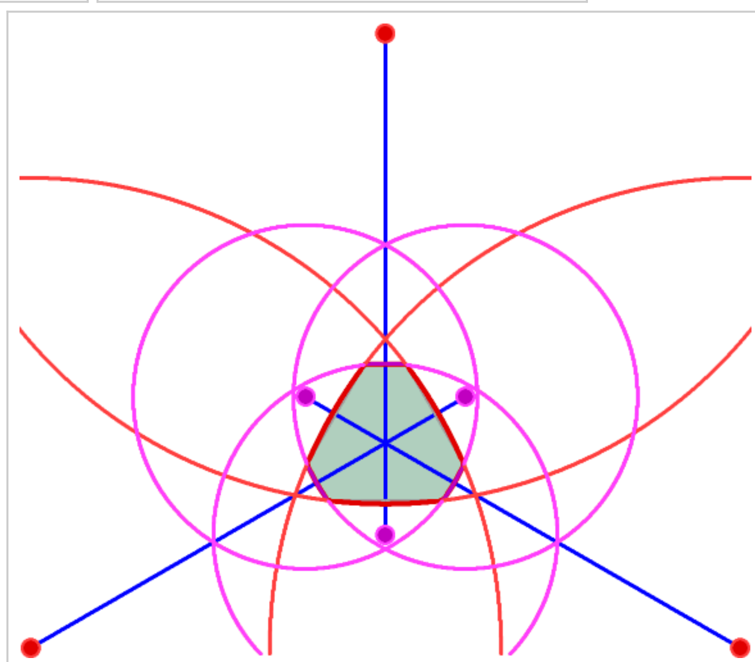
Back to Chapter Index

For the circular version of the panels I used arc radii that add 14° to the panels' corner angles (7° tangent-chord angle on either side of the corners), making the sum of the three corners at each ball vertex 372° and producing a smoothly round ball. I draw each panel shape using a Guide Framework that defines the locations of the circle centers for the arcs that form the circular patterns. This section demonstrates how I calculate the Guide Framework and the two arc radii, a longer one for the hex's long edge arcs and the square's edge arcs, and a shorter one for the hex's short edge arcs.

Using my edge arc radius formula from Chapter 5, I can calculate the radii, which is the first step toward calculating the Guide Framework:

$$\textbf{Arc Radius} = \frac{0.5\,s}{\sin 7°} \approx \frac{0.5\,s}{0.121869} \approx \mathbf{4.102755}\boldsymbol{s} \quad (s = \text{length of the corresponding hex side})$$





The "**Guide Framework**" (in blue) defines the circle center locations for the six arcs that form the hex pattern shape. The pattern has three long edge arcs (those circles are shown in red) and three short edge arcs (formed by the smaller, purple circles), so each of the three "**Framework Sticks**" is composed of a **Long Leg** and a **Short Leg**, which meet in the center of the framework. The length of each leg defines distance of the corresponding arc's circle center from the center of the figure.

The square's framework is shown farther on in this section.

For the formulas in this section, I will define the following variables:

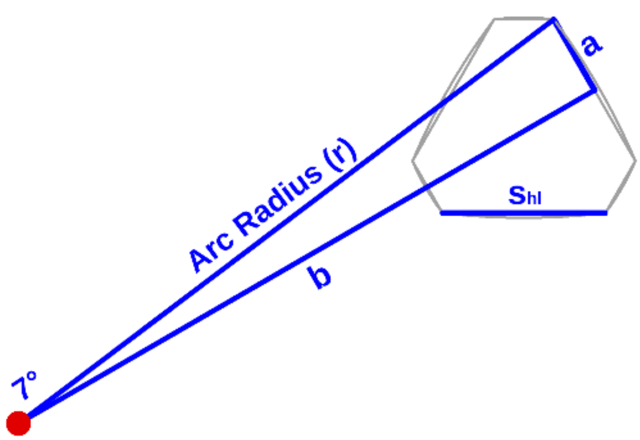
a_s = short arc length of the hexagon
d = diagonal of the square
g_L = long sagitta (height of apex of curve over hex's long side)
g_s = short sagitta (height of apex of curve over hex's short side)
h_{ch} = height of the circular hex
r = arc radius
s_{hl} = hexagon's long side length
s_{hs} = hexagon's short side length
s_t = side length of the starting triangle
w_{cs} = width of circular square
L_L = framework long leg
L_S = framework short leg

Calculating the guide framework

The guide framework for the hex pattern is composed of "Framework Sticks", each composed of a Long Leg and a Short Leg that are parallel to each other and connect in the center of the framework. The two leg lengths correspond to the longer circle radius forming the long edge arcs and the shorter radius forming the short edge arcs. Calculating the distance between the hexagon's center and the circle centers (yielding the two leg lengths), involves solving a right triangle as shown below. I will illustrate only the long leg calculation.

The hypotenuse of the right triangle is the arc radius, side a is half of the hex's long side, and I need to solve for side b, which extends from the circle center to the hexagon's opposite side. After that, I can subtract the apothem (center to edge) of the triangle from which the hex was derived to get the portion of b that is the framework stick's long leg.

To calculate the short leg, I will solve a similar right triangle corresponding to the hex's short side. In that case I would subtract from its side b the distance from the center to the short edge, which is the starting triangle's circumradius (center to corner) minus the height of the truncated corner.



$r \approx 4.102755 s_{hl}$ (calculated earlier) 4.10275452406253916500995933789958

$a = 0.5 s_{hl}$

$$b = \frac{a}{\tan 7°} \text{ or } \sqrt{r^2 - a^2} \ ▶$$

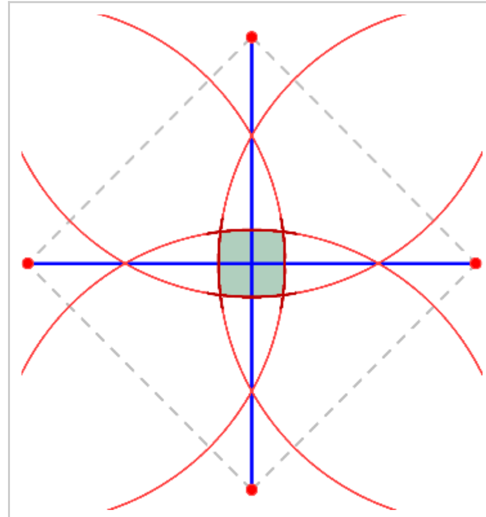
$$b = \frac{0.5 s_{hl}}{\tan 7°} \text{ or } \sqrt{(4.102755 s_{hl})^2 - (0.5 s_{hl})^2}$$

$\approx 4.072173 s_{hl}$ 4.0721732139872970119128306974898

Because the hexagon's long side is equal to the starting triangle's circumradius, and the apothem is half of the circumradius,

Starting Triangle Apothem = Hex "Apothem" (center to long side) = $0.5 s_{hl}$

Framework Long Leg ≈ $4.072173s_{hl} - 0.5s_{hl}$ ≈ $\mathbf{3.572173s_{hl}}$ 3.5721732139872970119128306974898



Because the circular square pattern's sides/arcs match those of the hex's long sides/arcs, its framework legs are the same length as the hex's long framework legs.

The dashed gray lines show how the square's Guide Framework (blue) forms the corners of a Guide Square. The corners define the circle centers for the arcs that form the pattern shape. **The distance between the center and each corner of the Guide Square is Framework Long Leg**. 2 × *Framework Long Leg* is the Guide Square's diagonal.

So, **the square pattern's Guide Framework is composed of two perpendicular lines of length 2 × *Framework Long Leg***. The Guide Square itself does not need to be drawn.

To calculate the Framework Short Leg (used only for the hex pattern), I will start with the short side b, which has the same ratio to the short side of the hexagon as the long leg has to the long side.

Short Side b ≈ $4.072173s_{hs}$

I want everything to be expressed in the same terms, so I will use the ratio of the hex's short side to the long to express the short side b in terms of the hex's long side:

s_{hs} ≈ $0.366025s_{hl}$, so 0.36602540378443864676372317075287

Short Side b ≈ $(4.072173)(0.366025)s_{hl}$ ≈ $1.490519s_{hl}$ 1.4905188449298756710337662249855

Now I need to calculate the hex's "circumradius" (center to short side) so I can subtract it from side b to get the Framework Short Leg (the distance from the hex's center to the circle center).

Starting Triangle Circumradius = s_{hl}, so the hex's "circumradius" (center to short side) = s_{hl} – height of the truncated corners. I know that

Side of Truncated Corner ≈ $0.366025s_{hl}$ 0.36602540378443864676372317075287

So,

Height of Truncated Corner ≈ $\frac{\sqrt{3}}{2}(0.366025s_{hl})$ ≈ $0.316987s_{hl}$ 0.31698729810778067661813841462348

And so

Hex "Circumradius" (center to short side) ≈ $s_{hl} - 0.316987s_{hl}$ ≈ $0.683013s_{hl}$ 0.68301270189221932338186158537652

Thus,

Framework Short Leg ≈ $1.490519s_{hl} - 0.683013s_{hl}$ ≈ $\mathbf{0.807506s_{hl}}$ 0.80750614303765634765190463960898
Full Framework Stick ≈ $3.572173s_{hl} + 0.807506s_{hl}$ ≈ $\mathbf{4.379679s_{hl}}$ 4.3796793570249533595647353370988

Expressing all remaining aspects of the design in common terms

In the end I need to express the framework in terms of the ball circumference, and the arc radii in terms of the respective framework legs, since that is the only way to draw them for a specifically sized beanbag. First, though, I need all aspects of the design, including the ball circumference, expressed in terms of s_{hl} like the Framework Legs and arc radius. With common terms I can express any aspect as a ratio of any other.

I calculated the long arc radius at the beginning of this section. I can calculate the short arc radius by multiplying the long radius by the ratio of the long hex side to the short (the ratios will be the same).

Long Arc Radius ≈ $4.102755s_{hl}$ 4.10275452406253916500995933789 58

Short Arc Radius ≈ $(4.102755s_{hs})(0.366025) \approx 1.501712s_{hl}$ 1.5017123812984233019354152015 27

Calculating the circumference of the ball requires the length of the short edge arc. I can use the arc length formula

Arc Length = $\dfrac{\theta^\circ r \pi}{180}$ (where θ in this case is 14)

to calculate the hex's short arc length, a_s, in terms of the hexagon's long side (s_{hl}):

$$\boldsymbol{a_s} \approx \frac{14^\circ(1.501712\, s_{hl})(\pi)}{180} \approx \mathbf{0.366938}\boldsymbol{s_{hl}}$$ 0.366937556602708082012758029841 45

I also need the height of the circular hex and circular square, which is the height of the straight-edged shapes plus the relevant sagittas, g_L and g_s (the heights of the apexes of the arcs over the edges). The circular hex's height includes both the large sagitta and small sagitta. The circular square's height/width include two large sagittas. The formula for the sagitta is the following (r = arc radius, c = chord, or, in this case, the hex side the arc spans). Note that it is simply the arc radius minus side b from earlier.

$$\text{Sagitta} = r - \frac{0.5c}{\tan 7^\circ} \quad \text{or } r - \sqrt{r^2 - (0.5c)^2}$$

So the large and small sagittas are

$$g_L \approx 4.102755 - \frac{0.5(1)}{0.122785} \approx 0.030581s_{hl}$$ 0.030581310075242153097128640406

$$g_s \approx 1.501712 - \frac{0.5(0.366025)}{0.122785} \approx 0.011194s_{hl}$$ 0.011193536368547630901648976541 5

I calculated the height of the hexagon in the straight-edged section, but in terms of s_t ($0.683013s_t$). I need to divide that by the ratio of s_t to s_{hl} to express it in terms of s_{hl}.

$$\text{Height of Straight-Edged Hexagon} \approx \frac{0.683013}{0.577350}s_{hl} \approx 1.183013s_{hl}$$ 1.183012701892219323381861585376 4

Height of Circular Hex, h_{ch} ≈ $1.183013s_{hl} + 0.030581s_{hl} + 0.011194s_{hl}$ **≈ $1.224788s_{hl}$** 1.224787548336009107380639202323 9

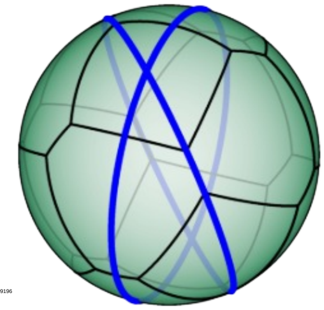
Width/Height of Circular Square, $w_{cs} \approx s_{hl} + 2(0.030581s_{hl}) \approx 1.061163s_{hl}$ 1.061162620150484306194257280812

I calculated the square's diagonal in the straight-edged section, but in terms of s_t ($d \approx \sqrt{2}(0.577350s_t) \approx 0.816497s_t$). The diagonal is the same for the circular square, so I just need express it in terms of s_{hl}.

$$\textbf{Diagonal of Square, } \boldsymbol{d} \approx \frac{0.816497}{0.577350}s_{hl} \approx \mathbf{1.414214}\boldsymbol{s_{hl}}$$ 1.414213562373095048801688724209 7

With those values I can calculate the ball circumference produced by the circular panels.

Expressing the ball circumference in the same terms



Here are the two ways to measure the circumference (previously described in the straight-edged pattern section):

Circumference A = $4h_{ch} + 2w_{cs}$
Circumference B = $4d + 4a_s$

Using the values I calculated previously,

Circumference A ≈ $4(1.224788s_{hl}) + 2(1.061163s_{hl}) \approx 7.021475s_{hl}$ 7.0214754336450050419110713709196
Circumference B ≈ $4(1.414214s_{hl}) + 4(0.366938s_{hl}) \approx 7.124604s_{hl}$ 7.1246044759032125232577870162046

Circumference B is greater than A by only 1.47% this time. I will still calculate the weighted average with A weighted twice since it occurs 6 times on the polyhedron and B occurs 3 times:

Weighted Average Circular Design Circumference ≈ $7.055852s_{hl}$ 7.0558517810644075356933099193479

Last step: Expressing the framework in terms of the ball size

Now that I have everything expressed in a common term, s_{hl}, I can finally express the framework legs in terms of the ball size and I can express the arc radii in terms of the respective framework legs.

To calculate the framework legs in terms of the Circumference, C, I divide the leg length expressions by the circumference expression. For the Diameter, D, I multiply the circumference expressions by π.

Hex and Square Framework Long Leg, $L_L \approx \frac{3.572173}{7.055852}C \approx 0.506271C$ 0.50627101090386259967661756523624
$\approx 1.590497D$ 1.5904972885810528515262872815482

Hex Framework Short Leg, $L_S \approx \frac{0.807506}{7.055852}C \approx 0.114445C$ 0.11444488462820860871769729114217
$\approx 0.359539D$ 0.35953920878891162082922431102874

Full Hex Framework Stick (sum of above) ≈ $0.620716C$ 0.62071589553207120839431485637841
$\approx 1.950036D$ 1.9500364973699644723555115925769

Full Square Framework Stick (Long Leg × 2) ≈ $1.012542C$ 1.0125420218077251993532351304725
$\approx 3.180995D$ 3.1809945771621057030525745630964

To calculate the long arc radius in terms of the Framework Long Leg and the short arc radius in terms of the Framework Short Leg, I will divide each radius expressed in terms of the hex's long edge by the associated Framework Leg expressed in the same terms to get the ratio between them:

Long Radius (for Hex and Square) ≈ $\frac{4.102755}{3.572173}L_L \approx 1.148532L_L$ 1.148531797953605323338360133231

Short Radius (for Hex only) ≈ $\frac{1.501712}{0.807506}L_S \approx 1.859692L_S$ 1.8596915877931520689126865610385

Cutting pattern adjustment

To make a cutting pattern, simply increase the arc radius by the desired seam allowance. The framework remains the same.

Damn that was a lot more math than I anticipated! I'll have to go over that a few times to make sure I did it all correctly, and made it reasonably easy to follow. It helps to have the whole design drawn in SketchUp so I can verify my calculations with actual measurements. I drew the SketchUp model using a unit triangle to start with, which is easy. These calculations are what allow the design to be drawn according to a specified final ball size, rather than a specified starting triangle. That is a lot more complicated.

I'm getting back to this chapter on 8/24/2020, after a week or two away from it.

Writing this chapter, and especially the Mathematics section, burned me out for about a couple weeks, and I felt intimidated by it and didn't want to face it again to check it over. My depression also flared up, which may have been part of the cause, and I have felt burned out in general. I completed and double-checked the dodecahedron chapter first, which is somewhat easier. For a while my brain simply bounced off the math, even in the dodecahedron chapter, and would not engage with it. I was gradually able to touch up the illustrations and text, though.

I finished the dodecahedron chapter yesterday, and today I was finally able to face this chapter. I reworked and made many improvements to the Mathematics section, and corrected my previously incorrect circular version circumference estimation. I also improved some of the illustrations throughout the chapter and tweaked the text. My depression is still bad, but at least my brain can face the math again.

Back to Chapter Index

How I Developed This Design

Back to Chapter Index

Comparative geometric analysis of 14-panel designs

Photos from (respectively) http://www.renegadejuggling.com/14-Panel-Suede-Leather-Ball-p115.html, http://www.higginsbrothers.com/html/juggling_balls.html, and http://www.flyingclipper.com/home/fly/page_256_94/dirtbag_14_footbag.html

I created this design in April, 2013, eight months after I wrote the first edition guide. It was my first design with more than twelve panels and my first with more than one panel shape. I often saw beanbags made according to the design shown above and I liked it, so I originally included it in the *Other Designs and Variations* chapter [now titled “Chapter 4 – Other Juggling Bag and Footbag Designs”]. I learned around that time (from the Dirtbag product page) that Flying Clipper holds a patent on this design (I don’t know if it is mathematically identical to the others or to mine) and that the goal for the design was to “produce the visual of a soccer ball”.

All I could determine about the design was that it appeared to be a cuboctahedron (shown below, third from the right) with the triangles’ corners truncated slightly, turning them into semiregular hexagons. I had always assumed that the amount of truncation was arbitrary and only intended to decrease the prominence of the vertices of the beanbag. I had no interest in designing it, both because of its apparent arbitrary nature and because I preferred the idea of curved edges to truncated corners for approximating a sphere.

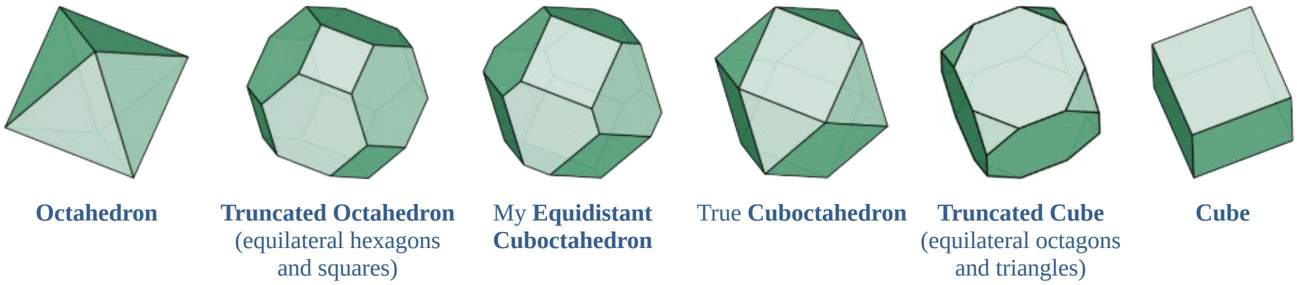
I occasionally tried to figure out the curves necessary to construct a spherical cuboctahedron, but I never succeeded. I always liked the look of these bags, though, and I couldn’t entirely give up the thought of making a cuboctahedron-based bag.

This truncated design, I later learned, is approximately a one-third transition point between a cuboctahedron and a truncated octahedron, the latter of which is shown below, second from the left, and is composed of squares and regular hexagons. Flying Clipper refers to the truncated triangles as truncated hexagons, so they may have come from the opposite direction from me and used the truncated octahedron as the basis for their design.

The illustrations below show how all these polyhedra are related, including the cube and octahedron, which are duals of each other. As the octahedron’s vertices are progressively truncated, it becomes the truncated octahedron (equilateral hexagons and squares), my polyhedron, and finally the cuboctahedron at the point when the octahedron is fully truncated, or rectified, meaning the truncation has reached the centers of its edges, reducing the hexagons’ short edges to their vanishing point, forming triangles. If the square faces are then further truncated, enlarging them and shrinking the triangles, the squares become octagons, forming the truncated cube. Then, at the point the triangular faces disappear and the octagonal faces become squares again, the shape becomes the cube.

If the cube is fully truncated, through the truncated cube stage, the result is the cuboctahedron. If the cuboctahedron's triangular faces are then further truncated, shrinking the square faces and converting the triangular faces into hexagons, the reverse progression of polyhedra is formed until the square faces shrink to nothing and become the vertices of the octahedron, and the hexagons become triangles again. That is the progression direction from which I formed my polyhedron. Flying Clipper may have approached it from the other direction. The interrelations of polyhedra is part of why I find the study of

Truncation progression between the Octahedron and the Cube, which are duals of each other



Around the time I was theorizing about the truncated design, I found a true cuboctahedral beanbag (shown below), which is the only one I have ever seen and is made by Juggling Thingies. I don't like the look of the true cuboctahedron as much as the modified one, though.

True cuboctahedral bags from http://jugglingthingies.homestead.com/files/index.htm

Except for its visual beauty, the true cuboctahedron (without curved edges) never seemed to me to be as good a design as the dodecahedron for three reasons. First, the angles that form each vertex have a sum of 300° while the dodecahedron's have a sum of 324°, so the cuboctahedron has sharper, more prominent vertices. Second, the edges of the cuboctahedron are longer than those of the dodecahedron by about 40% for the same circumference. (The Conclusion subsection discusses how the Equidistant design improves these geometric inferiorities.)

These two properties are due to, and can be summarized by, the fact that triangles and squares with three and four sides are farther from being round than pentagons with five and so, except in much larger numbers, cannot produce as good a sphere. Another way to look at it is that a dodecahedron has 30 edges and 20 vertices while a cuboctahedron has only 24 edges and 12 vertices, so the latter must have longer edges and sharper vertices to complete the solid.

As a result, I think the cuboctahedral beanbag will feel less smoothly spherical than the dodecahedral even though it has two more faces[3]. That brings me to the third reason: The cuboctahedron has two more panels to cut out and sew.

3 October, 2023: I finally decided to add the true cuboctahedron (with curved panel edges) to my guide. As part of my experimentation I made a cuboctahedral ball with straight edges, and so I can now confirm empirically that it is indeed more angular and has sharper, more prominent vertices than either the straight-edged dodecahedron or the straight-edged Equidistant Cuboctahedron. The dodecahedron is the most round of the three designs.

Designing the Equidistant Cuboctahedron

Back to Chapter Index

In April, 2013, I became interested in figuring out an optimal form of the cuboctahedron concept for a juggling beanbag. I wanted to discover if there is a non-arbitrary choice for the amount of truncation applied to the triangles, and a way to get the aesthetic attributes of the cuboctahedron without losing the geometric benefits of the dodecahedron. The process of designing this polyhedron was by far the most mathematically intensive of all my designs up to this time and of all projects of any kind I've done since college. Some of my failed attempts involved formulas of ridiculous complexity with inverse trig functions nested within trig functions nested within radicals within sums of fractions within radicals all part of huge fractions.

I began modifying the cuboctahedron simplistically by merely truncating the triangles in 2D based on fractions of the triangle's sides. Though this was an arbitrary method, I hoped it would lead to some kind of discovery about the design (and it did). I began with ⅙ of the side (giving it a vague connection to a hexagon), making the short side of the resulting hexagon ¼ the length of the long. I compared this visually to the beanbag photos and it looked roughly the same.

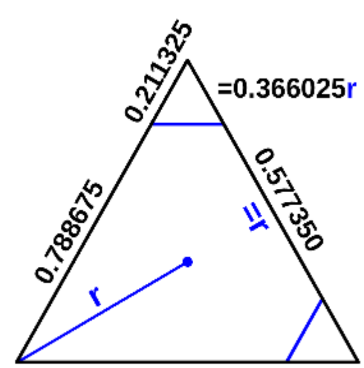
I then began constructing cuboctahedrons in SketchUp, and I tried to construct one that had the same truncation. I also observed the effect this had on the surface area of the faces. I began hoping for a design that had the same surface area for both face shapes as calculated by SketchUp (the square is much larger than the triangle). This still seemed somewhat arbitrary, but it at least made a bit more sense as an approach to designing a juggling bag with a uniform surface. I did not know enough math to construct these designs directly and had to do it partially by trial-and-error. I did not achieve precision.

At some point it occurred to me that by constructing cuboctahedrons with truncated triangles, I was bringing the triangular/hexagonal faces closer together in relation to the squares. I measured the true cuboctahedron in SketchUp and found that the distance between the triangles is 15.47% greater than between the squares. This gave me the idea that I stuck with, which was to construct a cuboctahedron in which both face types were the same distance from each other, giving the resulting ball a more constant diameter.

I had to do hours of thinking and trying to make SketchUp do what I wanted, as well as a lot of calculations of angles and distances of various aspects of the necessary polyhedron just to get the Equidistant Cuboctahedron drawn, but I had made use of SketchUp measurements of some of the dimensions along the way. Crude construction and measurements of the faces' dimensions weren't good enough for me; I wanted a mathematical definition.

I spent another couple of days trying to find a formula that would define the hexagon shape. The eventual result was a ratio between the circumradius of the standard cuboctahedron triangle and the circumradius of my larger triangle (before the truncation that converts it to a hex). This ratio can also be applied to the sides of the triangle so that it tells how much of each side to cut off.

$$\frac{1}{3-\sqrt{3}} = 0.788675 \longrightarrow$$



Though I cannot yet prove it mathematically, through measurement of multiple iterations of this panel shape, I found that **the circumradius of the full triangle is equal** (to as many decimal places as I bothered to test it) **to the length of the long side of the derived hexagon** and (obviously) to the square's width. There is probably a fascinating geometrical reason why this occurs at the point at which the polyhedron attains equal widths between all opposing faces, but I don't yet see it and I haven't felt up to the task of discovering it. However, it makes me feel that I have designed an important polyhedron and so I like it all the more.

8/5/2020 – Proof of the above assertion: For the Second Edition I decided to see if I could prove that the triangle's circumradius equals the derived hex's long side. I succeeded, and I was surprised and pleased by how the proof worked out. For the proof I will assume a unit equilateral triangle to eliminate the variable for the triangle's side.

A unit equilateral triangle's circumradius is

$$R = \frac{1}{\sqrt{3}}$$

I will define s_s to be the derived hexagon's short side, and s_L to be its long side. The short side of the hexagon is

$$s_s = 1-\frac{1}{3-\sqrt{3}}$$

The hexagon's long side is the triangle's side, 1, minus two short sides:

$$s_L = 1 - 2(s_s) \;=\; 1-2\left(1-\frac{1}{3-\sqrt{3}}\right)$$

I want to determine if $s_L = R$. I will begin by multiplying the above expression out. Then I will create common denominators for each term and add the terms together. Then I will factor out a common factor from the numerator and denominator, and finally cancel the factors because they match.

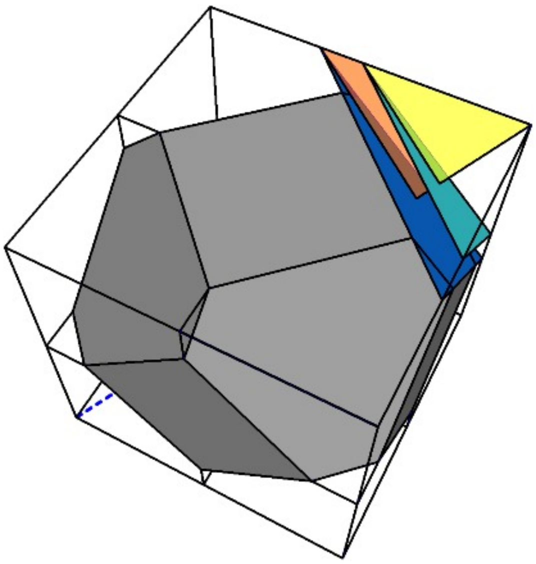
$$s_L = 1-2+\frac{2}{3-\sqrt{3}} \;\blacktriangleright\; \frac{3-\sqrt{3}}{3-\sqrt{3}}-\frac{2(3-\sqrt{3})}{3-\sqrt{3}}+\frac{2}{3-\sqrt{3}} \;=\; \frac{\sqrt{3}-1}{3-\sqrt{3}} \;\blacktriangleright\; \left(\frac{\sqrt{3}-1}{\sqrt{3}-1}\right)\left(\frac{1}{\sqrt{3}}\right) \;\blacktriangleright\; \left(\frac{\cancel{\sqrt{3}-1}}{\cancel{\sqrt{3}-1}}\right)\left(\frac{1}{\sqrt{3}}\right) \;\blacktriangleright\; \mathbf{\frac{1}{\sqrt{3}}}$$

So with some mathematical finagling I can convert the expression of the hexagon's long side to match the expression of the triangle's circumradius, proving that they are equal.

I later attempted to find out if my design is the same as the Renegade brand 14-panel bags, which I really like the look of (the other brands probably use an identical design), by measuring a high-resolution photo of them in Photoshop. My measurements tell me that the long side of the hexagon is a little under 2.75 × the length of the short side. Dividing 0.5774 by 0.2113 yields 2.73, so I would say the two designs are probably the same.

Explanation of the mathematical design process

Back to Chapter Index

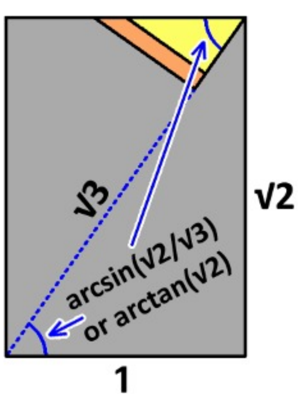


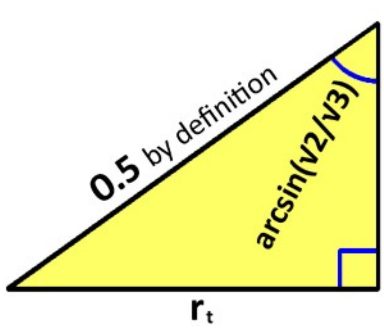
The method I came up with to find the definition of the hexagon shape is illustrated on the left. I use a cube to construct my cuboctahedra (an octahedron can also be used as a starting point, but a cube is easier to draw). I formed a hypothetical right triangle (illustrated in yellow) by drawing an imaginary line from the corner of the cube to the center of the triangular face of the true cuboctahedron, shown in teal blue (this line, if continued, would extend through the center of the cube to the opposite corner as shown by the blue dashed line on the lower left), drawing another line from the center of the triangular face to its corner which intersects the center of the cube edge (this is the triangular face's circumradius), and then using the cube edge as the third side. I was able to find the angles of this triangle and the length of the hypotenuse (which is half the length of the cube edge) all in the form of ratios of a unit cube.

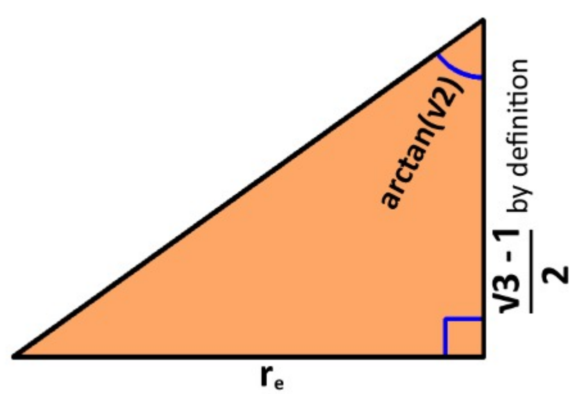
I then extended this triangle inward (orange) to where the proposed new triangular face (blue) would be which is defined such that its distance from the opposite triangular face is the same as the cube's width (because that is the distance between the cuboctahedron's square faces). I was able to calculate this triangle's dimensions because the angles are the same as the first and the side that is perpendicular to the plane of the triangular face has a length, by definition, equal to the diagonal of the cube minus the width of the cube, divided by two.

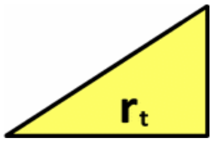
Following is some of my work and the derivation of the ratio formula. I included this both for the benefit of anyone who wants to understand it or check my work, and so that when I forget what I did and question whether I did it right, I can recheck my work. (My brain tends to be very faulty and I can't fully trust it.)

The gray rectangle below is the diagonal cross section of a unit cube (the short sides are the edges of the cube and the long sides are the diagonals of the cube's faces). Its dimensions can be found using the Pythagorean theorem. **The yellow and orange triangles are the ones from the diagram above.** r_t and r_e are the radii of the triangular faces for the true cuboctahedron and the equidistant version, respectively. I used two different expressions of the same angle to make the formulas simplify more easily (a trig function and its inverse cancel each other out).

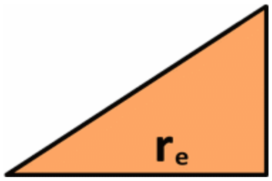








$$\frac{r_t}{0.5} = \sin\left(\arcsin\left(\frac{\sqrt{2}}{\sqrt{3}}\right)\right) = \frac{\sqrt{2}}{\sqrt{3}} \qquad \textbf{solve for } r_t \longrightarrow r_t = \frac{\sqrt{2}}{2\sqrt{3}}$$

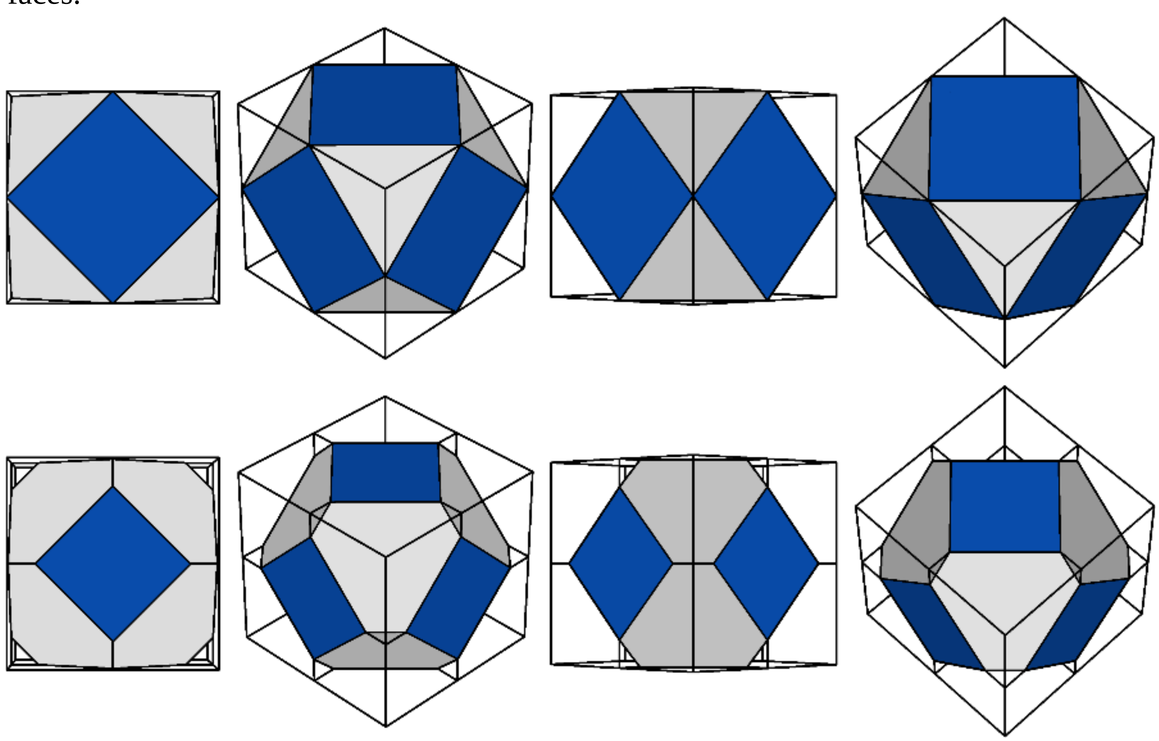


$$\frac{r_e}{\frac{\sqrt{3}-1}{2}} = \tan\left(\arctan\left(\frac{\sqrt{2}}{1}\right)\right) = \sqrt{2} \qquad \textbf{solve for } r_e \longrightarrow r_e = \frac{\sqrt{2}(\sqrt{3}-1)}{2}$$

$$\frac{r_t}{r_e} = \frac{\cancel{2}\cancel{\sqrt{2}}}{\cancel{2}\sqrt{3}(\cancel{\sqrt{2}}(\sqrt{3}-1))} = \frac{1}{\sqrt{3}(\sqrt{3}-1)} = \frac{1}{3-\sqrt{3}} = 0.788675$$

Because the truncation of the larger triangular face is due to its intersection with the adjacent face, which occurs at a point in line with the center of the cube edge, which is where the corner of the smaller triangular face intersects, the larger triangular (now hexagonal) face's new height will be that of the smaller face. Because the reduction in height will cause the same ratio of reduction to the other measurements (including the circumradius), I can infer that the ratio of the two triangles, yellow and orange, (which are based on the radii of the two triangular faces) gives me the ratio of this truncation. I sometimes doubt the validity of this reasoning, but my original, crudely but correctly constructed version yielded the same ratio (by using SketchUp measurements), so that gives me some confirmation.

Below is a comparison of the two cuboctahedron versions. The top row is the true cuboctahedron and the bottom row is my modified version. Each one is within the original cube framework from which I formed it. The equal width between all opposing faces can be seen in the third column. In those two images the top and bottom edges are square faces and the diagonal edges are triangular/hexagonal faces. Observe that in the true cuboctahedron the diagonal faces are farther from each other than the top and bottom faces.



A true cuboctahedron (top row) has 24 edges and 12 vertices, and my polyhedron (bottom row) has 24 long edges, 12 short edges, and 24 vertices. Basically, each vertex of the cuboctahedron has been turned into two vertices and a new edge.

To construct the Equidistant Cuboctahedron from a cube, the corners of the cuboctahedron's square faces must be offset from their normal points (the centers of each cube edge) by 0.133975 of the cube's edge length, and so must the corners of the triangles before truncation. That value is the hypotenuse of the orange triangle minus the hypotenuse of the yellow triangle which simplifies to

$$\frac{3-\sqrt{3}}{2}-0.5 = \frac{2-\sqrt{3}}{2} = 0.133975$$

Conclusion – The geometric benefits of the Equidistant design

The truncation of the triangles helps to solve the first two problems I mentioned concerning the cuboctahedron as a beanbag design leaving only the third (two more panels to sew) as the price to pay for the beauty of the 14-panel design. The new vertices are 330° as opposed to 300° (and 324° for the dodecahedron), and the edges are only 18% longer than the dodecahedron's as opposed to 40%. The triangles are now hexagons and so have a roundness more nearly that of the pentagon, and the solid now has more edges and vertices than the dodecahedron.

There is even the benefit of the two face shapes being more nearly the same size than before, which was one of my early design goals and will improve the uniform feel of the bag. The resulting beanbag is not only visually gorgeous, but also luxuriously spherical and uniform. I love it.

Designing the spherical version with curved panel edges

Back to Chapter Index

For the second edition of this guide, late in 2020, I designed curves for the edges of my 12 and 14-panel designs to produce better spheres. I worked on the two designs concurrently because they are similar and I assumed they would require similar curves.

During the writing of the first edition I had no idea of how to design curves for polyhedral faces to produce a sphere rather than a polyhedron. That motivated the conversion of the cuboctahedron to the equidistant version, as I believed that would create a closer approximation of a sphere.

Then, sometime later in 2013, I read an article on spherical geometry[4] which explained that for polygons to form a sphere, the sum of the angles meeting at each vertex must be 360°. I later learned the mathematics to calculate the radius of the arc that will form a specified angle to a polygon edge. (For a full explanation of this, read the "Curved-Edge Faces" section of Chapter 5). This spherical geometry article and the edge arc radius formula I created enabled me to design the curves for my panels. The curve I designed for the 14-panel bag turns a very angular bag (when made with my stiff design testing fabric) into a beautifully spherical one.

The corners of the Equidistant Cuboctahedron are composed of a 90° square corner and two 120° hexagon corners for a total of 330°. So I would need to add 30° to increase the sum to 360°. I began my experimentation by making a straight-edged version for comparison, as I had done with the dodecahedron, using a new fabric I bought for design testing. This fabric is thin, stiff, tightly woven, and non-stretch, and better manifests the effects of the panel shapes than softer fabrics. The resulting bag was surprisingly angular. Much more so than the straight-edged dodecahedron. I then made a bag using curves designed to add 10° to each corner. That bag was more spherical, but still significantly angular.

---

4 http://euler.slu.edu/escher/index.php/Spherical_Geometry

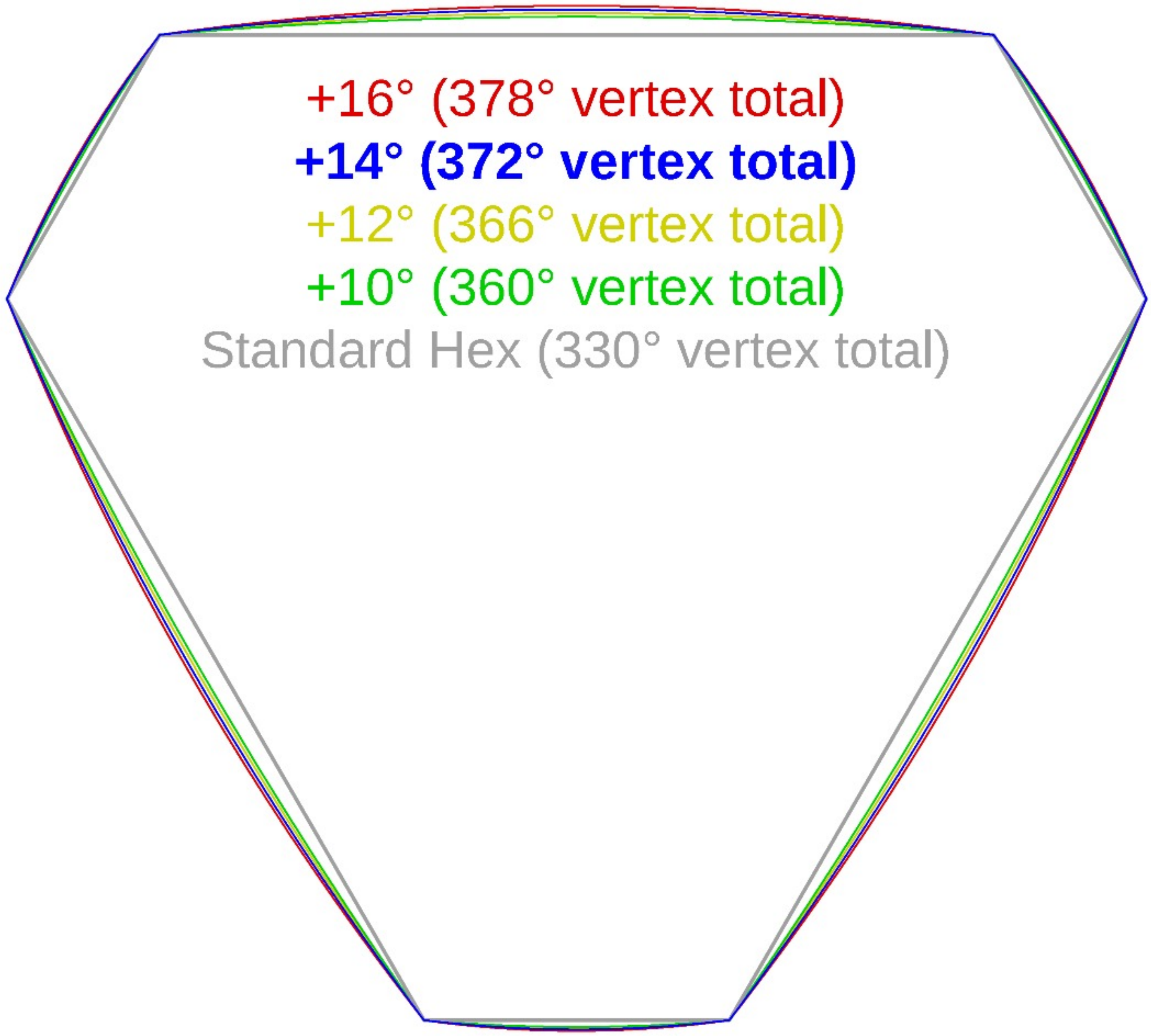
My next experiment was to add 16° to each corner to match the angle increase I had applied in my current dodecahedron experiment. (For most of my designs I found that a steeper arc than that which produces the mathematically correct angle works better due to the need for a higher apex.) 16° was slightly too much (and was too much for the dodecahedron). The edges bulged slightly too much and the vertices had a slight tendency to pull inward (it was so slight that I don't think anyone without my obsessive attention to detail would notice). I then made a bag with +14° and that was perfect. It amazed me how much difference a tiny panel bulge, or change in bulge, made in the finished bag.

To determine which design made the best sphere, I spent many days occasionally comparing the different bags. I examined them visually at different angles, I rotated them in my hands, feeling the overall shape for any angularity or inward puckering, I ran my fingers along the edges and vertices to feel the contours, and I prodded the vertices to see if they had a tendency to sink inward. (Later, during a period of doubt, I even tried rolling the bags back and forth at a fast speed between my hands on my hardwood floor to listen for the patter of edges and faces hitting the wood. This helped a bit, as I could sometimes hear a louder patter in the bag with a shallower curve.) It was a very long and nit-picky process. But I am severely depressed and for that reason unemployed and without other hobbies or activities, and this project is highly enjoyable for me, so I spent a lot of time at it.

I had also decided on an additional 14° for the pentagonal panels of the 12-panel design, but while that resulted in 366° vertices on the 12-panel bag, it results in 372° vertices on the 14-panel bag. The reason, I believe, it needs at least the same curve angle is that its edges are 18% longer than the 12-panel's edges, and so need a greater bulge to match the circular profile of the bag. The greater length will make the same curve produce a greater bulge, so that works out.

Despite that logic, I spent a long time in doubt about whether my curve choices were the best possible. I wasn't even sure if the +14° on the 12-panel was the best choice. Feeling or seeing the difference between an additional 14° and an additional 16° is very difficult, especially because of the slight differences in the bags themselves that I couldn't help but introduce even though I sewed them with great precision. I was also unsure if my jump from +10° to +14°, guessing that I needed a curve closer to +16° than to +10°, was indeed the best curve. So I finally had to make a +12° bag.

The +12° bag was noticeably angular, even though I ironed it impeccably before the bag was finished, and again after to further smooth it out (ironing the bags from the outside significantly reduces their angularity), and after breaking it in for a while. The most telling aspect of the bag's angularity is the short seams, which are overly prominent in the +10° and +12° bag and slightly sunken in on the +16° bag. They feel perfectly smooth and round on the +14° bag. So I am now convinced that the +14° curve is the best choice. Below is the hex panel shape with the four curves I tried.



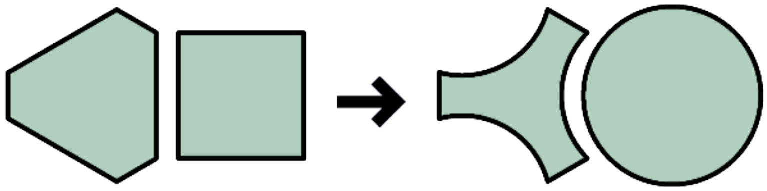
All the curves with which I experimented (I used the same curves for the square pattern)

Back to Chapter Index

14-Panel Polka Dot Variation

Back to Chapter Index

Note that I wrote this section for the first edition, before I had designed curved edges for my 14-panel design. So this discussion does not include a corresponding compensation in the panel shapes to produce a properly spherical beanbag.

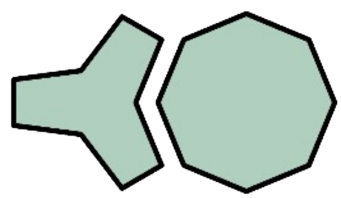


Super Mario Bros. mushroom source: http://mario.wikia.com/wiki/File:Mushroom-SM3DL.png

© & TM LucasFilm Ltd.

By converting the square panels of the 14-panel design into circles and making corresponding circular cuts in the long sides of the hexagons, you can get a Super Mario Bros. mushroom look. This is also the same structure as the body plates of the BB-8 droid from Disney's Star Wars universe.

Be warned that sewing a convex curve to a concave curve, especially such tight curves, is much more difficult than sewing two matching patterns together. You cannot simply lay the panels flat together and sew along the line; you will have to continually adjust the panels as you sew to keep the point of each pattern you are sewing matched up. This is especially difficult when sewing from the outside.

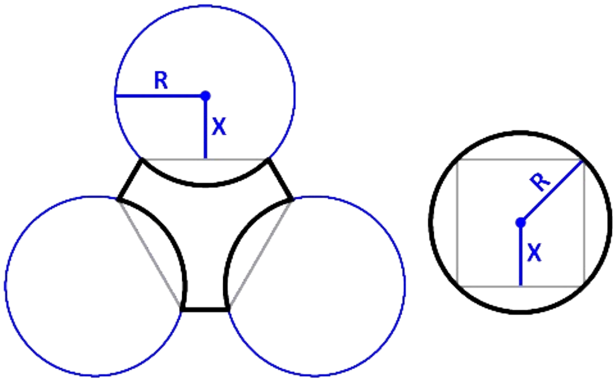


An alternative version of this concept that would be easier to assemble would have an octagon instead of a circle and corresponding 135° angular cuts in the sides of the hexagon as shown on the right.

Below are photos of my proof-of-concept bag followed by the bag that inspired this design (which has smaller polka dots). I'm not as happy with the look of my bag as I thought I would be, but it's still a fun shape. It was quite difficult and frustrating to make and it looked rather lumpy when it was done due to all the puckering of the seams caused by their curvature. I had to spend a lot of time ironing the seams and adjusting the seam allowances to make it look smooth and pretty like my others.

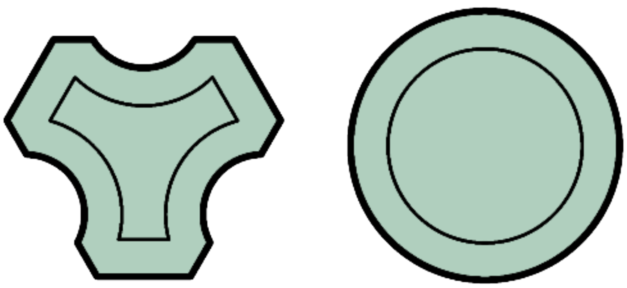
It might not have been so difficult, or so lumpy, if I had used a thinner, more flexible fabric than denim, but take warning that this is not an easy bag to make. It does have a great tactile quality to it, though. The six large, soft, round pads separated by the firmer, narrow, slightly recessed areas feel good in the hand and give it a good grip.

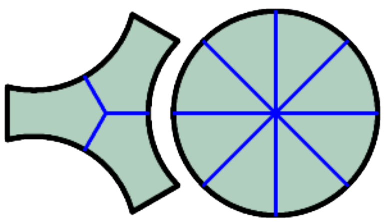
Third photo from http://www.footbagshop.com/reaper-14-net-red-white.html



To make a circumscribing circle as I did, the radius of the curves is equal to half the diagonal of the square. The length of X (from the hexagon's side to the center of the circle) is equal to half the square's side length.

The Super Mario Bros. mushroom appears to have slightly smaller spots than those produced by a circumscribed circle, and the footbag in the photo certainly does, so a smaller radius may be preferable.

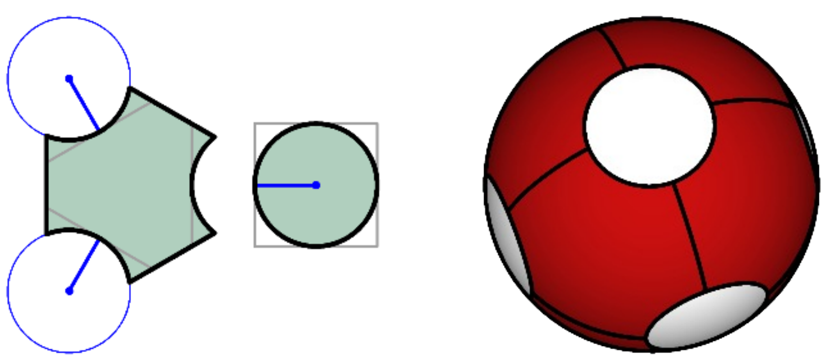

Cutting patterns


Stitching patterns

To draw cutting patterns for this design, increase the radius of the circle by the desired seam allowance and decrease the hex curve radius by the same amount.

You will need four markers on the circle corresponding to the corners of the original square so you can align those points with the corners of the tri-curve panels as you sew. Because of the difficulty of sewing concave and convex curves to each other, I recommend adding a second set of four markers in between the other four, and making corresponding marks in the center of each curve of the tri-curve panels. These markers will aid in progressing your stitching equally along each curve of a seam and preventing the seam from becoming distorted.

After recomposing this section for the Second Edition, I decided to see if I could emulate the design with smaller polka dots by converting the square into an inscribed circle instead of a circumscribed one. The resulting panel shapes and the 3D model of what the bag would look like are below. This is clearly not the way that bag was designed. The circular panels are much too small. So the design in the photo must be somewhere between the inscribed and circumscribed circle.

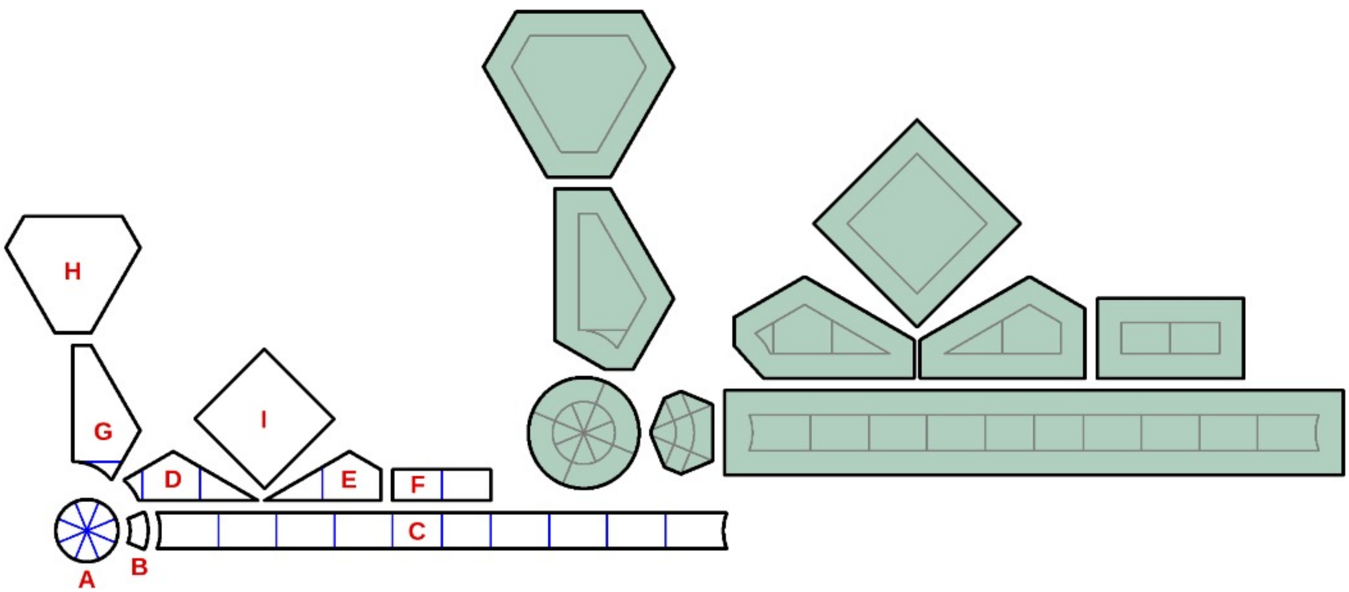

Pattern shapes and ball appearance when using circles inscribed within the squares instead of circumscribing circles

Back to Chapter Index

14-Panel-Derived Poké Ball

Back to Chapter Index

In December, 2016 I designed and made a Poké Ball by modifying the 14-panel design. Below are the patterns I used (stitching patterns on the lower-left, cutting patterns on the upper-right in template green). I had not yet designed curved panels, so I used the angular panel shapes. That in combination with the very stiff denims I used (except for the black stripe and button ring which are corduroy) made the Poké Ball significantly angular in places. I rounded it a bit with my hands for the photos. This would benefit greatly from my new curved-edge panels.



Pattern shapes. Stitching patterns on the left, cutting patterns on the right in template green.

I made the button using a square panel converted into a small circle (A) surrounded by eight tiny segments (B) forming the outer ring of the button. The band around the equator was one long rectangle with circular cutouts in each end that fit against the button's ring (C). The width of the band was equal to the length of the hex's short edge.

I created three different versions of a partial hex panel. Two were just the portion outside the short edge, forming a 30° angle at the corner where the short edge would have been. A mirror image pair of these lay opposite each other against the equatorial band in the same position as the hex of the 14-panel bag (D and E). Basically, the equatorial band replaced the middle of the hex panel. The second photo above shows two of these. One of them has a circular cutout where the partial long edge was (D), which allowed it to fit against the button ring. There were four of each of these panels on the Poké Ball, two left-handed panels and two right-handed.

The third modified hex was a half-hex with the half long edge converted into a circular cutout to fit against the button ring (G). There were two lefts and two rights of this panel which fit side by side above and below the button. These can be seen in the first and fourth photos. The reason I divided the panel in half is that the circular cutout would have been too tight a concave curve to assemble. The seam allowance would need to stretch too much to be able to fold down.

I also needed a square panel with the width of the equatorial band removed from it, forming two rectangles that would lay above and below the band (F). This pair was located opposite the button and can be seen in the third photo. The remaining panels were two hexes and four squares (H and I). There were 30 panels in all. I made the ball about the size of a baseball (I think I used the 3″ pattern sizes) and filled it with plastic pellets supplemented by steel BBs to give it a solid weight.

My geeky friend Chad liked it and wanted one. I lacked the motivation to make another, though.

Back to Chapter Index