The Complete Homemade Juggling Beanbag Guide

24-Panel Isovertex Deltoidal Icositetrahedron Chapter

Small file size version (150dpi images)

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This is part of a multi-document guide. Hyperlinks with the icon[1] open other guide documents[2], if they are saved to the same folder (**CTRL+Click** opens them in a new tab).

Visit my website to download those, and check back occasionally for revisions and corrections:

www.joshuaclifton.com/juggle

Compare the Last Edited date above on the right with the one on the web page to see if I have submitted changes.

Please contact me with your thoughts! Feedback on this project would be helpful and encouraging. You may also request custom patterns or other help with your project.

If this guide is useful to you, please **consider donating at my website** linked on the left. I am not monetizing the guide, and I am in need of income.

My website also provides blank **color arrangement diagrams** for experimenting with new arrangements in an image editor.

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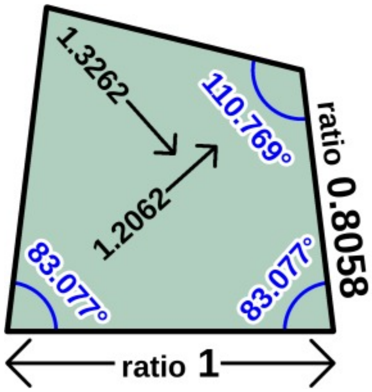
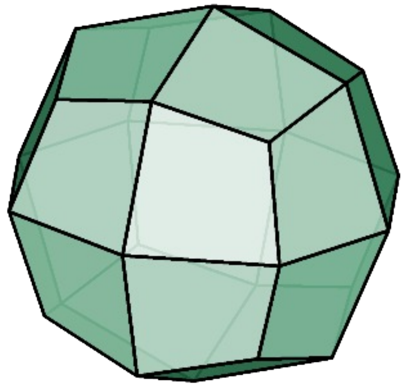
1 Icon from https://freesvg.org/vector-illustration-of-external-link-icon

2 **If the linked PDF does not open at the specified location**, keep it open, switch to the previous PDF's tab, and click the link again. **Cross-document links may not work in mobile PDF readers.** In that case you must open the document yourself and find the linked topic.

24-PANEL ISOVERTEX DELTOIDAL ICOSITETRAHEDRON INSTRUCTIONS





"Diamond Ring with Dual Caps" arrangement

"Patchwork Ball" arrangement (6 colors)

My original denim bag ("Diamond Ring")

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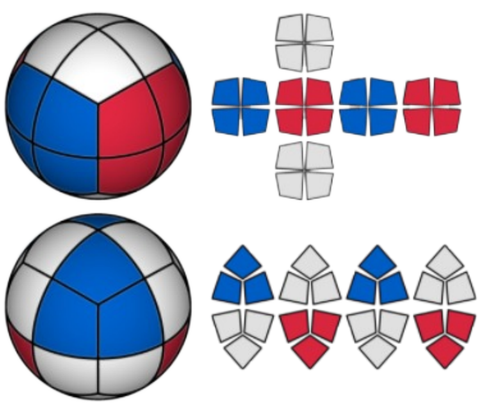
Design Notes



I was inspired to design this bag by a footbag design I found (the photo below[3] and the one in the Color Arrangements section are the specific photos that inspired me). I have found many footbag panel structures, but this one stood out to me. It has a comparatively reasonable number of panels, it simply looks attractive to me, and I discovered that it is **effectively a combination of the spherical cube and octahedron structures** as shown on the left and so can be made to look like either one of those but with more uniformity and roundness because of the greater number of panels. It's like a sleeper beanbag. Somebody will look at it and think it's just a standard 8-panel structure. Mundane. But on closer inspection it is suddenly much more interesting and impressive.

As can be seen on the right, this shape can also be conceptualized as a spherical cube with each face divided in half, and then half again to form four kite-shaped quarters from each of the cube's original faces.

I designed my own kite for the panel shape that produces a better sphere than the normal one by forming polyhedral vertices that all have equal sums of face angles forming them (see the "How I Developed This Design" section for more information) for which I coined the term "isovertex". It does not form a closed polyhedron, though. My polyhedron illustrations are my original modified version, which is still

3 Footbag photo from http://www.expo-star.com/lview.asp?mainid=12&Subid=0&pid=52

an improvement over the normal polyhedron. I also provide patterns for the original kite shape, and for the normal kite shape for the true deltoidal icositetrahedron.

Supplies

- **For the templates**
  - Cardboard or Template Plastic, X-Acto Knife or Scissors, Glue Stick or Adhesive Tape (to attach the pattern to the template material). **For drawing the pattern by hand:** Paper, Compass or Protractor, metric Ruler, fine-point Pencil.
- **For the beanbag**
  - Fabric, Needle and durable Thread, Scissors, Fabric Marker or soft Pencil, beanbag Filler, Funnel.
- **For your information**
  - Unless you are experienced with this sort of thing, I recommend that you browse through the **General Information and Techniques** chapter (in the ***01 – Homemade Juggling Beanbag Guide – Index & Supplementary Chapters*** document) before starting. You may find some tips there that will improve your experience and your beanbags.

Printing and Drawing the Pattern

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Later in this chapter I provide ready-to-print patterns. (When printing them, be sure to tell the Print Dialog to print only the page(s) you want so you don't print the entire document.) After those are sizing formulas, pre-calculated pattern measurements, and instructions for drawing the pattern yourself. Click the hyperlinks or look to the Chapter Index to locate those sections. At the end of the chapter there are ready-to-print patterns for the other two kite shapes (the one for the true deltoidal icositetrahedron and my original design).

Color Arrangements

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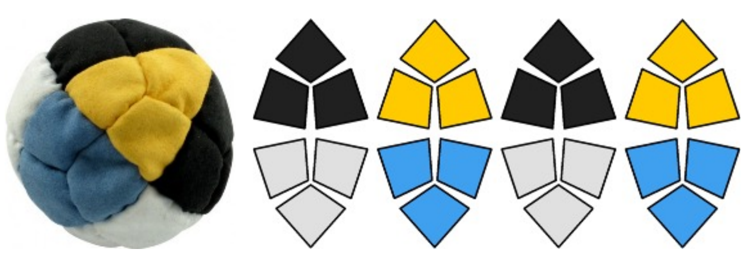
Following are some color arrangement ideas. The first five sets of arrangements (2 – 6 colors) are unique to this panel structure, but after those I have provided two types of **cube arrangements** so you can see how they would be laid out with my octahedron assembly method. Remember that this panel structure supports not only all the color arrangements of the cube, but also of the octahedron and 4-panel orange peel ball. (The latter two are easy to translate to this design because my assembly method uses a panel layout pattern that matches their layouts.) On the right is an example[4] of an octahedron arrangement and a diagram showing how the panels would be laid out for assembly.

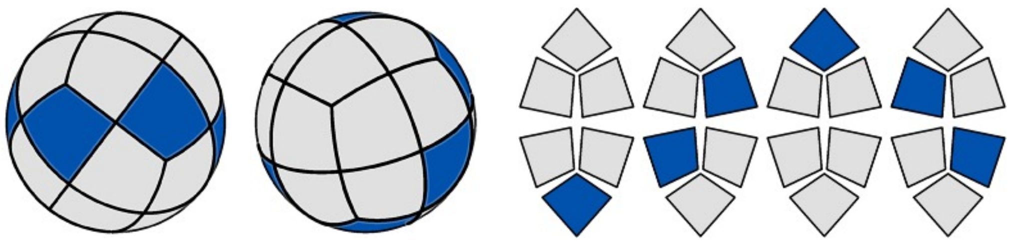


4 Footbag photo from http://www.jugglingstore.com/pyramid-footbag-765.html.

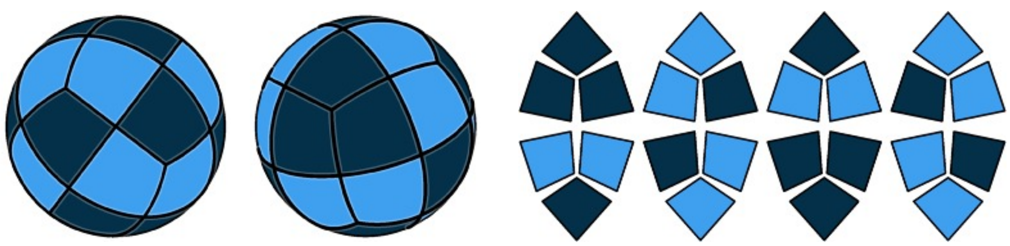
To help me figure out the arrangements and create my diagrams, I stuck colored thumbtacks into an all-white 24-panel beanbag I made using my design-testing fabric. I recommend this as a way to design new arrangements or to use as a reference to aid you in correctly assembling the bags.

I also provide printable blank color arrangement diagrams for the ball views and the assembly layout after the printable patterns. You can use those to experiment with color arrangements without having to make a beanbag. Look at the chapter index to locate them.

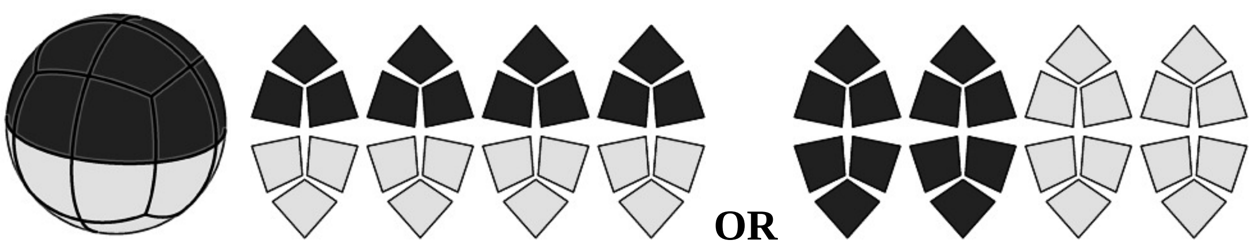
2 colors



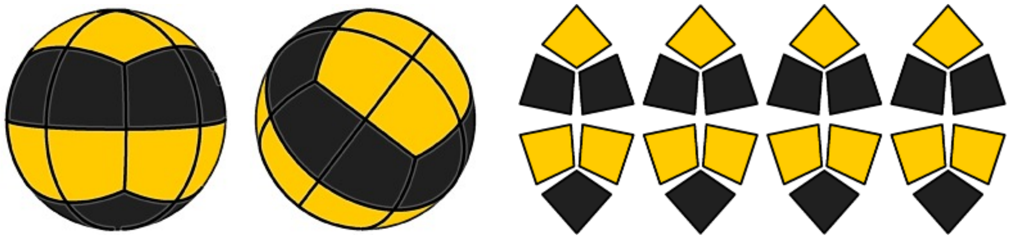
#1: Diamond Ring. Color A on a ring of 6 panels that join at their lateral corners, and color B on the remaining 18 panels.



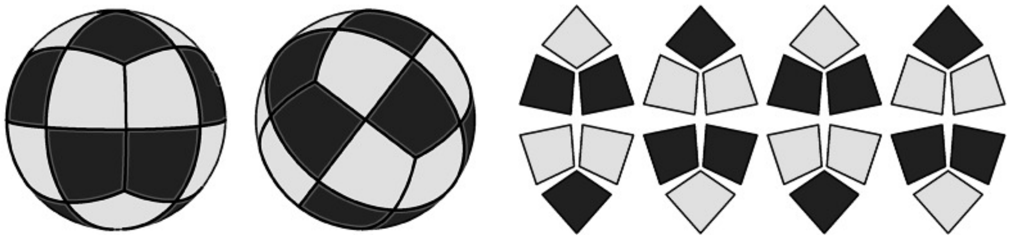
#2: Diamond Ring with Caps. Color A on two opposite patches of three panels and on the six panels touching the corners of each patch (those six form the same ring as above). Color B on the remaining 12 panels. My old denim beanbag shown at the beginning of this chapter used this arrangement.



#3: Hemispheres. Each hemisphere a different color.

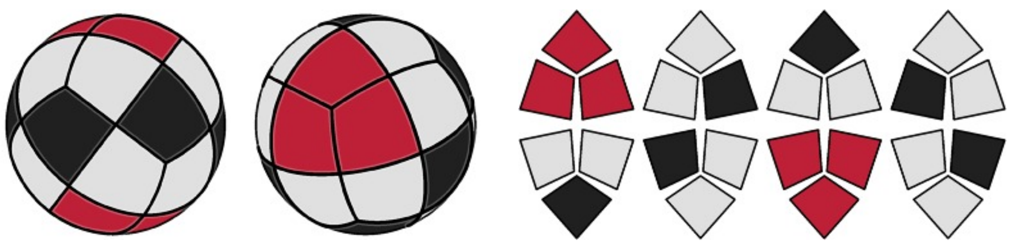


#4: Alternating Stripes. Color A on a patch of 4 panels, color B on the ring of 8 panels around that patch, color A on the adjacent ring of 8 panels, and color B on the final patch of 4 panels.

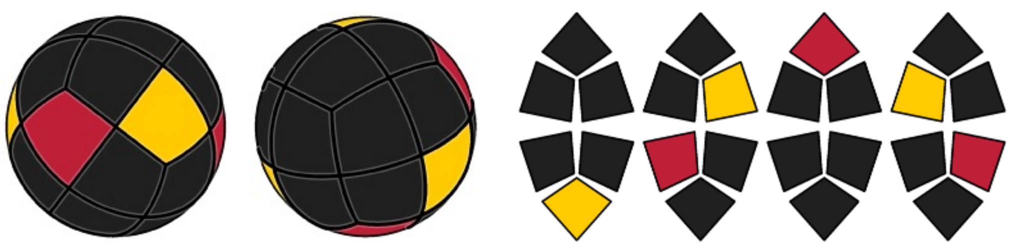


#5: Checker Ball. This arrangement is best conceptualized by laying the panels out in the 4-panel orange peel ball layout as I have. The two colors then form alternating horizontal stripes down each patch that also alternate from patch to patch. I think I originally found this arrangement on a manufacturer's website.

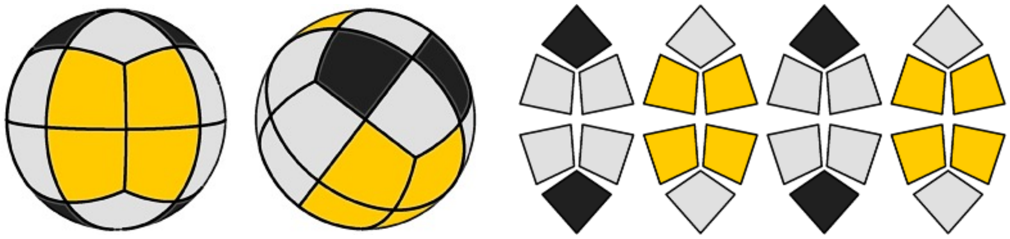
3 colors



#6: Diamond Ring with Caps (3-color variation). Same concept as the 2-color Diamond Ring with Caps arrangement. Color A on opposite triangular patches of three panels, color B on a ring of 6 panels between them, and color C on the remaining 12 panels. Each pair of color C panels lies adjacent to a side of the triangular caps.

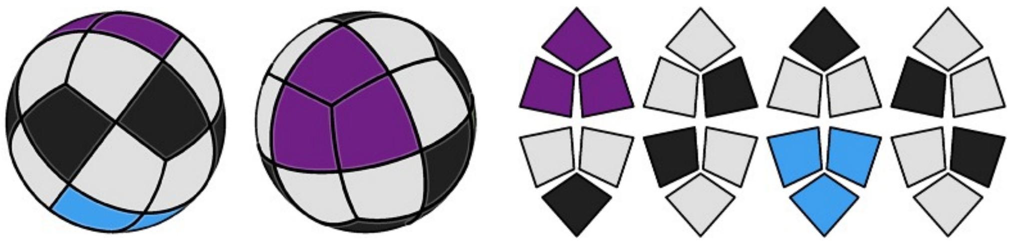


#7: Bi-Color Diamond Ring. Same concept as the 2-color Diamond Ring arrangement, but the ring has two alternating colors. All kites of the same color are oriented the same way.

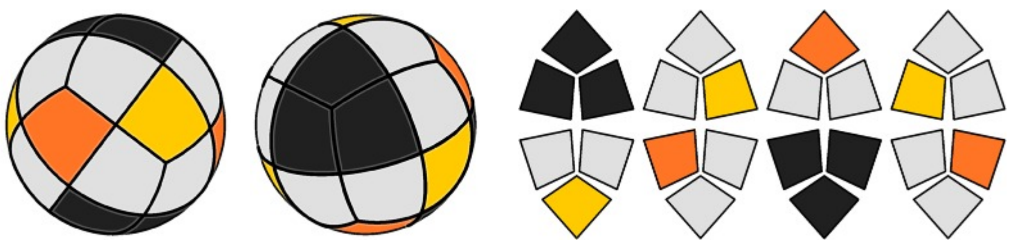


#8: This is an arrangement I came across on a footbag store's website[5]. I don't care for it much, but others evidently like it. I'm not going to try to describe it verbally, but the ball and assembly layout illustrations should give you the idea.

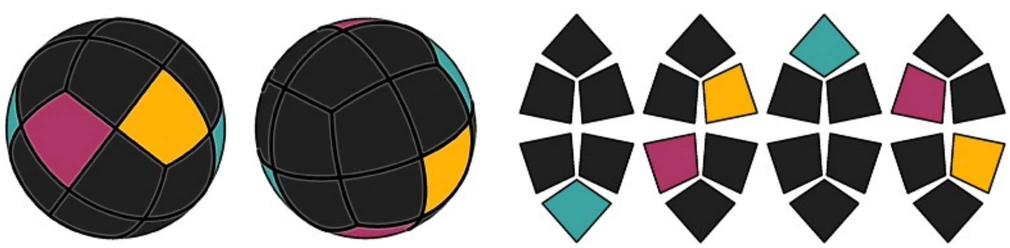
4 colors



#9: Diamond Ring with Dual Caps. Same concept as the 2-color Diamond Ring with Caps arrangement except that the two opposite patches of color A are now two unique colors. My new corduroy beanbag pictured at the beginning of the chapter uses this arrangement.

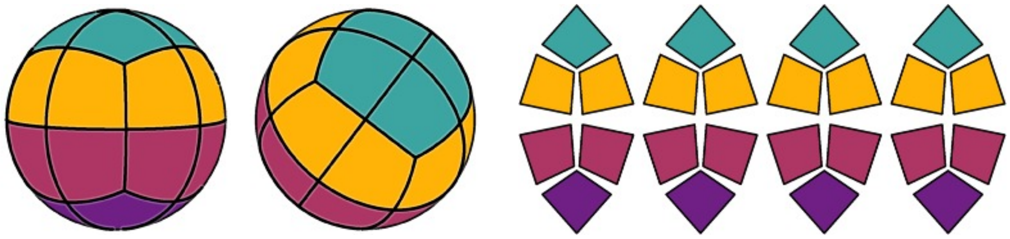


#10: Bi-Color Diamond Ring with Caps. A combination of the Diamond Ring with Caps and Bi-Color Diamond ring arrangements. Each triangular cap has three kites of the same color connected to its corners.



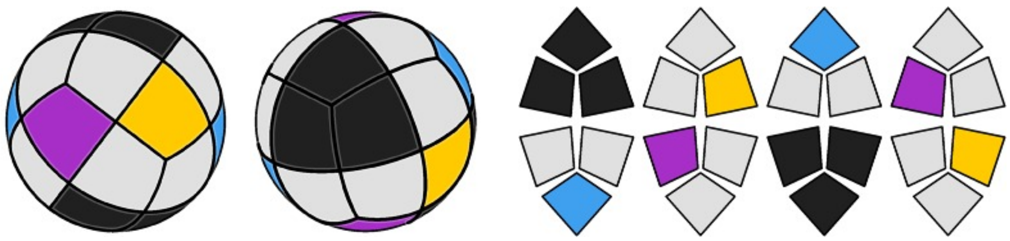
#11: Tri-Color Diamond Ring. Same concept as the 2-color Diamond Ring arrangement, but the ring has a repeating sequence of three colors. Each color will be opposite its match.

5 http://footbagcanada.com/detail.aspx?ID=93



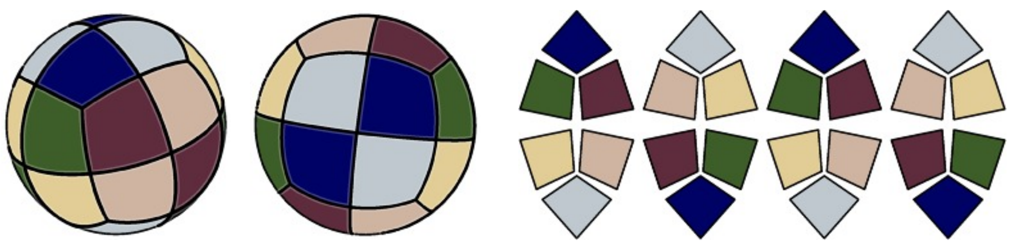
#12: Four Stripes. Same concept as the 2-color Alternating Stripes arrangement. Color A on a patch of 4 panels, color B on the ring of 8 panels around the patch, color C on the adjacent ring of 8 panels, and color D on the final patch of 4 panels.

5 colors



#13: Tri-Color Diamond Ring with Caps. A combination of the Diamond Ring with Caps and Tri-Color Diamond ring arrangements. Each cap has all three ring colors connected to its corners, and each color is opposite its match.

6 colors



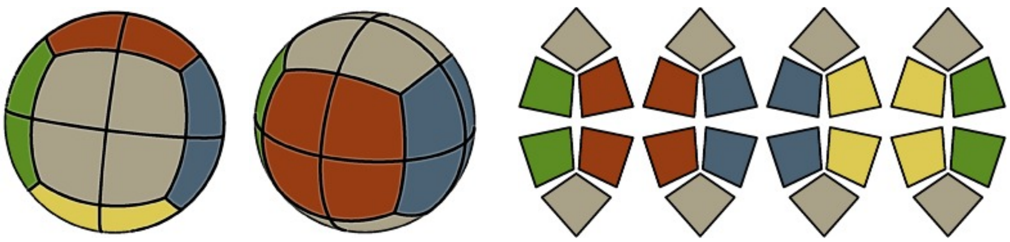
#14: Patchwork Ball. This arrangement is essentially the two-color checker pattern of the octahedron, but with each 3-panel patch's color replaced with a distinct triplet of colors which are sequenced in the same way for each matching patch in terms of clockwise/counterclockwise directions. To make the octahedron pattern more pronounced, I have selected colors such that the 3-panel patches alternate between dark, vivid colors, and light, pale colors. (One of my corduroy bags pictured at the beginning of the chapter has a similar color scheme.)

Since the corners of the triangular patches form the faces of a cube, I also arranged the colors so that those that lie together on the same cube face are a dark and pale shade of the same color (dark blue & pale blue at the top and bottom tips, green & yellow-beige at the inner and outer middles, and dark purple & pale purple on the remaining panels).

So, this arrangement can also be thought of as a 3-color cube with each color replaced with a checkered pattern of two colors. Each distinct pair of colors lies on opposite patches of four panels. For a clearer guide to how each pair of checkered colors should be positioned in the octahedron layout, look at the first cube arrangement below. I have found that, though I tried both in this diagram and in my corduroy bag to emphasize the octahedron, the cube persistently asserts itself in my eyes. Only with some effort, or in poor light, can I see the octahedron.

Cube Arrangements

3-color cube. This is how the 3-color cube arrangement would be laid out in the octahedron assembly method.



5-color cube. This is how the 5-color cube arrangement (a ring of four vivid colors around two opposite neutral colors) would be laid out. The assembly layout diagram positions the ring across the middle with the neutral caps on the top and bottom (as shown in the second ball view).

Cutting Out the Templates

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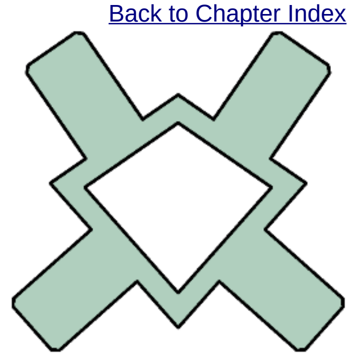

Because this design has so many panels, **I recommend making a combo type template as shown on the right,** or at least a stencil (interior) type (if you don't use a cutting template). The combo type includes the stitching template on the inside and the cutting template on the outside (actually just the corners of it because the tabs interrupt it). **Interior tracing is much faster and easier than exterior**, and for the cutting pattern you really only need the corners. My Ready-to-Print patterns include the combo type template. An X-Acto knife and steel ruler are the best tools for cutting out the interiors.

If you use a thick marker to trace the patterns, **remember to stitch on the *outside* of stencil type patterns, where the edges of the template were (inside the lines for exterior templates)**, so you don't change the size of the ball. If the marker soaks through the fabric you're using, however, you will need to stitch inside the patterns to **hide the lines within the seams**. In that case, cut out the template's interior slightly outside the lines, shifting the edges outward by the width of the marker lines. Then the edges of the pattern it produces will be correctly positioned for stitching inside them. For combo templates, shift the outer edges by the same amount to maintain the same seam allowance.

I recommend keeping the inner part that you cut out of stencil or combo templates for use in drawing the front stitching patterns. Step 2 of the Assembly instructions explains why.

Making the Panels

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Note that if you are using a separate stitching and cutting template and they are not translucent, you must be careful which pattern, cutting or stitching, you trace first so that the **second template doesn't hide the lines of the first** and prevent you from aligning the two. **Do not necessarily use them in the sequence below.**

1. You will need 24 panels, and **you will be tracing the patterns onto the back of the fabric (the side that will be inside the bag)**. If you use a cutting template, first trace that. If you are using a combo type like the one illustrated above, trace the inside and outside of it and skip Step 2.

   If you are using something like **corduroy, denim, or a striped fabric, or other woven fabric**, I recommend **orienting all the panels in the same way** in relation to the lines of the fabric, with the lines running either from end to end or from side to side. This will **produce a uniform and symmetric pattern** on the beanbag.

2. If you are using a separate stitching and cutting template, use the smaller, stitching template to trace the stitching pattern within each cutting pattern, being sure to center it well (centering it allows you to align the patterns more easily as you sew, but is not otherwise important).

3. Cut out the panels.

Assembly

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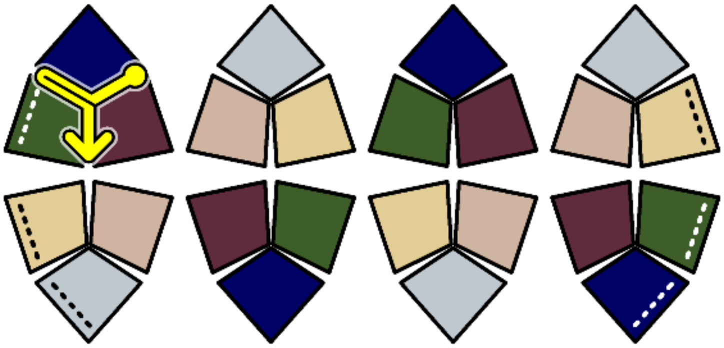
Following is an excellent method of assembling the panels that I developed. For the writing of the Second Edition I made many 24-panel bags and used this method for all of them. It really **makes the assembly simple and easy**. It will also make color layouts easy to arrange, and it gets the panels joined to each other as soon as possible to reduce the possibility of losing track of the arrangement during assembly.

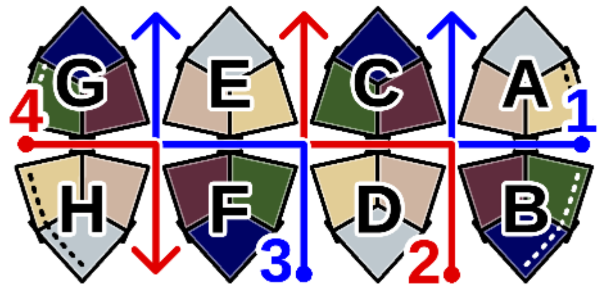
The method has two stages. Stage 1 is to form eight triangular 3-panel patches, and optionally iron their seam allowances. Stage 2, which begins after the photos, is to assemble the patches into a ball as you would an octahedron, following the numbered stitching pathways in the second diagram. The remaining octahedron seams can be ironed after assembly.

I am right-handed and so the diagram is oriented for stitching toward the left. In case you are left-handed or prefer the opposite orientation, I included a **left-handed stitching diagram below the instructions**.

I was able, barely, to turn my denim bag out through an opening of two long seams in a row. My instructions call for leaving three seams open (marked with dashed lines), just to be safe.



Stage 1: Arrange the panels, draw the front stitching patterns, and sew each group of three panels into a triangular patch.



Stage 2: Sew the patches together like an octahedron, following the numbered stitching pathways.

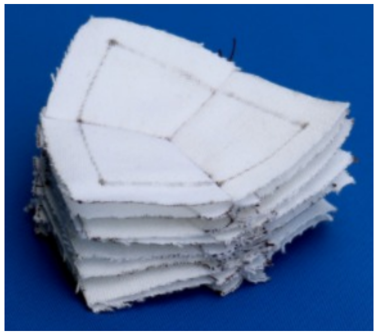
1. ***Stage 1:* Lay the panels out as shown above** (I prefer to place them front face up) and **arrange them according to your color pattern**.
2. Use the stitching template to **draw stitching lines on the *fronts*** of the six outer panel edges shown with dashed lines in the diagrams. My stitching pathway leaves these edges partially unsewn so the bag can be turned out between them. They will then be **sewn from the outside following the front stitching lines**. (If you use a thin or flexible fabric and don't need such a large opening, just skip marking the upper pair or two of panels.) Be sure to align the template as well as possible with the stitching patterns on the backs.

   If you want to **hide the stitching lines within the seams**, sketch them a millimeter or two nearer to the panel edges and stitch slightly inside them (toward the middle of the panels). **If you use a Stencil or Combo type template,** use the inner portion that you cut out of the template to draw these patterns, since the main template will cover the area near the edge.

   I have found it helpful to **add marks along the front stitching lines for each stitch** so that I can more easily keep the exterior stitches even with each other and not get a skewed seam. I space the stitch marks ⅛″ (3mm) apart. If you **make these marks on your template first**, you can more easily transfer them onto these and future panels.
3. **Sew each group of three panels together** as shown by the yellow arrow on the first group. Make sure you put them together correctly. Three obtuse angles (the angle between the short sides) come together in the center of each patch. Place two panels **front faces together**, sew along a short edge toward the obtuse corner, add the third panel and sew it to one of the first two, ending at the outer corner, double back to the center, and then stitch the third and last seam.

   If you are using the backstitch, you can make the duplicate stitches up to twice as long without causing the fabric to ripple as long as you're careful how tightly you pull them (if you pucker the fabric, wiggle it straight again). Tie off the thread and trim it after each patch. Place each finished patch in the original position among the others.
4. **When all eight patches are finished, consider ironing their seam allowances flat** (see the **General Information and Techniques** chapter under "Better Seams by Ironing"). I no longer iron this panel structure, but it's not too difficult to do. The photos below show how I fold the seam allowances at the intersections, followed by my ironed stack of "cloth potato chips" made with my design-testing fabric, and how the seams look from the front. I **fold the corners of the allowances over each other in a pinwheel manner**. This is fairly quick and easy to do, and

makes a great-looking vertex on the bag. Just fold an allowance, then fold the previous one over it like a flap of gift wrap, then proceed to each subsequent pair, holding the previous ones in place with your thumb. **Press the intersection against the iron** when the three flaps are folded.


5. ***Stage 2:*** Now **assemble the eight patches as you would an octahedron** design, following the numbered pathways in the second illustration at the beginning of the instructions (left-handed stitching pathways are diagrammed below). Place patches A and B **front faces together** and sew them together toward C and D, then add C and sew it to A.

   When you reach the midpoint of each patch's edge and have to **cross the perpendicular seam**, use the method I describe in the "Stitching Techniques" section of the **General Information and Techniques** chapter under "Crossing seam intersections while sewing patches of multiple panels together". The **retreating stitches** I recommend at the intersections help to **tightly close the 4-way vertices**, which can open up a bit if they are not cinched shut well. The same is true of the 4-way vertices at the corners of the patches (those corresponding to the vertices of the octahedron.) **To really close all the intersections well**, follow the second method in the "Closing seam intersections tightly" topic in the same section. In short, **stitch each panel corner to the one diagonally opposite it (the thread will form an X across the intersection) and cinch them together**.

6. **Continue adding patches in alphabetical order** as in the previous step and sewing them according to the pathways depicted in the diagram. Keep in mind that you can sew the first three paths with one very long thread by reversing the direction of path #2. **Sew the patches front faces together** so the bag will be inside out. At the end you should have an inside-out bag with up to three adjacent, parallel seams open, and these should have stitching lines on the fronts.

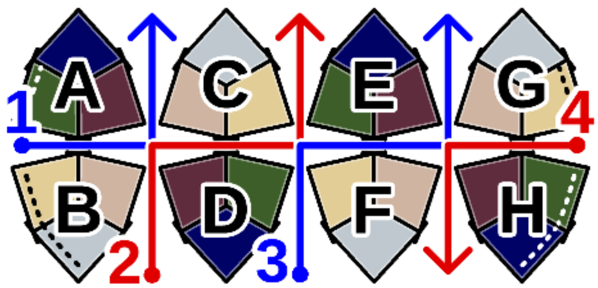
7. At this point, **consider ironing the remaining seam allowances** (those corresponding to the octahedron). The 4-way intersections can be folded in the pinwheel formation or with each pair of corners folded in opposite directions. The pinwheel formation produces somewhat flatter seams, though.

8. **Sew a few starter stitches** at one end of the final seams to make it easier to continue from the outside. If you don't need the entire opening to turn the bag out, continue to sew as much as you don't need. To **reduce the number of stitches you need to make from the outside**, you can make extra stitches and then loosen them to allow the panels to spread enough to turn the bag out. Then you can pull them tight again from the outside. If you want to do this, or if you want to be able to loosen the last several stitches enough to push a funnel between them, **your final thread will need several inches of extra length**.

9. **Turn the bag right side out through the opening**. A good method for this is to use the back end of a pen or other slender tool to push the fabric through the opening from the opposite side and then either invert the bag around the tool, or remove the tool and work the bag through with your fingers. **Be gentle so as not to pop any stitches**.

10. **Pull out the last stitch so that the thread is on the outside** where you can get to it. Continue sewing the opening closed following the front stitching lines. For help, see the "Stitching Techniques" section of the **General Information and Techniques** chapter under "Backstitch from the exterior Approaches". Fill the bag at some point during this final sewing with a funnel. I find it helpful to **put some filler in first to prevent the bag from collapsing** while I sew, and to hold the seam allowances in place and give me something to push the needle against.

    You can sew the entire opening closed before fully filling the bag, which prevents the filler from spilling back out while you sew. Just loosen the last several stitches enough to push the funnel between them, or at least to push some filler in with your fingers. Then re-tighten the stitches (see "Tips on finishing the bag").



Left-handed stitching pathways

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Ready-to-Print Patterns for the Isovertex Kite

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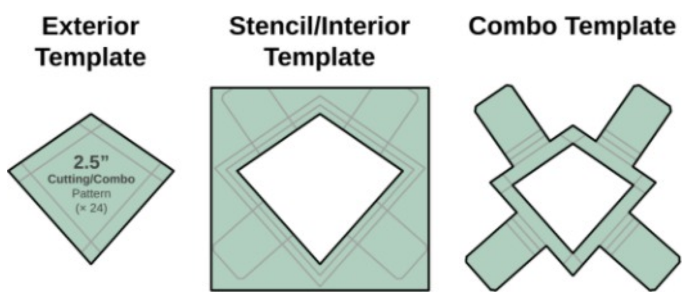
The pattern pages are 8.27″×11″ (210mm×279mm) to fit both "Letter" and "A4" sizes. ***Make sure the print is not being scaled to fit the printer margins*** *(select Default/None scaling/Actual size/Ignore printer margins). To verify correct sizing,* ***compare the centimeter grid to a ruler*** *and adjust the next print if necessary. (Note that PDF viewers and printers can both contribute to slight size inaccuracy.)*

On the following pages are patterns for the isovertex kite that produces a slightly better cloth sphere than the other kite designs. (The patterns for the other two kite shapes are at the end of the chapter.) The patterns are for beanbag diameters from 2″ – 3″ in ¼″ increments, and there is a 7″ pattern for scaling to larger sizes. The patterns are reduced by 1.93% from the mathematical calculation to account for the inflation in size I observed in my corduroy bag. **If you use a dense/stiff or completely non-stretch fabric, I recommend enlarging the pattern to about 102% to get the intended ball size.**

To make the templates, I recommend cutting out the portions of the paper with the patterns you want and gluing or taping them to your template material, and then cutting along the patterns.

The cutting patterns have 4mm, 6mm, and 8mm allowances so you can choose the amount that works best for your fabric and preference (the lighter, 6mm pattern is a hair under ¼"), and they include **tabs for the optional combo type template** (stitching pattern on the inside, cutting pattern on the outside, with the tabs to help you hold it down).

The examples on the right show the **three ways you can cut out the Cutting/Combo templates**. Remember that the cutting patterns have slightly different proportions from the stitching patterns (they are parallel, not proportional), so you should not use them as stitching patterns.



To produce other pattern sizes or correct an incorrectly sized beanbag, adjust the size scaling in the print dialog. For example, to reduce my 2.5″ pattern to the 2.3″ size recommended by the Juggling Store for small hands and numbers juggling, divide 2.3 by 2.5, multiply the result by 100, and that is your scale (92% in this case). If your beanbag ends up not being the expected size, see the **General Information and Techniques** chapter under "Adjusting/Scaling a Pattern to Produce an Accurate Ball Size" for help with correcting it.

The table below provides the scaling for the ⅛″ increments between my ¼″ sizes. The centimeter grid can be used to verify correct scaling.

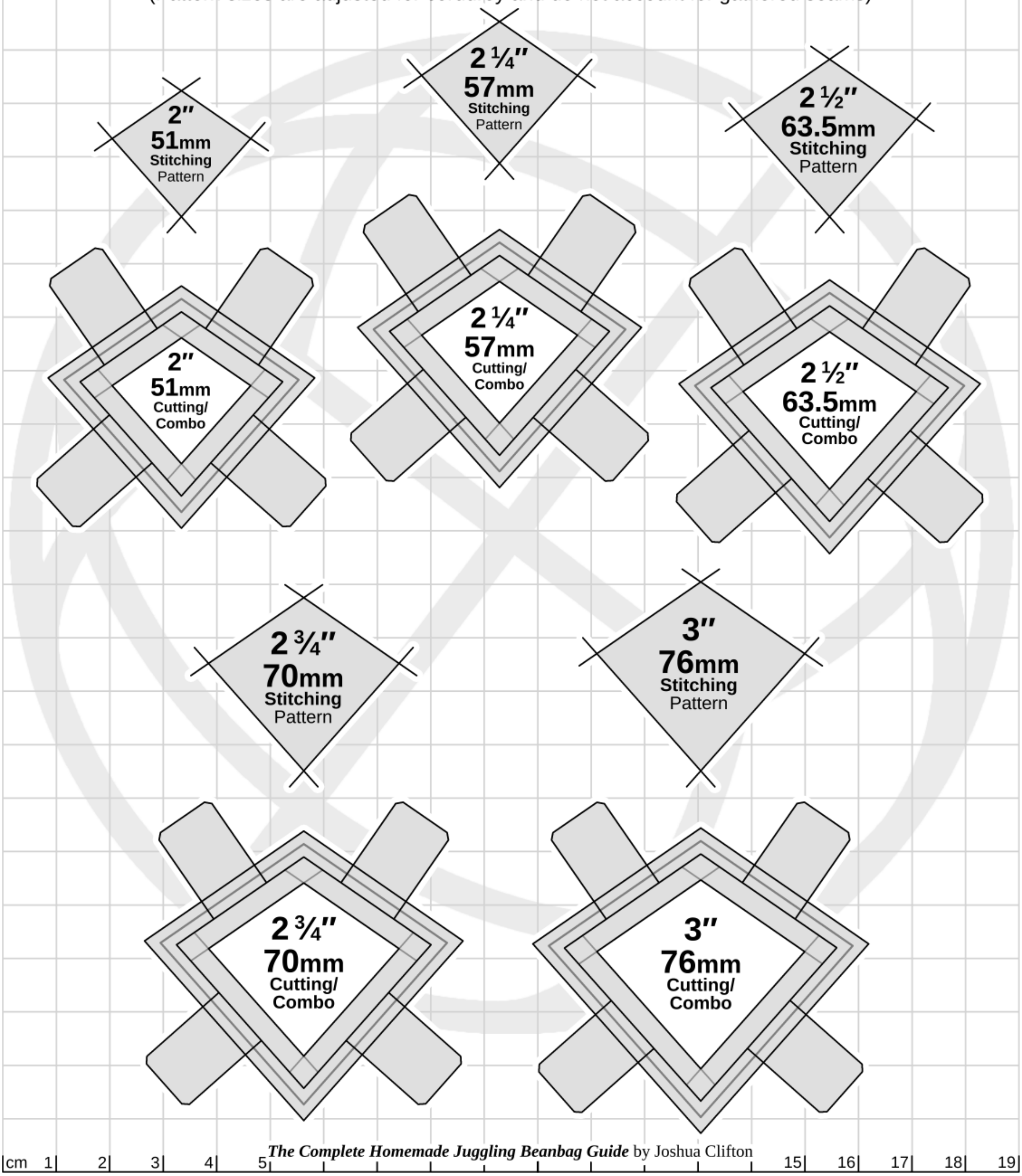
Target Diameter	Print this pattern size	At this scale
1¾″ (44.5mm)	2″	87.5%
1⅞″ (47.6mm)	2″	93.8%
2⅛″ (54.0mm)	2¼″	94.4%
2⅜″ (60.3mm)	2½″	95%
2⅝″ (66.7mm)	2¾″	95.4%
2⅞″ (73.0mm)	3″	95.8%

Deltoidal Icositetrahedron (24 Panels)

Isovertex kite *(forms the best cloth sphere)*

Kite angles: 110.769°, 83.077°

(Pattern sizes are adjusted for corduroy and do not account for gathered seams)



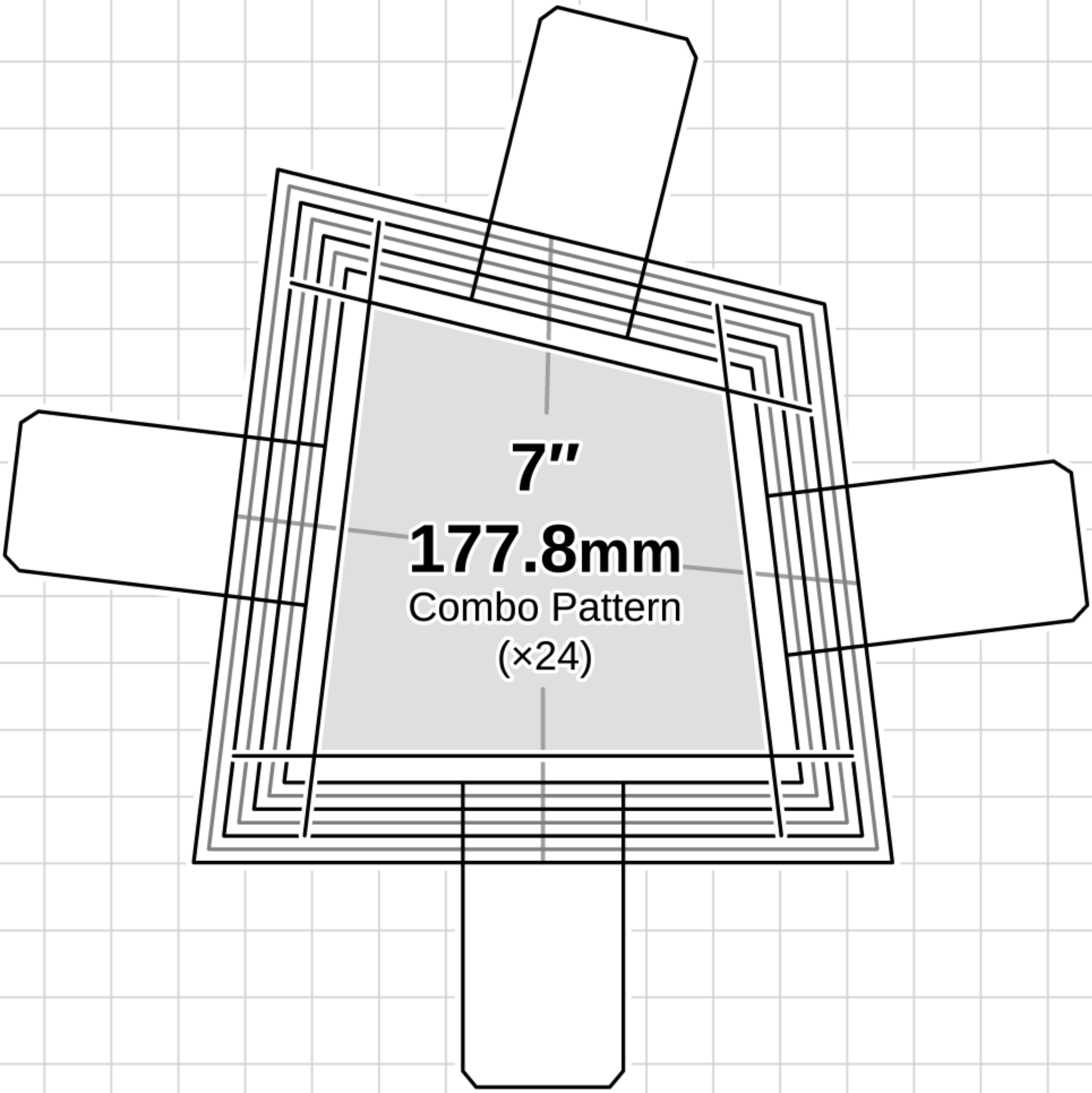
Deltoidal Icositetrahedron (24 Panels)

***Isovertex kite** (forms the best cloth sphere)*

Kite angles: 110.769°, 83.077°

(Pattern sizes are adjusted for corduroy and do not account for gathered seams)

Extra large and versatile pattern for scaling to larger sizes in the Print Dialog. Print twice if you want both a stitching template and a cutting template (or cut out a combo template). The inner pattern (filled with gray) is the stitching pattern. Each dark pattern outside of that marks a 4mm seam allowance interval (at 100% scaling). Use those or the lighter, half-intervals between them to cut out the amount of allowance you want for the cutting template.



cm 1 2 3 4 5 15 16 17 18 19

The Complete Homemade Juggling Beanbag Guide by Joshua Clifton

Blank Color Arrangement Diagrams

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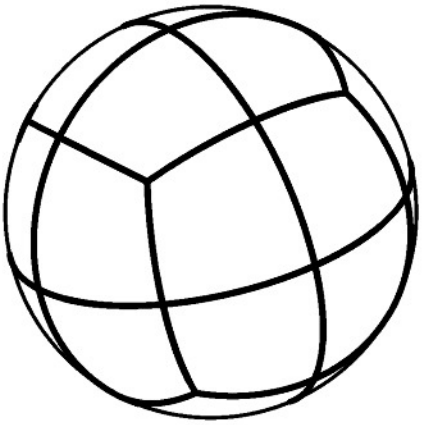
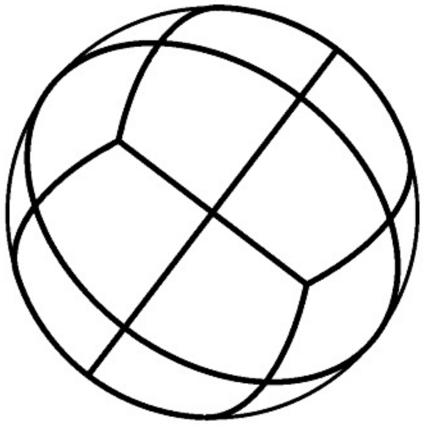
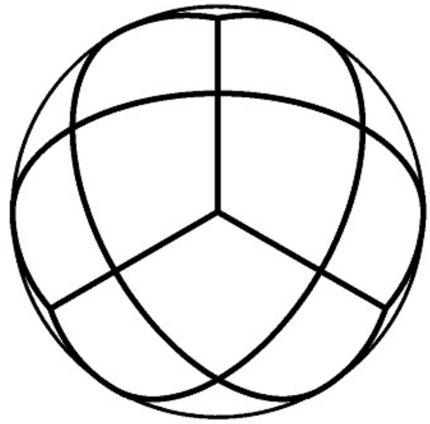
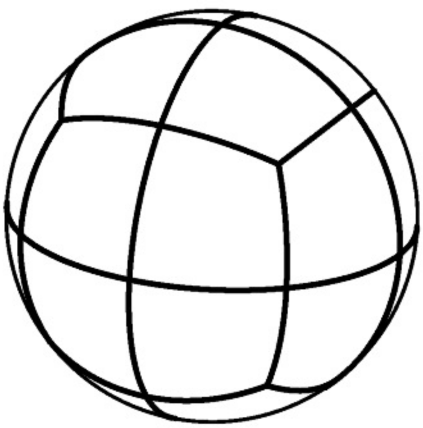
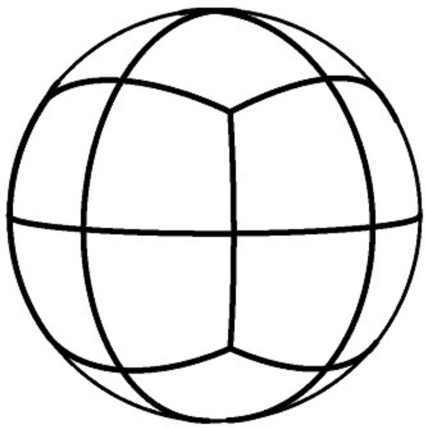
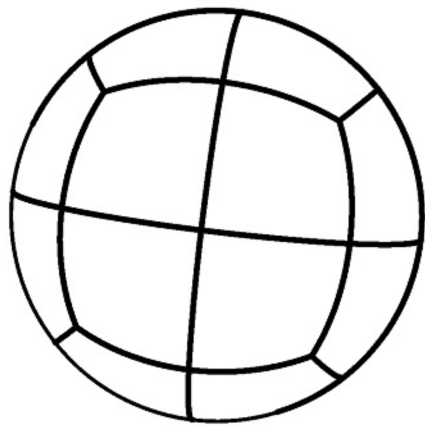
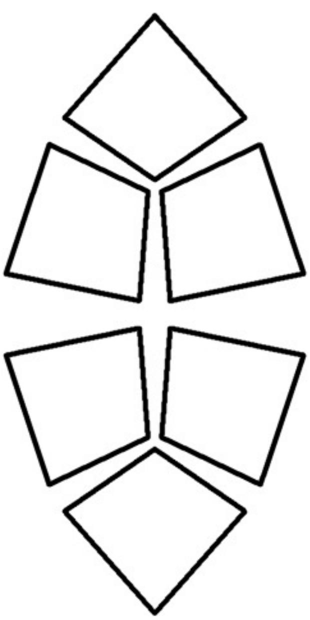
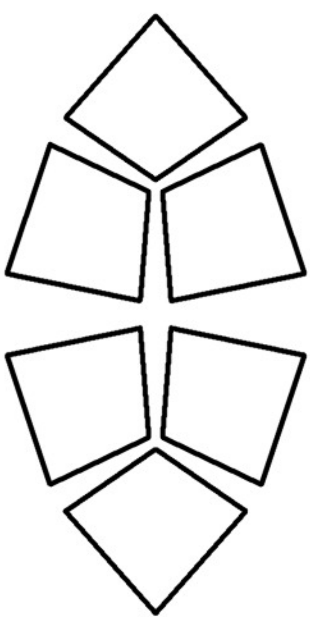
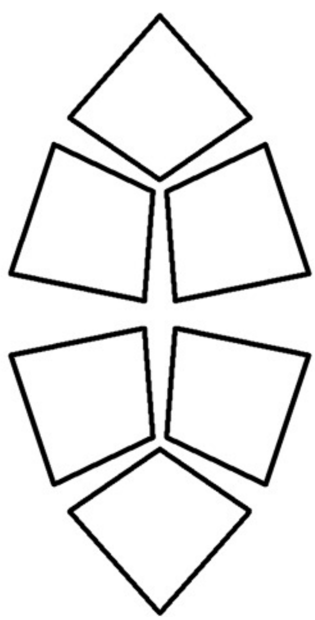
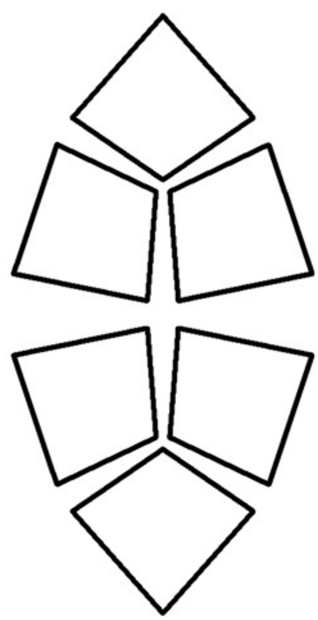
These are the ball and assembly layout diagrams I used for my color arrangement illustrations. You can use these to experiment with your own arrangements. I also offer PNG format diagrams for download on my website that you can use in an image editor. If they are unavailable, you can capture a screenshot of these pages or export the images and then color them in an image editor. Or you can just print them and color them by hand. Note that I did not end up using the ball diagram on the top right.

Sizing Formulas for Drawing the Pattern

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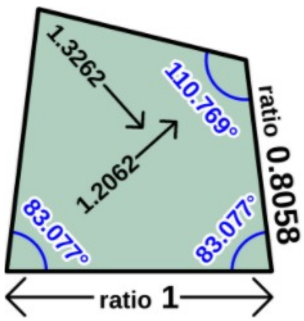
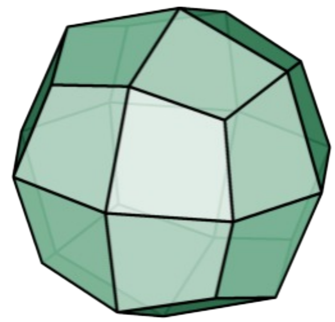
The next section has a table of pre-calculated pattern measurements for all ⅛″ diameter increments from 1 ¾″ – 3″. Following that are the drawing instructions. If you do not need to create a custom size, skip to that. I provide printable measuring tapes at the end of the **General Information and Techniques** chapter in case you care to measure your beanbags. The “Mathematics” section has explanations of all the formulas and ratios, and expresses their values in higher precision.

Design summary





On the left is the generic definition of the panel shape. I define the shape ratios in terms of the long sides. The short sides are 80.58% the length of the long. The kite’s short diagonal (from obtuse corner to opposite) is 120.62% of the long side, and the long diagonal is 132.62% of the long side.

The circumference of the bag can be measured three ways. I calculated a weighted average of them and defined the circumference as *Kite Long Side* × 8.0090. So to calculate the size of the kite, divide the target bag circumference by that value to get the length of the kite’s long side. To draw the kite using the protractor method, that is all you need. Just measure out the angles from the ends of the long side and from the end of the adjacent long side and the resulting short sides will be the correct length and meet at the correct angle. To use the compass method, use the formulas below to calculate the remaining dimensions.

Adjusting for the influence of fabric attributes on beanbag size

I calculated an adjustment factor of **1.0193**, meaning I expect a beanbag made of the same corduroy I used to be 1.93% larger than the mathematical prediction. My new isovertex ball was larger than the mathematical prediction by 0.164190% – 4.285741% depending on whether I filled it loosely or overfilled it. I target the average of those two extremes when sizing my patterns. The moderately filled size was 2.224963% larger.

I use the adjustment factor to adjust the pattern size to produce a more accurate finished size when using my fabric and stitching techniques. If you gather the seams, use a different fabric, or do anything else that changes the size of the bag, you may need to figure out your own adjustment factor. For help, see the **General Information and Techniques** chapter under “Adjusting/Scaling a Pattern to Produce an Accurate Ball Size”.

The bag I made with my design testing fabric (fairly thin, stiff, tightly-woven, non-stretch), moderately tightly filled, had an inflated size of -0.274% (slightly smaller than the mathematical prediction). So if you are using a fabric like this, I recommend that you use the Base value in the measurement tables rather than the Adjusted value. My denim bag from years ago measured -1.17% – +1.58% for an average of +0.205%. This is almost exactly the same as the design testing fabric, so if you use a thick, firm denim or similar fabric, use the Base sizing values for that, as well.

As I understand it, the bag size is affected by fabric attributes as follows. The folding of the fabric at the seams will cause thick, firm fabrics to significantly shrink the bag size unless the fabric has some stretch. Folding thin fabric doesn't consume as much of its size, but my design testing fabric, though fairly thin, has no stretch at all, and so ended up producing about the same size bag as the denim, which stretches a little. Corduroy is a softer, more loosely woven fabric than denim and flexes and compresses more easily, and so not as much of the panels' size is consumed by the folding. My denim and design testing fabric bags have very prominent seams while the corduroy bag has much more subtle seams.

Sizing formulas

Below are the formulas to calculate the pattern construction elements (*Diameter* and *Circumference* refer to your target ball size, π = 3.1416). The value in orange is the adjustment factor. **Don't forget to multiply the final result by 25.4 if you need to convert inches to millimeters.**

- **Kite Long Side = *Diameter* × π ÷ 8.0090 ÷** 1.0193 **(≈ *Diameter* × 0.3923 ÷** 1.0193**)**
  = *Circumference* ÷ 8.0090 ÷ 1.0193
- **Kite Short Side = *Long Side* × 0.8058**
- **Acute Angles = 83.077°** (round to **83°** if drawing by hand)
- **Obtuse Angle between Short Sides = 110.769°** (round to **111°** if drawing by hand)
- **Long Diagonal** (between the two matching corners) **= *Long Side* × 1.3262**
- For double-checking: **Short Diagonal = *Long Side* × 1.2062**

Table of Pre-Calculated Pattern Measurements

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The table below has stitching pattern measurements for each ⅛″ diameter increment from 1¾″ to 3″. The values in the **Adjusted** columns account for my 1.0193 adjustment factor. The adjusted values decrease the **Base** pattern size so that you will get a more accurate finished size when using corduroy or something similar (a soft, flexible, moderately thick fabric). If you are using a firm denim or a thin, but non-stretch fabric, use the Base value instead. I attempt to explain why in the "Adjusting for the influence of fabric attributes on beanbag size" topic above.

To draw the cutting pattern, increase the Long Diagonal by the desired allowance × 3.0160, the Long Side by the allowance × 2.2575, and the short side by the allowance × 1.8190. The angles remain the same.

Finished Diameter	**Long Side** (mm)		**Short Side** (mm)		**Long Diagonal** (mm)		**Short Diagonal** (mm) (for double-checking)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
1¾″ (44.5mm)	17.436	**17.106**	14.049	**13.783**	23.124	**22.686**	21.032	**20.633**
1⅞″ (47.6mm)	18.681	**18.327**	15.052	**14.767**	24.776	**24.307**	22.534	**22.107**
2″ (50.8mm)	19.927	**19.549**	16.056	**15.752**	26.428	**25.927**	24.036	**23.581**
2⅛″ (54.0mm)	21.172	**20.771**	17.059	**16.736**	28.079	**27.548**	25.538	**25.055**
2¼″ (57.2mm)	22.417	**21.993**	18.063	**17.721**	29.731	**29.168**	27.041	**26.529**

Finished Diameter	**Long Side** (mm)		**Short Side** (mm)		**Long Diagonal** (mm)		**Short Diagonal** (mm) (for double-checking)	
	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**	Base	**Adjusted**
2⅜″ (60.3mm)	23.663	**23.215**	19.066	**18.705**	31.383	**30.788**	28.543	**28.002**
2½″ (63.5mm)	24.908	**24.437**	20.070	**19.690**	33.034	**32.409**	30.045	**29.476**
2⅝″ (66.7mm)	26.154	**25.658**	21.073	**20.674**	34.686	**34.029**	31.547	**30.950**
2¾″ (69.9mm)	27.399	**26.880**	22.077	**21.659**	36.338	**35.650**	33.050	**32.424**
2⅞″ (73.0mm)	28.644	**28.102**	23.080	**22.643**	37.990	**37.270**	34.552	**33.898**
3″ (76.2mm)	29.890	**29.324**	24.084	**23.628**	39.641	**38.891**	36.054	**35.371**

How to Draw the Panel Shape

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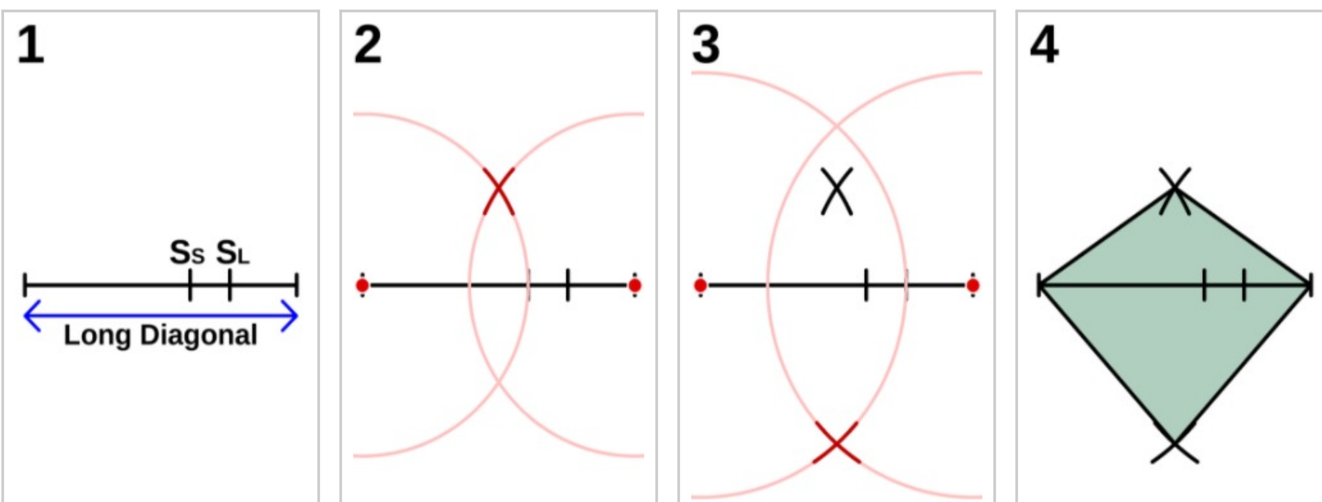
The panel shape is a kite quadrilateral. I have provided two methods of drawing it. For hand drawing, I recommend using the compass method which involves drawing a line for the kite's long diagonal, and using a compass to draw intersecting arcs that mark the locations of the other two corners. For drawing on a computer, it is simpler to use a protractor tool and measure the angles.

There is a separate set of illustrations for each method. Their numbers correspond to the step numbers. To conserve your template material, I recommend that you draw the pattern on paper and then glue or tape the pattern to your template material before cutting it out.



Manual directions (Compass method)

(The terms in bold refer to columns in the pattern measurement table above.

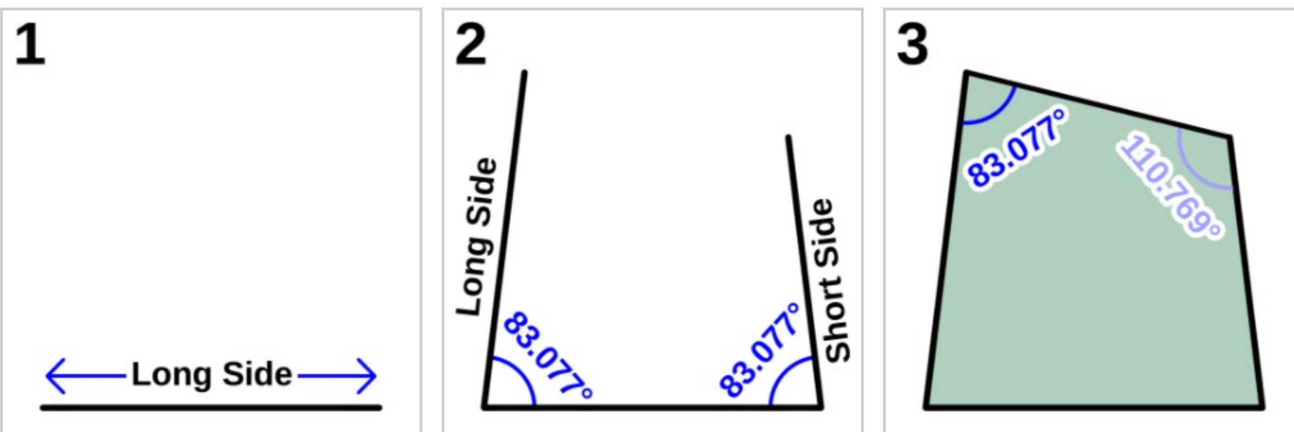
1. Draw a horizontal line the length of **Long Diagonal** and mark each end of it. Then mark two points along it, one the distance of **Long Side** from one end (S_L) and the other the distance of **Short Side** from the same end (S_S). These will be used to extend a compass to the correct radii.

2. Place a compass on the end of the line and extend it to point S_S (so that its radius is equal to **Short Side**) and draw a small arc above the line. Then place the compass on the other end of the line and draw a second small arc using the same radius to form an X.

3. Place a compass back on the first end of the line and extend it to point S_L (so that its radius is equal to **Long Side**) and draw a small arc below the line. Then place the compass on the other end of the line and draw a second small arc to form another X.

4. Draw lines connecting each end of the first line to both of the arc intersections, forming the kite shape. To ensure you drew the kite correctly, measure the sides and the Short Diagonal and make sure they match the values from the table. You can also check to see that the angles between the sides are correct. Any error you make will be compounded many times in the juggling bag, so be as precise as you can.

5. To draw a cutting pattern, multiply the desired allowance by 3.0160 and add that to the **Long Diagonal** length, multiply the allowance by 2.2575 and add that to the **Long Side** length, and multiply it by 1.8190 and add that to the **Short Side** length. Or, just draw it around the stitching pattern.



SketchUp directions (Protractor method)

(The terms in bold refer to columns in the pattern measurement table above.)

1. Draw a line the length of **Kite Long Side**. (If you are drawing this by hand, I recommend marking the ends of each line you draw and then extending it on both ends to aid in accurately aligning a protractor to it.)

2. Use the Protractor tool to measure an 83.077° angle on both ends of the line (round it to 83° if you are drawing this by hand). Draw the next two sides of the kite at the marked angles. Draw a **Long Side** at the left end and a **Short Side** at the right (the short side does not need to be measured; it can be longer).

3. Measure the same angle as before at the end of the second **Long Side**. Draw a line at that angle so that it meets the Short Side from Step 2, completing the kite shape. If you have some excess on the other short side, just erase it. To ensure you drew the kite correctly, measure the Short Sides and the diagonals and make sure they match the values from the table. You can also check

to see that the angle formed at the intersection of the two short sides is 110.769° (or 111° if you rounded the angles).

4. To draw a cutting pattern, multiply the desired allowance by 2.2575 and add that to the **Long Side** length. (The short side will increase by allowance × 1.8190, but that does not need to be measured.) Or, just draw the cutting pattern around the stitching pattern, using its edges as guides.

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Mathematics Behind the Relationship Between the Pattern Parameters and the Ball Size

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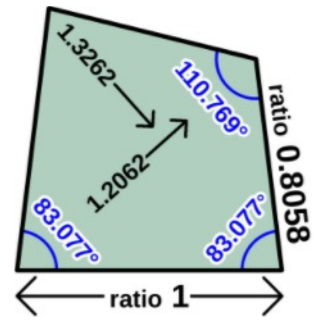
This section describes the math involved in drawing patterns to produce specified beanbag sizes, and creating the pattern sizing formulas. *(The numbers in tiny, right-justified typeface are my computer calculator's unrounded values which I display rounded to six places for brevity.) Note that the isovertex kite shape does not form a closed polyhedron, so the polyhedron illustrations are my original design.*

On the right is the generic definition of my isovertex kite shape. I define the shape ratios in terms of the long sides. The calculations that follow show the derivation of the specific corner angles and how the kite's dimensions and the polyhedron circumference are calculated from them.



Isovertex kite calculations (for producing uniform vertices)

There are two ways to calculate the isovertex kite angles. I originally used the algebraic method, but I later discovered the weighted average method.

Calculating the Isovertex Kite Angles – Algebraic Method

I will define ***a* to be the three acute angles** (which must be equal since they all form 4-way vertices, and all vertices must have the same angle sum to be Isovertex) and ***b* to be the obtuse angle**. I will create two equations defining the necessary properties of the angles, perform a substitution, and solve.

$\mathbf{3a + b = 360°}$ (a quadrilateral's angles must sum to 360°)
$\mathbf{3b = 4a}$ ► $b = \frac{4}{3}a$ (definition of an isovertex kite forming 3-way and 4-way vertices)

Substitution into the second equation: $3a + \frac{4}{3}a = 360°$

Solve: $\frac{13}{3}a = 360°$ ► $a = \frac{3(360°)}{13}$ ► **Acute angle, *a* ≈ 83.076923°** 83.076923076923076923076923076923

So the **Obtuse angle, *b*** ≈ 360° – 3(83.076923°) ≈ **110.769231°** 110.769230769230769230769230769230

Calculating the Isovertex Kite Angles – Weighted Average Method

There are two vertex sums on the normal polyhedron (all three acute kite corners are the same angle).
4-way vertex sum ≈ 326.315768° (4 × 81.578942°) 326.3157675274023128769366912106 4
3-way vertex sum ≈ 345.789523° (3 × 115.263174°) 345.7895230633447960268924447760 7

To calculate the count of each vertex type on the polyhedron, take the count of each type of corner on the face (3*a* and 1*b*), multiply each by the number of faces (72*a* and 24*b*), and divide each by the number meeting at the corresponding vertex type (72*a*/4 and 24*b*/3). There are 18 4-way and 8 3-way vertices. So

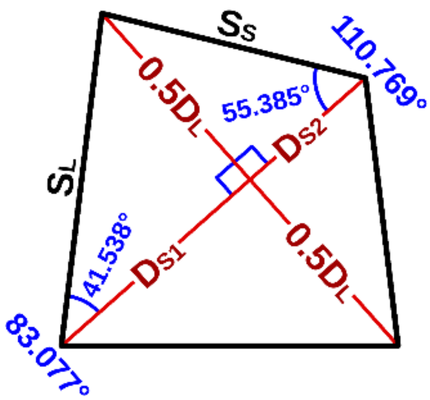
Weighted average ≈ $\frac{326.315768°(18)+345.789523°(8)}{26}$ ≈ **332.307692°** 332.307692307692307692307692307692307 69

Then simply divide that by 4 to get the acute angles and by 3 to get the obtuse angle.

Acute angle = $\frac{332.307692°}{4}$ ≈ **83.076923°** **Obtuse angle** = $\frac{332.307692°}{3}$ ≈ **110.769231°**

Calculating the polygon circumference in terms of the kite's long side

To produce a ball of a desired size, I need to know the relationship between the kite's side lengths (the long side by choice) and the size of the ball that kite will produce. To do that, I will first calculate the ratio between the kite's short and long sides, and then calculate the diagonals in terms of the long side, since the diagonals are part of the circumference measurements. Finally, I will express the circumference (which is a weighted average of three measurement methods) entirely in terms of the long side.



The ratio of the short side to the long, and the kite's diagonals, can be calculated using the corner angles as follows:

Long Diagonal = D_L = 2(sin 41.538462°) **= 1.326245S_L** 1.3262453164815904047535709853335

Ratio of short side to long (S_S / S_L)

Ratio of 0.5D_L to S_L = sin 41.538462° = 0.663123 0.66312265824079520237678549266677

Ratio of 0.5D_L to S_S = sin 55.384615° = 0.822984 0.82298386589365639457961742343938

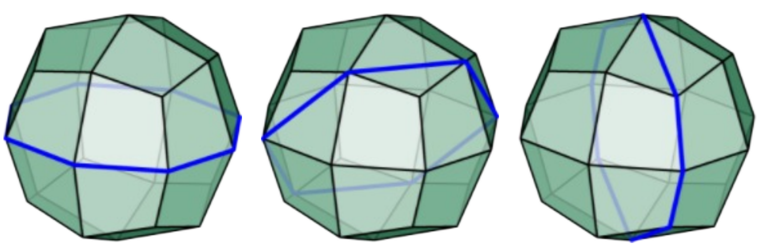
Ratio of the two = $\frac{0.663123}{0.822984}$ **= 0.805754** 0.80575414138979244929034943433255

Short Diagonal = D_{S1} + D_{S2} = cos 41.538462° + (cos 55.384615°)0.805754 **= 1.206231S_L**

1.2062312704272734492085369846028

There are three ways to measure the polyhedron's circumference (illustrated below). I use the first as the basis of comparison. The three circumferences of this version of the solid are nearly equal and calculating the weighted average it not important, but I did it for my original design, and it was easy to do again for this one.

To determine the count of each type of circumference, take one of the dimensions of the face that composes that circumference (for instance, the long side or the short diagonal), calculate the sum of those on all faces (48 long sides or 24 short diagonals) and divide by the number of those along each circumference path, remembering to count each face's side twice along the circumference because polygon edges are composed of two conjoined face sides (48 long sides ÷ 16 or 24 short diagonals ÷ 4).

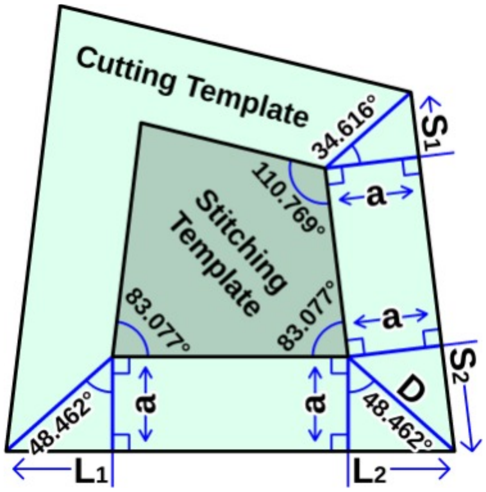


Circumference	Relative Proportion	Frequency
Long Side × 8	1	3
Long Diagonal × 6	0.994684 0.99468398736119280356517823900013	4
Short Diagonal × 4 + *Short Side* × 4	1.005993 1.0059927059085329492494432094676	6
Weighted Average	**1.001130** 1.0011301680689206853659517086774	

So I define the circumference as *Long Side* × 8.009041 (the weighted average × 8) 8.0090413445513654829276136694192

Cutting pattern calculations

To draw the cutting pattern directly (rather than using the stitching pattern as a guide), and make it perfect, more trigonometry is needed. The diagram below shows the amount to extend each of the two side lengths or the long diagonal to get a seam allowance *a*.



Long Side Length increase, $L_1 + L_2$

$$= (\tan 48.461538° \times 2)a \approx \mathbf{2.257533}\boldsymbol{a}$$ 2.2575332835382440461848369729097

Short Side Length increase, $S_1 + S_2$

$$= (\tan 34.615385° + \tan 48.461538°)a \approx \mathbf{1.819017}\boldsymbol{a}$$ 1.8190168136494367389779638430996

Long Diagonal increase, $2D$

$$= 2\frac{1}{\cos 48.461538°}a \approx \mathbf{(3.016033)}\boldsymbol{a}$$ 3.0160332709876332685007040216398

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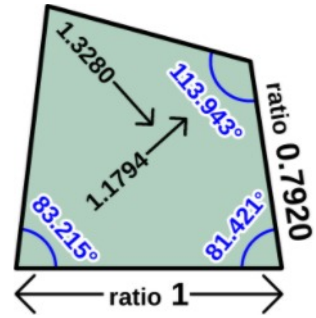
Original kite based on a modified polyhedron

This version of the kite forms a closed polyhedron and was my original design. I designed it by designing a polyhedron that formed a better ball than the true deltoidal icositetrahedron and then measuring the resulting kite face's angles in SketchUp. The next section, "How I Developed This Design", explains the long and complicated process that lead to this design, and the method I developed for drawing the polyhedron.



The three polyhedron circumferences are as follows. I use the first as the basis of comparison.

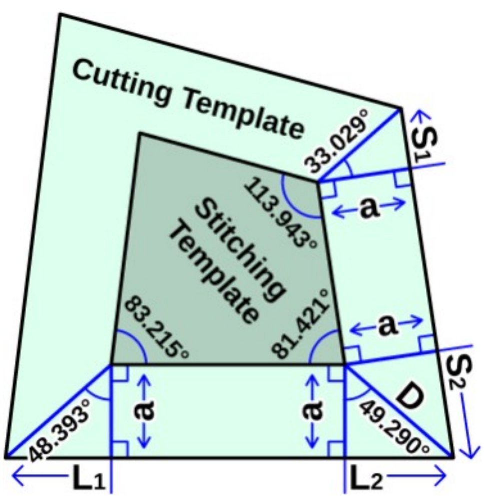
Circumference	Relative Proportion	Frequency
Long Side × 8	1	3
Long Diagonal × 6	0.996036 0.9960361402575132790476638588013	4
Short Diagonal × 4 + *Short Side* × 4	0.985709 0.98570855312574497733772503904943	6
Weighted Average	**0.992184** 0.99218429844496330617053890598593	

Kite dimensions in terms of the Long Side

- **Long Side =** $\dfrac{\textbf{\textit{Target Circumference}}}{\textbf{7.937474}}$ 7.9374743875597064493643112478874
- **Short Side =** ***Long Side*** **× 0.792014** 0.79201389494929989410551857045456
- **Long Diagonal (between the 81.421° corners) =** ***Long Side*** **× 1.328048** 1.3280481870100177053968851811735
- **Short Diagonal (from obtuse corner to opposite) =** ***Long Side*** **× 1.179403** 1.1794032113021900605699315076443
- **Angles: 83.215°, 81.421°, 113.943°**

Cutting pattern calculations

To draw the cutting pattern directly (rather than using the stitching pattern as a guide), and make it perfect, trigonometry is needed. The diagram below shows the amount to extend each of the two side lengths or the long diagonal to get a seam allowance *a*.



Long Side Length increase, $L_1 + L_2$

$= (\tan 48.3925° + \tan 49.2895°)a \approx \mathbf{2.288207}a$ 2.2882065750909126293570816053011

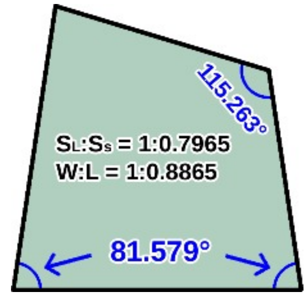
Short Side Length increase, $S_1 + S_2$

$= (\tan 33.0285° + \tan 49.2895°)a \approx \mathbf{1.812291}a$ 1.8122914019863513750162356048106

Long Diagonal increase, $2D$

$$= 2\frac{1}{\cos 49.2895°}a \approx \mathbf{3.066369}a$$ 3.0663685032510972050877228198208

True deltoidal icositetrahedron kite shape



Just for completeness, here are the dimensions of the true Catalan solid's kite shape. The three acute angles are equal to

$$\textbf{Acute angles} = \arccos\left(\frac{1}{2}-\frac{\sqrt{2}}{4}\right) \approx \mathbf{81.578942°}$$

81.578941881850578219234172802659

$$\textbf{Obtuse angle} = \arccos\left(-\frac{1}{4}-\frac{\sqrt{2}}{8}\right) \approx \mathbf{115.263174°}$$

115.26317435444826534229748159202

The three polyhedron circumferences are as follows. I use the first as the basis of comparison.

Circumference	Relative Proportion	Frequency
Long Side × 8	1	3
Kite Width × 6	0.979922 0.9799222236572823958924823800704	4
Kite Length × 4 + *Short Side* × 4	0.972327 0.9723271044783686134288468117923	6
Weighted Average	**0.981050** 0.98105011703841086647253926084888	

Kite dimensions in terms of the Long Side

- **Long Side =** $\dfrac{\textbf{\textit{Target Circumference}}}{\mathbf{7.848401}}$ 7.848400936307286931780314086791
- **Short Side =** ***Long Side*** **× 0.773459** 0.77345908033901357840024124631569
- **Long Diagonal (between the lateral corners) =** ***Long Side*** **× 1.306563** 1.3065629648763765278566431734272
- **Short Diagonal (from obtuse corner to opposite) =** ***Long Side*** **× 1.171195** 1.1711951286177236484574523772689

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How I Developed This Design

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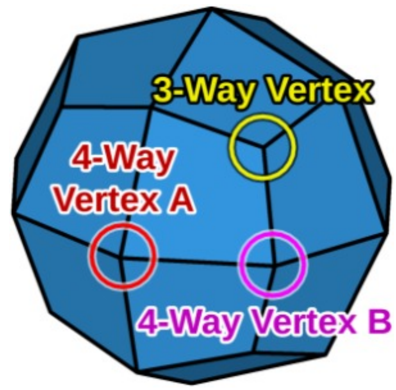
My new Isovertex modification in July, 2022

For the Third Edition of this guide I wrote the chapter on the 30-panel rhombic triacontahedron. I modified its rhombus panel shape so that the 3-way and 5-way vertices on the polyhedron would have **equal sums of face angles forming them, resulting in a rounder, more uniform sphere. I coined the term "isovertex" to describe this modification.** Though the face shape did not fit together into a closed polyhedron, it did produce an excellent sphere – much better than that produced by the normal rhombus. For a full discussion of this modification, see the Isovertex section of Chapter 5.



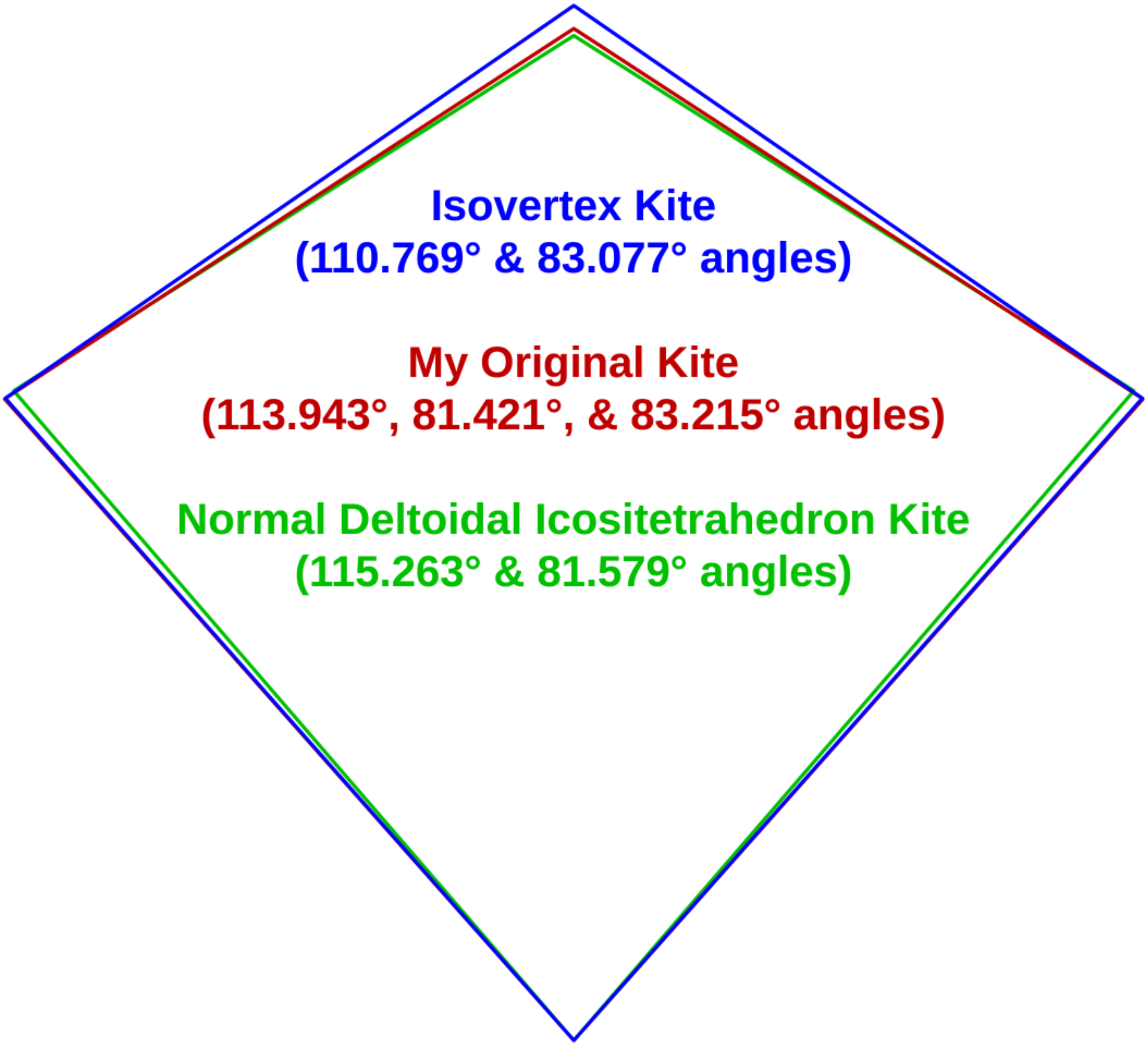
Back in late 2013 when I was working on the deltoidal icositetrahedron, I had the same idea for creating a better sphere with less prominent vertices by creating a kite shape that would form equal vertex sums. The normal kite has three 81.579° angles and a 115.263° angle. On the polyhedron three obtuse angles form the 3-way vertices and four acute angles form the 4-ways. This results in the 3-way vertices having a sum of 345.790° while the 4-way vertices have a sum of only 326.316°. So the 4-ways are sharper and farther from the center.

An Isovertex kite has 83.077° acute angles and a 110.769° obtuse angle resulting in both types of vertices having the same sum of 332.308° (which I later discovered is the weighted average of the two vertex sums). **But my idealized kite shape did not quite fit together into a closed polyhedron, and it did not occur to me that it might still work for making beanbags, so I abandoned that approach** and worked instead on designing an improved polyhedron. I succeeded and published that kite shape in my guide.

After my success with the isovertex rhombus of the rhombic triacontahedron, **I decided to experiment with an isovertex kite shape** to see if it would work as well for this panel structure. I made a beanbag with that shape and another with my original shape. The difference between them is very slight, but **the isovertex version does seem a little more uniformly round**.

Because of the slight improvement in beanbag shape and because this approach is my new preferred approach to converting a polyhedron with unequal vertices into a sphere, **I decided to edit this chapter to use the isovertex kite shape**. But I also left in the patterns and calculations for my original design, and I added patterns and calculations for the normal kite.

On the next page is a comparison of all three kite shapes. The remainder of this section describes the development of the original polyhedron nine years ago.



The new isovertex kite shape compared to my previous kite and to the one that forms the true Catalan solid.

Original kite development essay begins on the next page

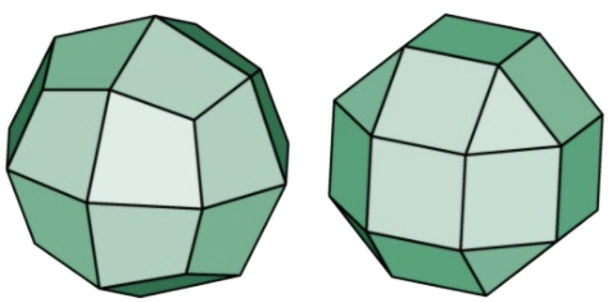
Discovering this panel structure

Footbag photo from http://www.expo-star.com/lview.asp?mainid=12&Subid=0&pid=52.
CG illustrations made in SketchUp and Photoshop by me.

I developed this design in October, 2013, about a year after publishing the first edition guide. In my lineup of footbag panel structures in Chapter 4 you can see examples of 24-panel footbags (one of which is duplicated above). When I first looked at it I saw it as an octahedron with each triangle divided into three kites[6] (the footbags had octahedron color arrangements). But weeks later I was examining it and thinking about how I might design the kites because I liked the look of this structure, and I realized that I also saw a cube structure within its seams, with each face of the cube composed of a checkered pattern of the two colors.

Excited, I took my spherical cube and octahedron SketchUp models, overlaid them on a sphere, and indeed this formed the 24-panel shape! That means that this structure supports all the color arrangements of the cube and octahedron (and the 4-panel orange peel ball) in addition to its own unique arrangements. The CG illustrations above show how both the octahedron and cube fit into the seam structure.

After attempting and failing to figure out how to draw the polyhedron, I did some web research and learned that this is (or is a variation of) a named solid, the “deltoidal icositetrahedron”. This is a Catalan solid or Archimedean dual, being the dual of the rhombicuboctahedron, which is a 26-face Archimedean solid also used as a footbag design (see the footbag lineup for an example). The two solids are shown below, oriented so as to display their relationship to each other. Where this solid has a vertex, the other has a face aligned center to vertex, and vice versa. The three-way vertices correspond to the triangular faces and the four-way vertices correspond to the square faces.



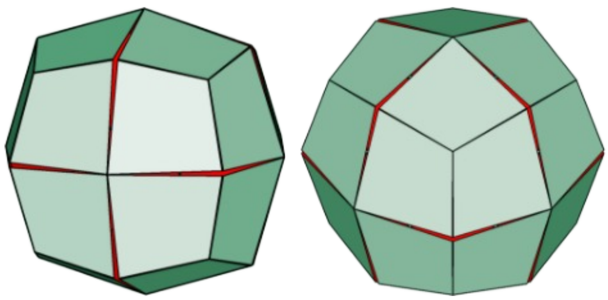
The Deltoidal Icositetrahedron (left) and Rhombicuboctahedron (right) are duals of each other.

6 I did not know before researching this polyhedron that “kite” is the name of a polygon: http://en.wikipedia.org/wiki/Kite_(geometry). According to Wikipedia, “Kite quadrilaterals are named after the wind-blown, flying kites, which often have this shape and which are in turn named for a bird.”

First attempts at designing the kite shape

Lacking knowledge of how to draw a deltoidal icositetrahedron and finding no help on the web, I tried to design an ideal variation of it by designing the kite shape in 2D. My goal was uniform vertices on the polyhedron. I would have liked to design a spherical form of the solid (with faces having circular edges) instead of a true polyhedron, but other than the fact that two of the edges should be rounded using the curve derived from the spherical octahedron and the opposite two with a curve derived from the spherical cube, I didn't know how I would draw it.

By trial-and-error I settled on a shape whose obtuse angle was 111° and whose acute angles were all 83°. The cardboard model I made looks great, but when I assembled the face shape into a polyhedron in SketchUp (which I did not figure out how to do until a day or so later), the faces did not fit together. The illustrations below show this version of the solid, which I redesigned by forming it around a cube and using trial-and-error to make the angles almost exactly 111° and 83°. The red is the gaps between the faces.



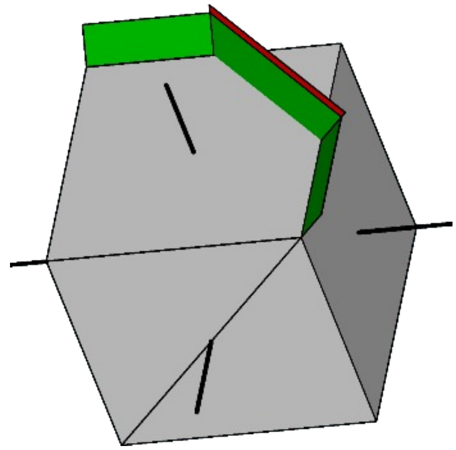
My first panel shape did not form a closed solid.

Except for the fact that my kite shape does not quite form a closed polyhedron, it is theoretically ideal. The 4-way vertices are 83°×4 = 332° and the 3-way vertices are 111°×3 = 333°. The ratio of the circumferences as calculated by the kite width (between the two opposite acute angles, 6 of which circumscribe the polyhedron), the long side of the kite (8 of which circumscribe the polyhedron), and the kite length (obtuse angle to opposite corner, 4 of which plus 4 short sides circumscribe the polyhedron) is 1.0061 : 1.0 : 1.0042. So it has almost perfectly uniform circumferences and vertices.

I have tried many different and more sophisticated methods of designing a polyhedron that retains as nearly as possible these attributes but has a face shape that fits together. I lacked sufficient math and primarily used trial-and-error. I even tried constructing a true deltoidal icositetrahedron by first building a rhombicuboctahedron (which took me a long time to figure out how to draw) and connecting the centers of the faces. The four points for each face did not lie on the same plane, however, and so resulted in a 48-face polyhedron. I don't know what I did wrong there.

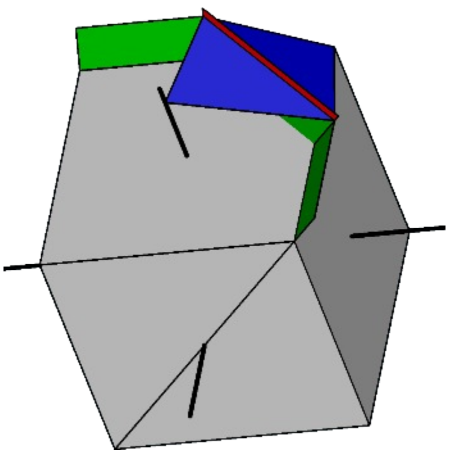
Constructing deltoidal icositetrahedra around cubes

Over the course of four days and many hours of experimentation I formed what I thought was an optimized polyhedron, adjusted it, rethought it and corrected it, arrived on what I thought was a final design, returned to thinking my original was best, and rethought it again. I finally settled on one of my earlier designs in which I had corrected my originally flawed method of forming the shape around a cube. There are two ways (that I know of) to form the kite shape in reference to a cube so that it forms a reasonable approximation of the deltoidal icositetrahedron, and these are depicted below.

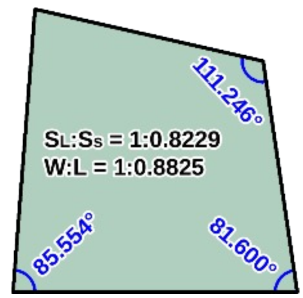


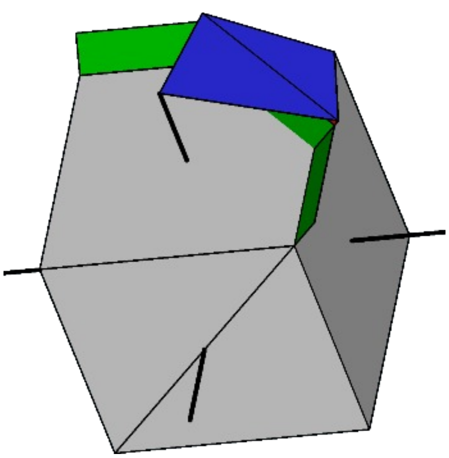
Since I want the vertices and the dimensions to be as uniform as possible, I first drew guides on the cube to mark equal distances from the center. The lines protruding from each face have the length (end to end) of the cube's diagonal so that their endpoints are the same distance from the center as the cube's corners. This is like overlaying an octahedron on the cube, which makes sense as the icositetrahedron is related to both the cube and the octahedron, which are duals of each other.

The tops of the green walls on the edges are also the same distance from the center. The walls extend to the centers of each edge and I connected them across the top face of the cube. The diagonal line across the foremost cube face shows how I constructed the walls. The entire wall is not actually needed, but only a stick in the center of each edge as I used for the faces. But I needed the wall during my experimentation and I left it there. The red strip will be explained later.



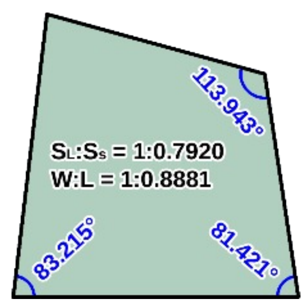
One way to fit a kite onto this framework is to make its corners meet the corner of the cube and the top of my green walls (at its corners). This results in the icositetrahedron having equal diameters at those vertices. But as you can see it does not meet the octahedron vertex (the stick protruding from the center of the face), but lies below it. The kite on the right shows the resulting dimensions (S_L and S_S stand for Long Side and Short Side, and W and L stand for the kite's Width and Length).





The other way to form the kite is to make it meet the octahedron vertex and the cube's corner. This configuration causes it to lie above the green walls at the top of the red strip (which I made to provide a reference for this version of the kite). This results in the kite dimensions shown on the right.

This is the version I chose for my beanbags.



To determine the altitude of the red strip, extend the central wall to a higher altitude than you know you'll need, draw a line from the octahedron vertex stick to the cube's corner which will intersect the wall, and draw a line from the intersection point to each edge of the wall, making sure it is parallel to the wall's top/bottom. That line forms the top of the red strip.

I also tried making the kite meet the octahedron vertex and the edge centers, but that, of course, causes the obtuse corner of the kite to fall beneath the surface of the cube and it no longer fits together as a polyhedron.

If you want to draw this shape yourself, you don't necessarily need to draw the walls as I drew them, which were a result of inexperience and design experiments. What you need are simply the reference points, for which lines like the ones on the cube faces are sufficient. (I left the full walls in place for the illustrations because I thought they would make the structure's design easier to understand.) Once you have drawn the first face, rotate copies of it or of the entire model until you have formed the entire icositetrahedron around the cube.

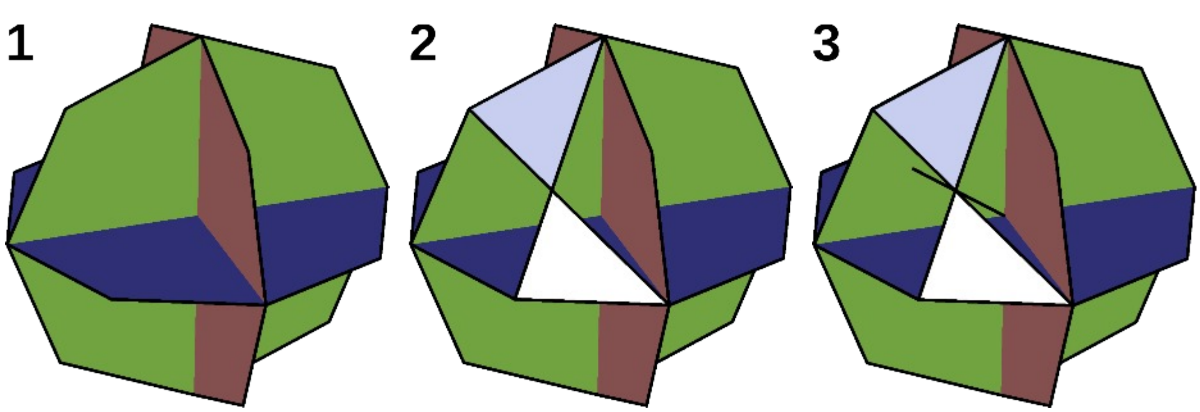
I have not been able to find any information on the web telling me what the angles on the deltoidal icositetrahedron kite are supposed to be, but my high-res Photoshop measurements of a net I downloaded from Wolfram MathWorld are 115.3° and 81.6° (all three acute angles match). The ratio of the long side to the short side is supposed to be approximately 1:0.7735 according to Wikipedia and Wolfram MathWorld[7].

Constructing a true deltoidal icositetrahedron using octagons

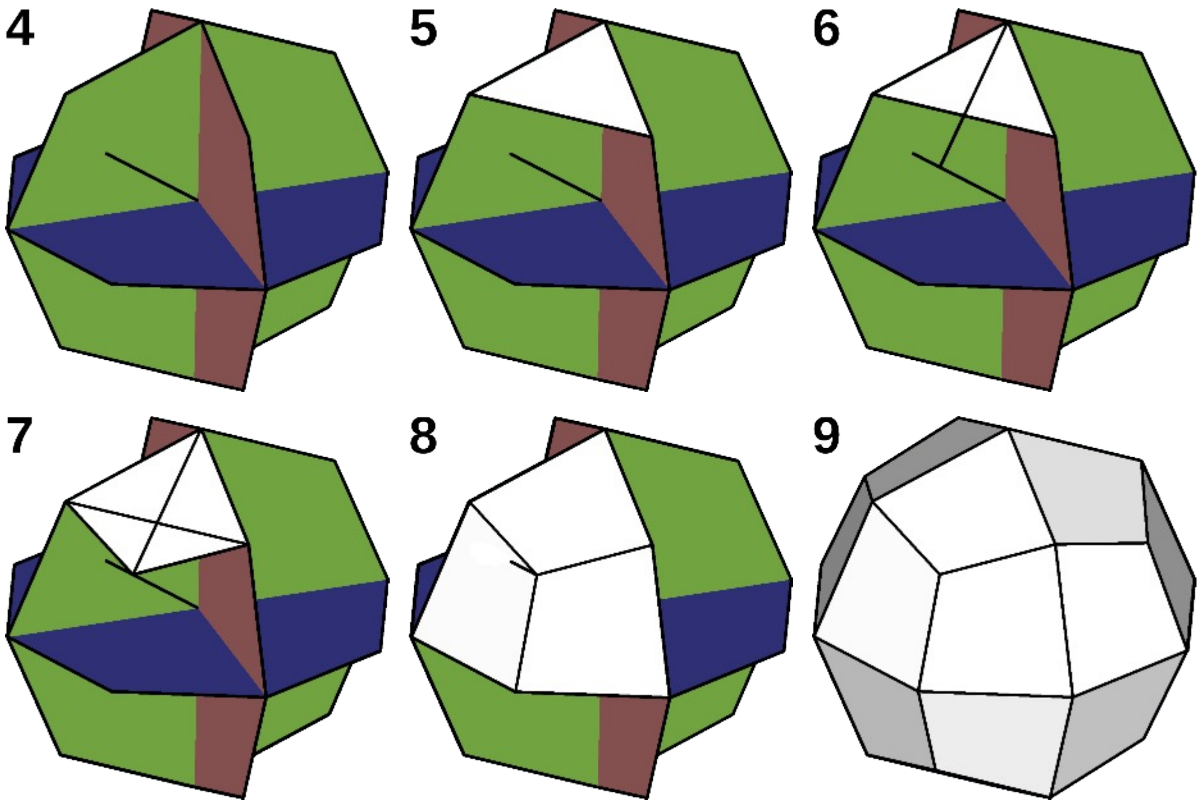
A few days after my cube method experiments I finally realized the correct way to draw the shape, and it produces a kite that matches my measurements of Wolfram's kite. My method is illustrated below.

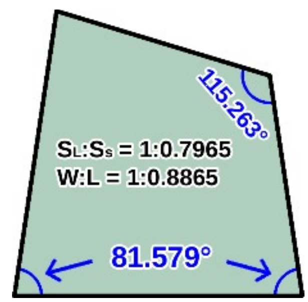
First, draw an octagon and rotate two copies of it (both centered on the first) so that they are all at right angles to each other (Illustration 1). Then draw a line from the center of the figure (easier to find if you draw the figure centered at the origin) that extends through the center of the space between the three octagons and make it longer than you know you'll need (Illustration 4). To find the center of that space, draw two lines between two pairs of opposite corners as shown in Illustration 2; their intersection marks the center. The kite will meet the octagon corners and will meet the line at a point that lies on the same plane as the three other kite corners (approx. 0.9473 of the octagon circumradius).

To find the point on the line where the kite will intersect, first form the upper triangle of the kite by drawing a line across two of the octagon corners (Illustration 5), then draw a line through the triangle's center that extends from the octagons' intersection to the center line (making sure it lies on the same plane as the triangle). Then complete the kite. Draw another line to complete the triplet of kites. Then rotate copies of the figure until you have completed the solid.



7 http://mathworld.wolfram.com/DeltoidalIcositetrahedron.html





Though I have not been able to find any source that will tell me the angles of the kite face, I have found the ratio of the short and long side and the ratio of the length and width[8]. My method produces a kite shape that matches these ratios. Also, as I said, this kite matches the angles of Wolfram's kite as measured in Photoshop. Edit 8/7/2022: Wikipedia does give the kite angles. Either I overlooked that in 2013 or that information was added since then.

Below are three different profiles of each version of the icositetrahedron. The first two are the versions I formed using the cube (in the same order as I presented them before) and the third is the octagon version. Except in the left-most profile, which looks a little rounder in the top version, the middle and bottom versions make a much more uniform shape. The bottom shape forms a uniform octagon in the right-most profile, but has sharper vertical vertices in the middle profile.

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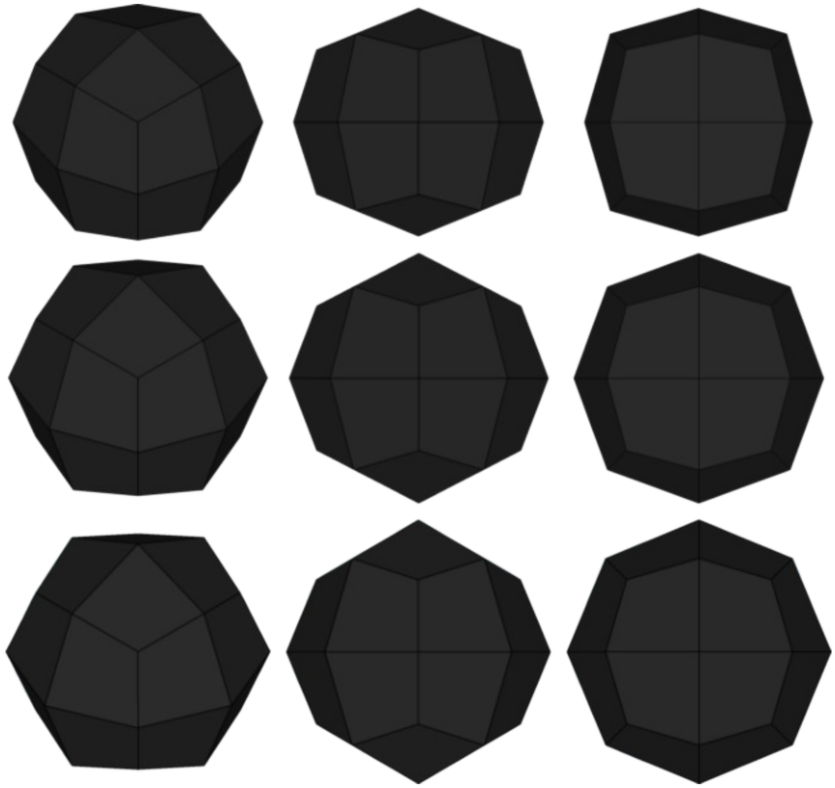
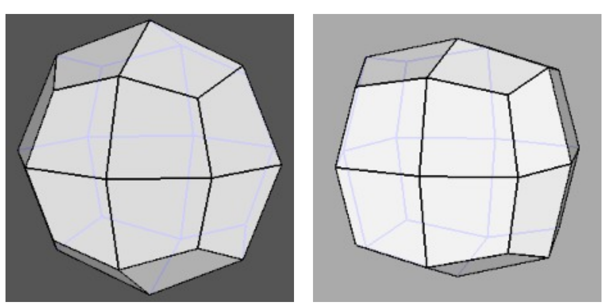
8 http://en.wikipedia.org/wiki/Deltoidal_icositetrahedron#Dimensions, http://mathworld.wolfram.com/DeltoidalIcositetrahedron.html, http://dmccooey.com/polyhedra/DeltoidalIcositetrahedron.html



Profile comparisons of my two cube-method designs (first two rows) and the octagon method (bottom row)

The second version down (my chosen version) has the following vertices: 341.8° × 8, 332.9° × 6, and 325.7° × 12. The true version has 345.9° × 8 and 326.4° × 18. My vertices are slightly more uniform, but it's not a huge difference. Visually, however, the top and bottom vertices, those corresponding to the cube faces, are significantly sharper in the true version. I made a beanbag according to both designs and the true version did seem to have slightly greater bulges on those vertices when loosely filled. It was hardly noticeable, though.

Back near the beginning of my research I found a web page titled *Platonic and Catalan Polyhedra as Archetypes of Forms Belonging to the Cubic and Icosahedral Systems*[9] that showed (among many other polyhedra) two forms of the deltoidal icositetrahedron (shown below). The one on the left with the sharper vertices is the true Catalan solid while the other is some alternate version that looks more cubic. I liked the more cubic version for a beanbag because of its blunter vertices and it influenced my work. My first version above is probably that one or very close. My second version is about halfway between the two, which is preferable because I don't want the beanbag to be too cubic.



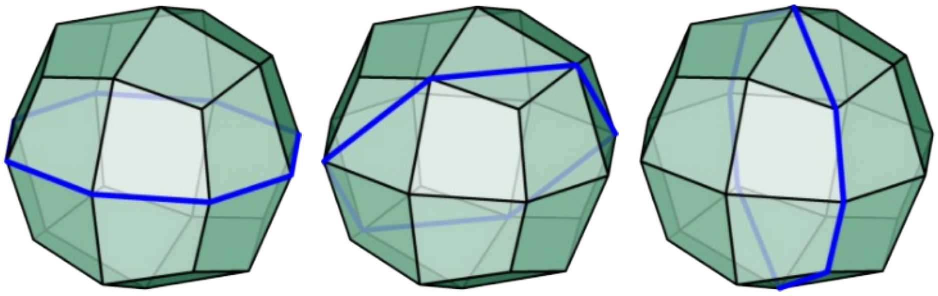
Deltoidal icositetrahedron images from
http://www.mi.sanu.ac.rs/vismath/zefiro2009april/__cubic&icosahedral_forms_from_archetypal_Platonic&Catalan_solids.htm

9 http://www.mi.sanu.ac.rs/vismath/zefiro2009april/__cubic&icosahedral_forms_from_archetypal_Platonic&Catalan_solids.htm

In my chosen version the diameter between the “cube edges” is 3.53% greater than the diameter between the “cube faces”. The diameter between the 3-way vertices (the “cube corners”) matches the cube face diameter. In the third version down (the true version) the cube face and cube edge diameters match, but are 5.56% greater than the cube corner diameter.

Below is a comparison table of the second and third version in terms of their circumference uniformity. There are three simple ways to measure the circumference of this polyhedron:

- Long Side × 8
- Kite Width × 6
- Kite Length × 4 + Short Side × 4



I prefer the first method because it is the easiest to use in sizing the panel, so I used it as the basis of comparison. As you can see, my icositetrahedron has a slightly more uniform circumference.

	My Chosen Version	**True Version**
Long Side Circumference	1	1
Kite Width Circumference	0.9960	0.9799
Kite Length/Short Side Circumference	0.9851	0.9723

Based on my analysis, the cardboard models I constructed, and on the beanbags I made of each design, I think my design is slightly better than the true icositetrahedron, but the difference is so small that it is hard to decide.

Circular kite edge experiments

For the Second Edition guide I wanted to design curved edges for the kite, as I had for the 12 and 14-panel designs, to improve the bag’s roundness. I had great difficulty in determining the best curves. In the end I made ten 24-panel beanbags, two of them out of corduroy, and ended up not using curves at all. The curved edges made only a minor improvement in stiff fabrics, and for soft fabrics like my corduroy they produced a poor shape with concave vertices. In addition, they severely complicate an otherwise very simple pattern shape.

My first few experiments used angles that I had rounded correctly to 83°, 81.5°, and 114°. I first tried curves that added 10° to the corners, since that would make the obtuse and opposite vertices amount to 372° which matches my 14-panel design, and the lateral vertices 366° which matches my 12-panel design. This seemed to make the 3-way vertices slightly too flat, so I tried 7° curves. Then I tried 8° at the obtuse corners and 10° at the opposite corners, making the lateral corners 9° greater.

In the end, after many days of experimenting and much observation of the different beanbags, I decided that the differences were too small to reliably discern. At this point I nearly decided to give up and stick with the straight-edged version. But though that version is not very angular, the curved versions felt so much smoother that I could easily pick them out by feel alone. After a few more days of consideration, I settled on the 10° curves.

I did also try one more curve design. For the alternate rounded angle version mentioned previously I made a bag with curves that added 10° to the obtuse corner and 12° to the opposite corner, which represents the upper limit. Beyond that there will almost certainly be some inward puckering of the vertices. There did seem to be a bit of flatness at the cube faces, but, again, it was hard to tell.

While I spent a few days making the corduroy bag with the 10° curves, I wrote the instructions and Mathematics section, and made the illustrations, for the circular design. When the corduroy bag was done (it took a long time due to my depression sapping my motivation), it had a very poor shape. The cube faces were very flat and had a tendency to sink inward. Even the octagon faces sank in more easily than I liked. Apparently the curvature, which seemed to work very well for the design testing fabric, was far too much for the softer corduroy.

So I decided that I should do what I had almost decided to do previously and stick to the straight-edged design. Frustrating… At least that does make this chapter, and the design, much simpler. I kept all my write-up for the circular design in a separate document in case I ever want to experiment with that again.

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Ready-to-Print Patterns for the Original and Normal Kite Shapes

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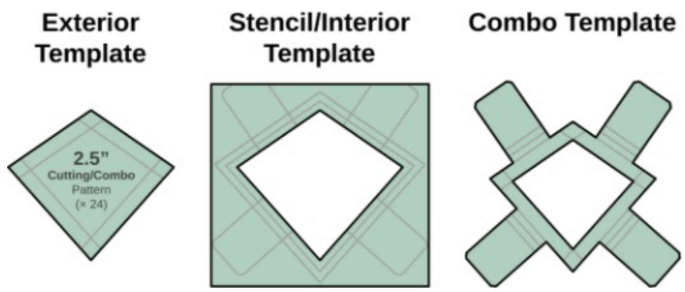
The pattern pages are 8.27″×11″ (210mm×279mm) to fit both "Letter" and "A4" sizes. ***Make sure the print is not being scaled to fit the printer margins*** *(select Default/None scaling/Actual size/Ignore printer margins). To verify correct sizing,* ***compare the centimeter grid to a ruler*** *and adjust the next print if necessary. (Note that PDF viewers and printers can both contribute to slight size inaccuracy.)*

On the following pages are patterns for my original kite design followed by the normal kite that produces a true deltoidal icositetrahedron. The patterns are for beanbag diameters from 2″ – 3″ in ¼″ increments, and there are 7″ patterns for scaling to larger sizes. The patterns are sized using my inflation-corrected sizing so as hopefully to produce accurate finished sizes (they are reduced by 1.93% from the mathematical calculation to account for the inflation in size I observed in my corduroy bag).

To make the templates, I recommend cutting out the portions of the paper with the patterns you want and gluing or taping them to your template material, and then cutting along the patterns.

The cutting patterns have 4mm, 6mm, and 8mm allowances so you can choose the amount that works best for your fabric and preference (the lighter, 6mm pattern is a hair under ¼″), and they include **tabs for the optional combo type template** (stitching pattern on the inside, cutting pattern on the outside, with the tabs to help you hold it down).

The examples on the right show the **three ways you can cut out the Cutting/Combo templates**. Remember that the cutting patterns have slightly different proportions from the stitching patterns (they are parallel, not proportional), so you should not use them as stitching patterns.



To produce other pattern sizes or correct an incorrectly sized beanbag, adjust the size scaling in the print dialog. For example, to reduce my 2.5″ pattern to the 2.3″ size recommended by the Juggling Store for small hands and numbers juggling, divide 2.3 by 2.5, multiply the result by 100, and that is your scale (92% in this case). If your beanbag ends up not being the expected size, see the **General Information and Techniques** chapter under "Adjusting/Scaling a Pattern to Produce an Accurate Ball Size" for help with correcting it.

The table below provides the scaling for the ⅛″ increments between my ¼″ sizes. The centimeter grid can be used to verify correct scaling.

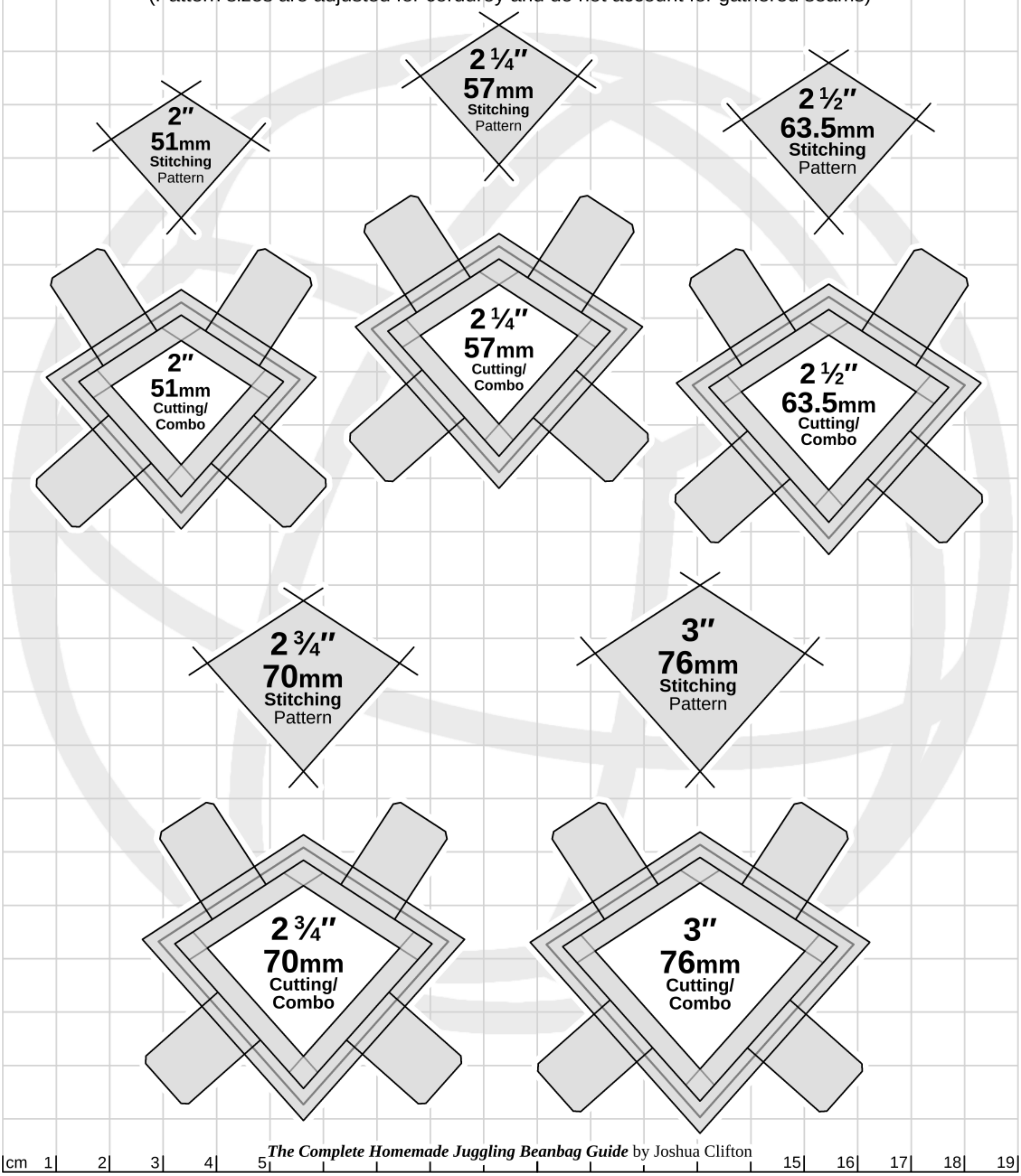
Target Diameter	Print this pattern size	At this scale
1¾″ (44.5mm)	2″	87.5%
1⅞″ (47.6mm)	2″	93.8%
2⅛″ (54.0mm)	2¼″	94.4%
2⅜″ (60.3mm)	2½″	95%
2⅝″ (66.7mm)	2¾″	95.4%
2⅞″ (73.0mm)	3″	95.8%

Deltoidal Icositetrahedron (24 Panels)

My *original kite design* (forms a modified polyhedron)

Kite angles: 113.943°, 81.421°, 83.215°

(Pattern sizes are adjusted for corduroy and do not account for gathered seams)



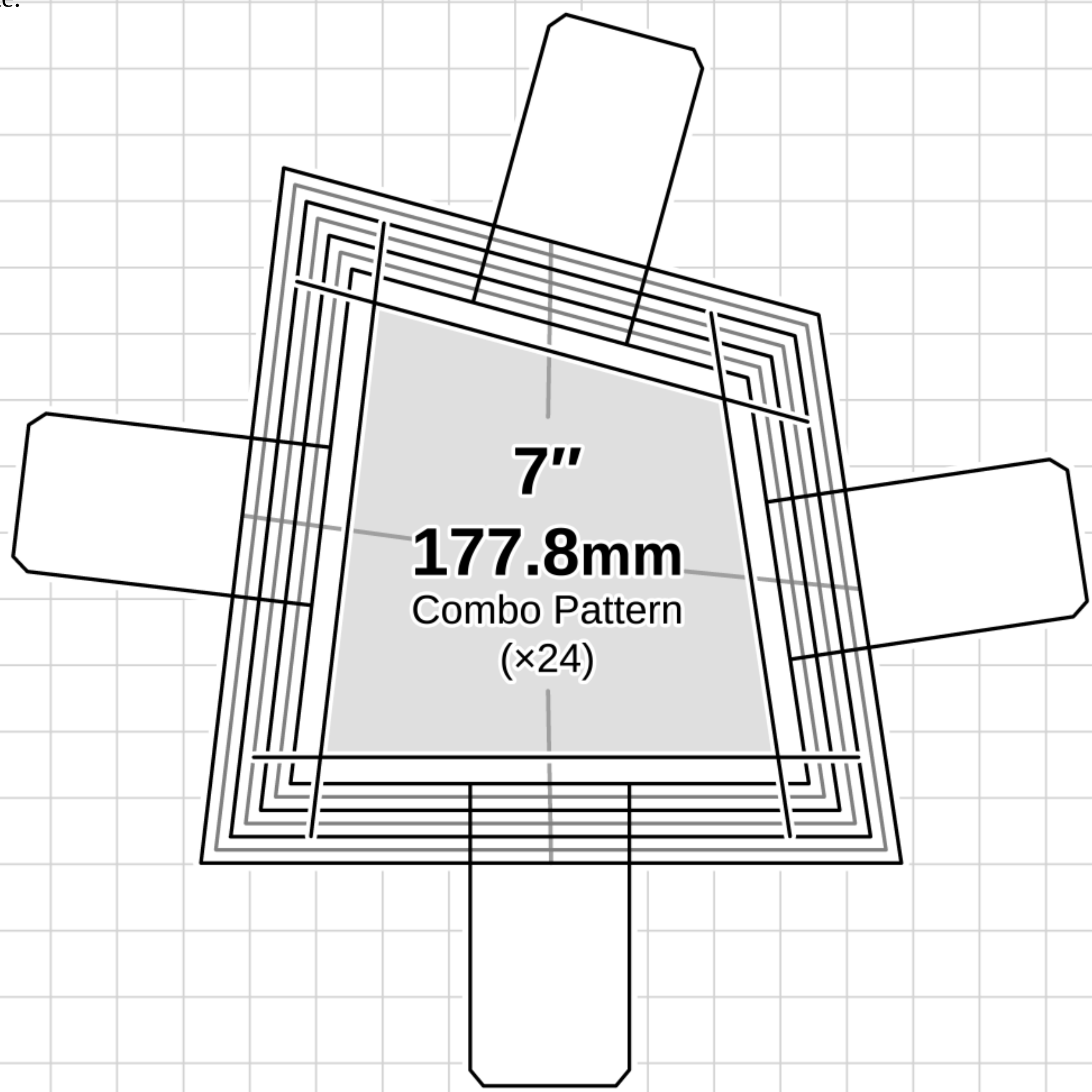
Deltoidal Icositetrahedron (24 Panels)

My *original kite design* (forms a modified polyhedron)

Kite angles: 113.943°, 81.421°, 83.215°

(Pattern sizes are adjusted for corduroy and do not account for gathered seams)

Extra large and versatile pattern for scaling to larger sizes in the Print Dialog. Print twice if you want both a stitching template and a cutting template (or cut out a combo template). The inner pattern (filled with gray) is the stitching pattern. Each dark pattern outside of that marks a 4mm seam allowance interval (at 100% scaling). Use those or the lighter, half-intervals between them to cut out the amount of allowance you want for the cutting template.



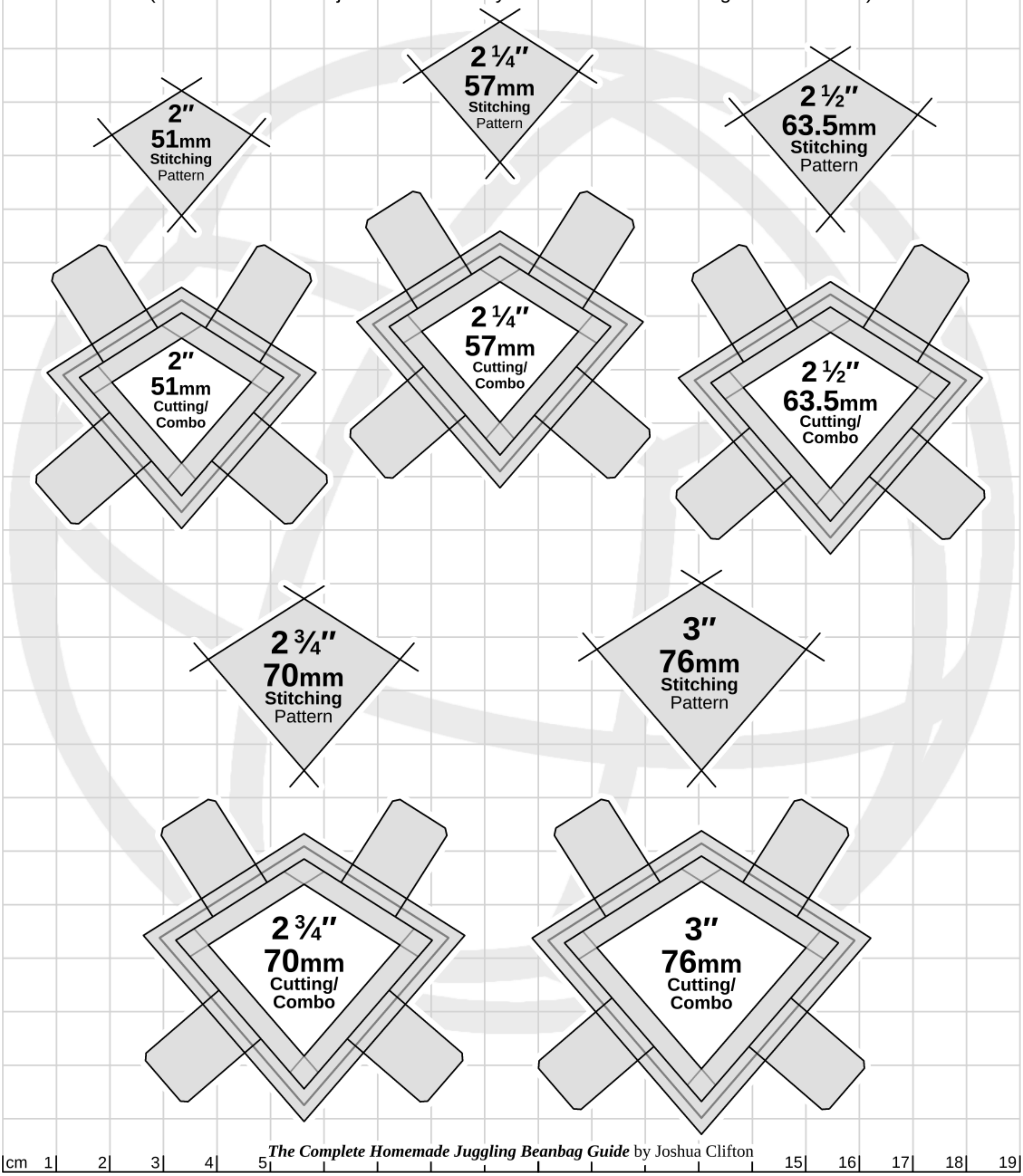
cm 1 2 3 4 5 15 16 17 18 19

Deltoidal Icositetrahedron (24 Panels)

***Normal kite** (forms the true deltoidal icositetrahedron)*

Kite angles: 115.263°, 81.579°

(Pattern sizes are adjusted for corduroy and do not account for gathered seams)



Deltoidal Icositetrahedron (24 Panels)

Normal kite *(forms the true deltoidal icositetrahedron)*

Kite angles: 115.263°, 81.579°

(Pattern sizes are adjusted for corduroy and do not account for gathered seams)

Extra large and versatile pattern for scaling to larger sizes in the Print Dialog. Print twice if you want both a stitching template and a cutting template (or cut out a combo template). The inner pattern (filled with gray) is the stitching pattern. Each dark pattern outside of that marks a 4mm seam allowance interval (at 100% scaling). Use those or the lighter, half-intervals between them to cut out the amount of allowance you want for the cutting template.